Chapter 9
Enhanced Mode Selection in Modal Domain Substructuring on the Round Robin Structure

Jure Korbar, Miha Pogačar, and Gregor Čepon

Abstract Dynamic substructuring (DS) techniques present an indispensable tool for estimating assembly dynamics based on the individual subcomponents' dynamics. The reliability of DS is heavily subject to the boundary conditions at the interface, the experimentalist's ability to obtain a sufficiently accurate representation of the interface, the domain where dynamic substructuring is performed, and the considerations regarding the weakening of the compatibility and equilibrium constraints. This paper addresses the Thin & Thick Wing challenge on the Round Robin benchmark structure with the objective of estimating the dynamics of the assembled Frame and Thick Wing. DS is performed in the modal domain, which is known to be highly sensitive to the selection of the participating modes in the substructuring methodology, as well as the weakening formulation. In this work, the modal-based substructuring is augmented by a novel methodology, addressing the problem of selecting appropriate modes for the substructuring procedure. This enhanced approach aims to improve the accuracy and robustness of modal-based substructuring, facilitating its utilization due to the reduced amount of guesswork within the existing approaches.

Keywords Dynamic substructuring · Modal substructuring · Round robin structure · Virtual point transformation · Mode selection

Introduction

Dynamic substructuring techniques allow to obtain the dynamic model of a structure by establishing relations between the individual subcomponents with known dynamic models in terms of compatibility and equilibrium constraints at the interface. Modal substructuring, also known as component-mode synthesis (CMS), refers to the subdomain of dynamic substructuring, where the substructures' dynamics are represented in the modal domain [1]. The accuracy of modal substructuring is known to be sensitive to the selection of modes used to describe the dynamics of the individual substructures. Due to the reduction of physical degrees of freedom (DoFs) to a smaller set of generalized coordinates, the established physical constraints generally cannot be satisfied exactly, as it often occurs that the total number of the available substructures' modes is lower than the number of constraints. In addition, for multi-point connections, some of the physical constraints may be redundant, resulting in interface stiffening and spurious eigenfrequencies [2]. Both issues are addressed by weakening the interface compatibility and equilibrium constraints. Weakening is performed by transforming the constraint equations using a suitable reduction basis, which may consist of vibration modes [3] or dominant singular vectors [4]. Regardless of the weakening procedure, the accuracy of modal substructuring remains subject to the modes considered in the modal reduction and weakening procedures. In this work, a framework for assessing whether the selected modes are appropriate for estimating coupled dynamics is proposed. The framework is based on neglecting one constraint at a time and performing modal substructuring considering the remaining constraints. By analyzing the stability of the calculated eigenfrequencies, the appropriateness of the selected modes for modal substructuring can be assessed. The proposed approach is demonstrated on the Round Robin benchmark structure with the objective of estimating the dynamics of the assembled Frame & Thick Wing by decoupling the Thin Wing from the assembled Frame & Thin Wing and coupling the Thick Wing. Two distinct representations of interface connection are considered, namely the equivalent multi-point connection, where the constraints are defined for the physical DoFs, and the connection via virtual points, where the constraints refer to the virtual point DoFs.

Jure Korbar · Miha Pogačar · Gregor Čepon
Faculty of Mechanical Engineering, University of Ljubljana, Aškerčeva cesta 6, 1000 Ljubljana
e-mail: jure.korbar@fs.uni-lj.si; miha.pogacar@fs.uni-lj.si; gregor.cepon@fs.uni-lj.si

Walter D'Ambrogio, et al. (eds.), *Dynamic Substructuring & Transfer Path Analysis, Vol. 4*, Conference Proceedings of the Society for Experimental Mechanics Series, https://doi.org/10.13052/97887-438-0149-8_9

