Chapter 8
Investigating the Propulsive Efficiency of Bio-Inspired Fish-like Elastic Caudal Fin through Dynamic Analysis and Experimental Validation

E. Paifelman, S. Milana, A. Culla, G. Costantino Muniz, F. Passacantilli, and E. Ciappi

Abstract The present work focuses on the dynamic analysis of bio-inspired fish-like robotic fins in order to evaluate their propulsive efficiency. This study relates the effect of the main factors on which this efficiency depends, particularly materials and stiffness in a typical biomimetic range. Numerical and experimental modal analysis techniques are employed to examine the vibrational properties of different fins. This analysis is essential to characterize the elastic materials and understand how the respective stiffness levels affect the dynamic behavior of the fin. A numerical analysis is then conducted using an analytical model that simulates the coupled dynamics between the fluid and the elastic fin. This model takes into account the two-ways interaction between the hydrodynamic forces and elastic deformations of the fin, allowing a detailed evaluation of propulsive efficiency parameters. Key performance indicators, such as elastic deformation, thrust, and efficiency, are calculated and analyzed in relation to changes in materials and stiffness. Hydrodynamic tank experiments are ongoing to directly measure the performance of the robotic fins, comparing the experimental data with the predictions of the analytical model. This validation phase is critical to confirm the accuracy of the model and to further refine the predictions regarding propulsive efficiency. The results of this paper provide valuable insights for the future design of underwater robots, with potential applications in areas such as marine exploration, environmental monitoring, and bioinspired robotics.

Keywords Fish-like · Bionspired robot · FSI model · Modal analysis · Biomimetic propulsion

Introduction

The use of biomimetic thrusters in the marine environment offers significant advantages for marine transportation, particularly through reduced noise compared to conventional thrusters and the ability to provide a wide range of propulsive efficiency. Although the applicability of such systems is currently limited to small marine vehicles, such as AUVs (Autonomous Underwater Vehicles) or ROVs (Remotely Operated Vehicles), interest in their performance is growing. Numerous studies in the literature point out that these thrusters are often designed on a small scale, with dimensions on the order of a centimeter, mainly for educational purposes or laboratory testing [1, 2]. The objective of this study is to overcome these limitations by proposing a systematic analysis on elastic propulsion system intended for larger robotic prototypes, with dimensions of the order of a meter. This allows them to cope with more complex operating conditions, such as higher cruising speeds and greater operational depths, than the micro-robots currently in use. Specifically, the paper presents an analysis of the propulsive behavior of different types of elastic thrusters designed to move an AUV with fish-inspired morphology. The analysis is based both on numerical results and on a preliminary experimental campaign, with the aim of identifying the most suitable materials and the relative frequencies peculiar to the systems under consideration.

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Mathematical Evaluation of Bio-Propulsion Efficiency

The proposed two-way model represents the cross section of the oscillatory rectangular caudal fin. This involves coupling the one-dimensional Euler-Bernoulli plate equation with potential flow theory. The hydroelastic treatment is developed entirely in the time domain, to be interfaced with control logics [3, 4], and from this has been derived an integro-differential final equation of motion with the presence of convolutional memory effects terms [5]. The model is developed and validated in [6, 7], where all coupling terms (mass, stiffness, added damping and memory effects) are developed in closed form through complex analysis, by making the model semi-analytical [8]. The coupled equation of motion is here formulated in nondimensional form, choosing the half-length $L/2$ and the flapping period $1/f_\alpha$ as the characteristic length and time scales, respectively so, the equation of motion for the fin displacement w is

$$2R\frac{\partial^2 w}{\partial t^2} + S_d \frac{8\pi^2}{3\sigma^2}\frac{\partial^2 w}{\partial x^2} = \tilde{F}_f(x,t) \tag{1}$$

where the dimensionless hydrodynamic load is given by $\tilde{F}_f(x,t) = \frac{4F_f(x,t)}{\rho d L^2 f_\alpha^2}$ with ρ the fluid density and d the fin width, the reduced frequency $\sigma = \frac{\pi L f_\alpha}{U}$ with U the inflow speed, the stiffness $S_d = \frac{E h_b^3}{\rho U^2 L^3}$ with E the Young modulus and h_b the fin thickness and the inertia ratio $R = \frac{\rho_b h_b}{\rho L}$ with ρ_b the material density. Using the modal approach $w(x,t) = \sum_{i=1}^{N} \phi_i(x) q_i(t)$ where N is the modes number and $\phi_i(x)$ is the fixed-free beam eigenfunction, equation (1) can be decoupled and solved by imposing the initial conditions $q_i(t=0) = q_{i0}$ (see [6] for further details). Once the solution $w(x,t)$ is found from equations (1), one can compute the propulsive parameters fundamental to