Primal Formulation of Modal Substructuring

Dynamic substructuring in the modal domain allows to combine individual component models to estimate assembly dynamics or remove parts of the structure based on the individual substructures' modal parameters. The governing linear equation of motion for a substructure s in the physical domain can be written as:

$$\mathbf{M}^{(s)}\ddot{\boldsymbol{u}}^{(s)} + \mathbf{C}^{(s)}\dot{\boldsymbol{u}}^{(s)} + \mathbf{K}^{(s)}\boldsymbol{u}^{(s)} = \boldsymbol{f}^{(s)} + \boldsymbol{g}^{(s)}, \tag{1}$$

where $\mathbf{M}^{(s)}$, $\mathbf{C}^{(s)}$, and $\mathbf{K}^{(s)}$ are the mass, damping, and stiffness matrices, respectively, $\boldsymbol{u}^{(s)}$ is the displacement vector, $\boldsymbol{f}^{(s)}$ is the external force vector, and $\boldsymbol{g}^{(s)}$ is the connecting interface force vector. The velocity and acceleration vectors are denoted as the first and second time derivatives $\dot{\boldsymbol{u}}^{(s)}$ and $\ddot{\boldsymbol{u}}^{(s)}$ of displacements $\boldsymbol{u}^{(s)}$. The equations of motion for N individual substructures can be combined into a single uncoupled equation of motion:

$$\mathbf{M}\ddot{\boldsymbol{u}} + \mathbf{C}\dot{\boldsymbol{u}} + \mathbf{K}\boldsymbol{u} = \boldsymbol{f} + \boldsymbol{g}, \tag{2}$$

where $\mathbf{M} = \mathrm{diag}\left(\mathbf{M}^{(1)}, \dots, \mathbf{M}^{(N)}\right)$, $\mathbf{C} = \mathrm{diag}\left(\mathbf{C}^{(1)}, \dots, \mathbf{C}^{(N)}\right)$, $\mathbf{K} = \mathrm{diag}\left(\mathbf{K}^{(1)}, \dots, \mathbf{K}^{(N)}\right)$, $\boldsymbol{u} = \left\{\boldsymbol{u}^{(1)\top}, \dots, \boldsymbol{u}^{(N)\top}\right\}^\top$, $\boldsymbol{f} = \left\{\boldsymbol{f}^{(1)\top}, \dots, \boldsymbol{f}^{(N)\top}\right\}^\top$, and $\boldsymbol{g} = \left\{\boldsymbol{g}^{(1)\top}, \dots, \boldsymbol{g}^{(N)\top}\right\}^\top$. Coupling the individual dynamic models requires satisfying the constraints regarding the compatibility of interface displacements and equilibrium of interface forces:

$$\mathbf{B}\boldsymbol{u} = \mathbf{0}, \tag{3}$$

$$\mathbf{L}^\top\boldsymbol{g} = \mathbf{0}. \tag{4}$$

The physical DoFs are reduced to a smaller set of generalized DoFs $\boldsymbol{\eta}$ using a block-diagonal reduction matrix $\mathbf{R}_\mathrm{m}$:

$$\boldsymbol{u} = \mathbf{R}_\mathrm{m}\boldsymbol{\eta}, \tag{5}$$

where $\mathbf{R}_\mathrm{m} = \mathrm{diag}\left(\mathbf{R}_\mathrm{m}^{(1)}, \dots, \mathbf{R}_\mathrm{m}^{(N)}\right)$, $\boldsymbol{\eta} = \left\{\boldsymbol{\eta}^{(1)\top}, \dots, \boldsymbol{\eta}^{(N)\top}\right\}^\top$, and $\mathbf{R}_\mathrm{m}^{(s)}$ denotes the reduction matrix. In this study, a set of mass-normalized mode shapes $\mathbf{\Phi}^{(s)}$ is considered as a reduction matrix. Pre-multiplying Eq. (2) with $\mathbf{R}_\mathrm{m}^\top$ and considering the reduction in Eq. (5) yields the following equation of motion and corresponding compatibility and equilibrium constraints:

$$\mathbf{M}_\mathrm{m}\ddot{\boldsymbol{\eta}} + \mathbf{C}_\mathrm{m}\dot{\boldsymbol{\eta}} + \mathbf{K}_\mathrm{m}\boldsymbol{\eta} = \boldsymbol{f}_\mathrm{m} + \boldsymbol{g}_\mathrm{m}, \tag{6}$$

$$\mathbf{B}_\mathrm{m}\boldsymbol{\eta} = \mathbf{0}, \tag{7}$$

$$\mathbf{L}_\mathrm{m}^\top\boldsymbol{g}_\mathrm{m} = \mathbf{0}, \tag{8}$$

where $\mathbf{M}_\mathrm{m} = \mathbf{R}_\mathrm{m}^\top\mathbf{M}\mathbf{R}_\mathrm{m}$, $\mathbf{C}_\mathrm{m} = \mathbf{R}_\mathrm{m}^\top\mathbf{C}\mathbf{R}_\mathrm{m}$, $\mathbf{K}_\mathrm{m} = \mathbf{R}_\mathrm{m}^\top\mathbf{K}\mathbf{R}_\mathrm{m}$, $\mathbf{B}_\mathrm{m} = \mathbf{B}\mathbf{R}_\mathrm{m}$, $\mathbf{L}_\mathrm{m} = \mathrm{null}\left(\mathbf{B}_\mathrm{m}\right)$, $\boldsymbol{f}_\mathrm{m} = \mathbf{R}_\mathrm{m}^\top\boldsymbol{f}$, and $\boldsymbol{g}_\mathrm{m} = \mathbf{R}_\mathrm{m}^\top\boldsymbol{g}$. Eq. (7) enforces exact/strong compatibility of physical DoFs in the reduced DoF space. If the number of constraints exceeds the combined number of the individual substructures' modes, the set compatibility equations is overdetermined and generally cannot be satisfied exactly. In addition, the number of modes which can be calculated for the coupled structure decreases with the increasing number of constraints and decreasing number of modes of the individual substructures. Therefore, it can be reasonable to reduce the number of constraints by weakening the compatibility conditions, i. e. approximately satisfying Eq. (7) by pre-multiplication with a weakening matrix $\mathbf{W}$:

$$\widetilde{\mathbf{B}}_\mathrm{m}\boldsymbol{\eta} = \mathbf{0}, \tag{9}$$

$$\widetilde{\mathbf{L}}_\mathrm{m}\boldsymbol{g}_\mathrm{m} = \mathbf{0}, \tag{10}$$

where $\widetilde{\mathbf{B}}_\mathrm{m} = \mathbf{W}\mathbf{B}_\mathrm{m}$ and $\widetilde{\mathbf{L}}_\mathrm{m} = \mathrm{null}\left(\mathbf{W}\mathbf{B}_\mathrm{m}\right)^\top$. Substructuring in the primal formulation is performed by expressing the dual set of generalized coordinates $\boldsymbol{\eta}$ using a unique set of generalized coordinates $\boldsymbol{\xi}$:

$$\boldsymbol{\eta} = \widetilde{\mathbf{L}}_\mathrm{m}\boldsymbol{\xi}, \tag{11}$$

therefore, the coupled equation of motion is written as:

$$\widetilde{\mathbf{M}}_\mathrm{m}\ddot{\boldsymbol{\xi}} + \widetilde{\mathbf{C}}_\mathrm{m}\dot{\boldsymbol{\xi}} + \widetilde{\mathbf{K}}_\mathrm{m}\boldsymbol{\xi} = \widetilde{\boldsymbol{f}}_\mathrm{m}, \tag{12}$$

where $\widetilde{\mathbf{M}}_\mathrm{m} = \widetilde{\mathbf{L}}_\mathrm{m}^\top\mathbf{M}_\mathrm{m}\widetilde{\mathbf{L}}_\mathrm{m}$, $\widetilde{\mathbf{C}}_\mathrm{m} = \widetilde{\mathbf{L}}_\mathrm{m}^\top\mathbf{C}_\mathrm{m}\widetilde{\mathbf{L}}_\mathrm{m}$, $\widetilde{\mathbf{K}}_\mathrm{m} = \widetilde{\mathbf{L}}_\mathrm{m}^\top\mathbf{K}_\mathrm{m}\widetilde{\mathbf{L}}_\mathrm{m}$, and $\widetilde{\boldsymbol{f}}_\mathrm{m} = \widetilde{\mathbf{L}}_\mathrm{m}^\top\boldsymbol{f}_\mathrm{m}$.

Modal Domain Virtual Point Transformation

Performing experimental dynamic substructuring requires collocated response measurements on the adjacent sides of the interface, which proves to be challenging. Furthermore, coupling rotational DoFs is subject to one's ability to measure the rotational DoFs. By coupling multiple closely spaced translational DoFs to implicitly account for rotational DoFs, one may encounter over-stiffening of interface. The virtual point transformation (VPT) addresses all of the above issues by performing a geometric transformation of the measured DoFs into a single virtual point (VP) [5]. The VPT assumes that the local motion of the interface can be described using a set of interface deformation modes (IDMs). This allows to couple experimental models with collocated DoFs and directly couple rotational DoFs.[1]

The VPT is typically applied when performing frequency-based substructuring, however, there are no assumptions regarding modal substructuring that would prohibit applying the VPT in the modal domain. Furthermore, modal-domain VPT (M-VPT) can introduce the advantages of the classical VPT to modal substructuring. Denoting by $\boldsymbol{u}_b$ the subset of displacements pertaining to the measured interface DoFs, which are to be transformed to the VP DoFs $\boldsymbol{q}_b$, the relation between the measured and VP DoFs can be written as:

$$\boldsymbol{u}_b = \mathbf{R}_b \boldsymbol{q}_b, \tag{13}$$

where $\mathbf{R}_b$ is the IDM matrix transforming the VP DoFs to the measured DoFs. Eq. (13) is typically overdetermined (the number of measured DoFs exceeds the number of IDMs) to mitigate measurement errors. The objective of VPT is to transform measured DoFs to VP DoFs, therefore, the VP DoFs $\boldsymbol{q}$ are calculated as a least-squares solution of Eq. (13):

$$\boldsymbol{q}_b = \mathbf{T}_b \boldsymbol{u}_b, \tag{14}$$

where $\mathbf{T}_b = \mathbf{R}_b^+$, with $\star^+$ denoting the Moore-Penrose inverse of $\star$. Since the interface DoFs are being replaced with VP DoFs, the reduction of physical DoFs to generalized DoFs is now modified at interface:

$$\boldsymbol{q}_b = \mathbf{T}_b \boldsymbol{u}_b = \mathbf{T}_b \mathbf{R}_{\mathrm{m},b} \boldsymbol{\eta}, \tag{15}$$

where $\mathbf{R}_{\mathrm{m},b}$ denotes the section of the reduction matrix $\mathbf{R}_{\mathrm{m}}$ pertaining to the interface DoFs.

Mode Selection Framework

Successfully performing modal substructuring is greatly influenced by the selection of the individual substructures' modes involved in the reduction to generalized coordinates and the weakening formulation. Without prior knowledge of the coupled structure's modal parameters, it is difficult to know whether a certain combination of selected modes is appropriate. This work proposes a framework for assessing whether a particular subset of substructures' modes is appropriate for modal substructuring.

A prerequisite for applying the described framework is that the number of physical interface DoFs is larger than the minimum number of interface DoFs required to sufficiently couple the substructures. This can be achieved by selecting enough measurement locations at the interface to achieve some level of redundancy. A six step framework for assessing the validity of the selected modes for modal substructuring is described in the following:

1. Select a subset of modes for each substructure to perform DoF reduction.
2. Establish compatibility equations for interface displacements.
3. Remove compatibility equations pertaining to a single DoF at the interface.[2]
4. Perform modal substructuring by considering the remaining compatibility equations.
5. Obtain a set of eigenfrequencies for all possible combinations of compatibility equations with one missing DoF by repeating steps 2–4 for each interface DoF.
6. Assess the stability of the set of all eigenfrequencies calculated in step 5.

If the calculated eigenfrequencies do not significantly deviate within the frequency range of interest, the selected modes can be considered as a valid subset of modes for modal substructuring.

[1] The VPT typically assumes the motion of the interface can be described using rigid IDMs, therefore, the measured interface DoFs are transformed into a set of three translational and three rotational DoFs. However, the set of IDMs can be extended to include flexible interface motion [6].

[2] When performing substructuring using a transmission simulator, there are two coupling equations per interface DoF; one for decoupling the transmission simulator and one for coupling the substructure missing from the assembly.

Round Robin Structure Case Study

A case study of the proposed framework was performed on the Round Robin structure, addressing the Thin & Thick Wing challenge, where the objective is to estimate the dynamics of the Frame & Thick Wing assembly by decoupling the Thin Wing from the Frame & Thin Wing assembly and coupling the Thick Wing. For brevity, the Frame, Thin Wing and Thick Wing substructures are denoted as A, TS, and B, respectively. Frame & Thin Wing and Frame & Thick Wing assemblies are respectively denoted as ATS and AB. The substructuring procedure can be conceptually outlined in the equation form: $\mathrm{ATS} - \mathrm{TS} + \mathrm{B} = \mathrm{AB}$.

The substructures involved in this case study are shown in Fig. 1, along with the sensor placement. Both Thin and Thick Wing are mounted to the Frame at four connection points. Three triaxial accelerometers are placed in the proximity of each connection point to facilitate the application of M-VPT. A VP is located at each connection point for a total of four VPs, depicted as green spheres in Fig. 1, with six rigid IDMs per VP. The four triaxial accelerometers in the corners of TS and B were also included in the substructuring procedure, therefore, the extended interface formulation was considered.

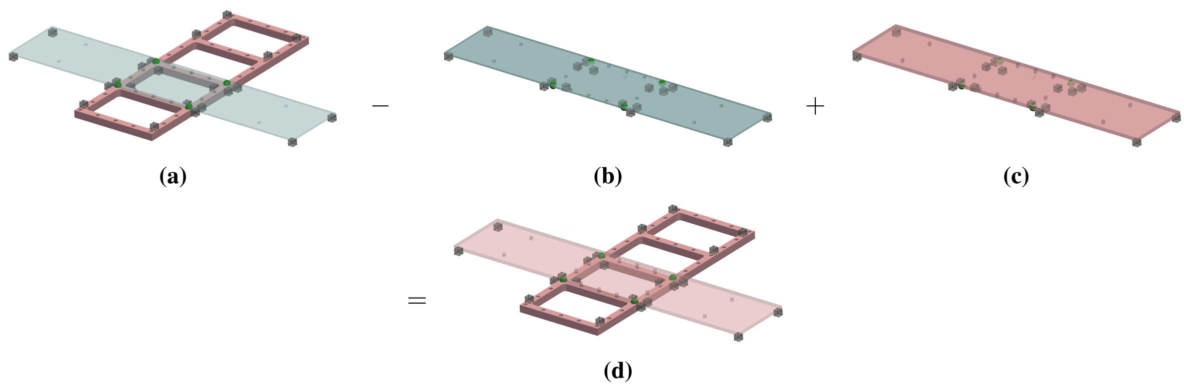

Fig. 1 Substructures used in the Thin & Thick Wing challenge: (a) ATS, (b) TS, (c) B, (d) AB.

To clarify the substructuring procedure, the matrices $\mathbf{W}$, $\mathbf{B}_\mathrm{m}$, and generalized coordinates $\boldsymbol{\eta}$ are written in a block form as:

$$\mathbf{W} = \begin{bmatrix} \boldsymbol{\Phi}_b^{(\mathrm{TS})+} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Phi}_b^{(\mathrm{TS})+} \end{bmatrix}, \quad \mathbf{B}_\mathrm{m} = \begin{bmatrix} \boldsymbol{\Phi}_b^{(\mathrm{ATS})} & -\boldsymbol{\Phi}_b^{(\mathrm{TS})} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Phi}_b^{(\mathrm{TS})} & -\boldsymbol{\Phi}_b^{(\mathrm{B})} \end{bmatrix}, \quad \boldsymbol{\eta} = \begin{Bmatrix} \boldsymbol{\eta}^{(\mathrm{ATS})} \\ \boldsymbol{\eta}^{(\mathrm{TS})} \\ \boldsymbol{\eta}^{(\mathrm{B})} \end{Bmatrix}. \tag{16}$$

The proposed mode selection framework is performed by removing a single row in $\boldsymbol{\Phi}_b^{(\mathrm{TS})}$ and the corresponding rows in $\boldsymbol{\Phi}_b^{(\mathrm{ATS})}$ and $\boldsymbol{\Phi}_b^{(\mathrm{B})}$. The number of modes taken into consideration for modal substructuring for substructures ATS, TS, and B, are denoted as $M^{(\mathrm{ATS})}$, $M^{(\mathrm{TS})}$, and $M^{(\mathrm{B})}$, respectively . The results of the mode selection framework are shown in Fig. 2, where two distinct subsets of modes S_1 and S_2 are tested, which are defined as follows:

$$S_1 = \begin{cases} M^{(\mathrm{ATS})} = 25 \\ M^{(\mathrm{TS})} = 22 \\ M^{(\mathrm{B})} = 22 \end{cases} \quad \text{and} \quad S_2 = \begin{cases} M^{(\mathrm{ATS})} = 30 \\ M^{(\mathrm{TS})} = 18 \\ M^{(\mathrm{B})} = 18 \end{cases}. \tag{17}$$

Modal substructuring is performed on each subset of modes with two distinct interface representations, namely the equivalent multi-point connection (EMPC), where the constraints are defined directly on the physical set of coordinates, as well as the M-VPT representation, where the constraints refer to the VP DoFs. It can be seen that the more stable eigenfrequency results are generally in line with the reference eigenfrequencies, which are not known in practice. The stability of the eigenfrequencies can therefore be used as a guide whether the subset of modes used within modal substructuring is appropriate to estimate the assembly dynamics in the frequency range of interest.

The results also allow to compare the EMPC and the M-VPT approaches for describing the interface DoFs. One significant advantage of M-VPT over the EMPC is the improved accuracy of rigid-body dynamics. All substructures in this study have free boundary conditions, therefore, each substructure has six rigid-body vibration modes. The M-VPT approach accurately predicts six rigid-body modes for the coupled structure, while the EMPC approach typically predicts only four eigenfrequencies at 0 Hz. The M-VPT also generally outperforms the EMPC in terms of more accurate estimation of the coupled eigenfrequencies.

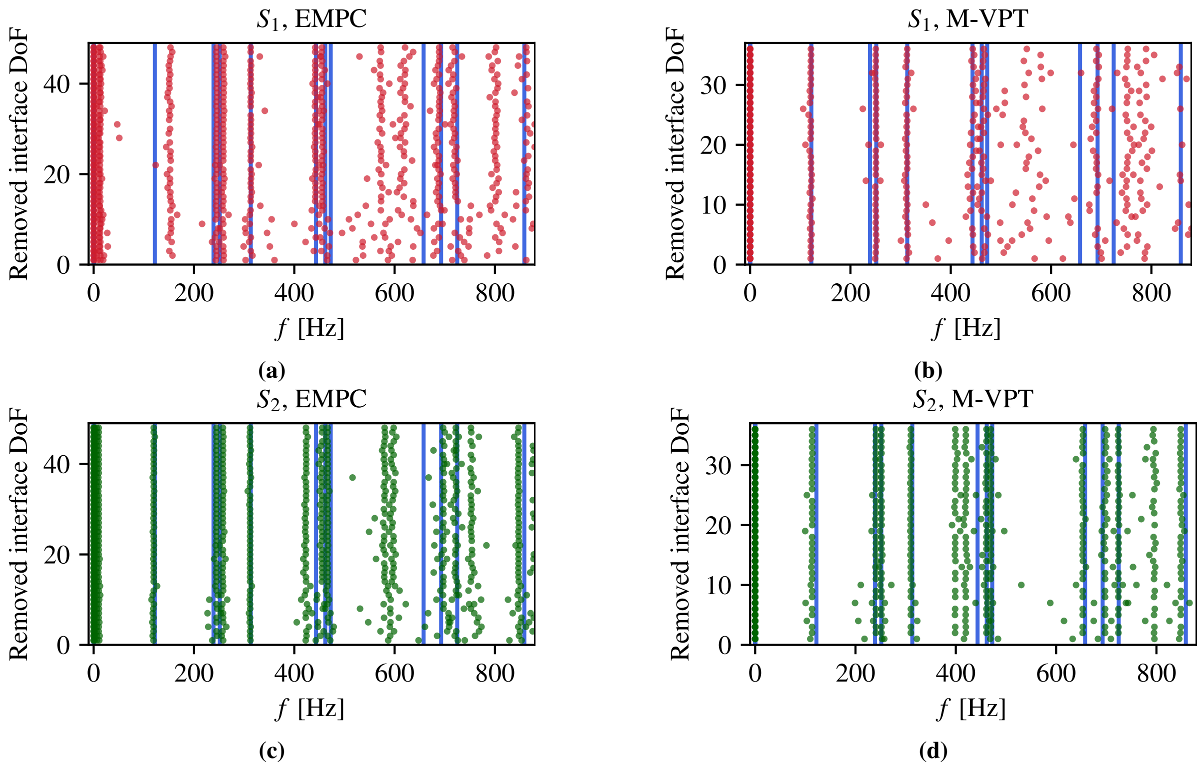

Fig. 2 Application of the mode selection framework: (a) mode subset S_1 with EMPC , (b) mode subset S_1 with VPT, (c) mode subset S_2 with EMPC, (d) mode subset S_2 with VPT.

Conclusion

This work proposes a framework for assessing whether the selected subset of substructures' modes is appropriate for performing modal substructuring. By removing a single interface DoF, the stability of the coupled eigenfrequencies can be assessed, with higher stability indicating that the subset of modes considered for substructuring is appropriate.

References

1. De Klerk, D., Rixen, D.J., and Voormeeren, S.N. "General framework for dynamic substructuring: history, review and classification of techniques". *AIAA journal*, 46(5):1169–1181 (2008).
2. Rixen, D.J. "How measurement inaccuracies induce spurious peaks in frequency based substructuring". In *Proceedings of the Twenty Sixth International Modal Analysis Conference, Orlando, FL. Society for Experimental Mechanics, Bethel, CT* (2008).
3. Allen, M.S., Mayes, R.L., and Bergman, E.J. "Experimental modal substructuring to couple and uncouple substructures with flexible fixtures and multi-point connections". *Journal of Sound and Vibration*, 329(23):4891–4906 (2010).
4. Pogačar, M., Čepon, G., and Boltežar, M. "Weakening of the multi-point constraints in modal substructuring using singular value decomposition". *Mechanical Systems and Signal Processing*, 163:108109 (2022).
5. van der Seijs, M., van den Bosch, D., Rixen, D.J., and de Klerk, D. "An improved methodology for the virtual point transformation of measured frequency response functions in dynamic substructuring". *COMPDYN* (2013).
6. Pasma, E.A., van der Seijs, M., Klaassen, S.W.B., and van der Kooij, M.W. "Frequency based substructuring with the virtual point transformation, flexible interface modes and a transmission simulator". In *Dynamics of Coupled Structures, Volume 4*, pages 205–213. Springer (2018).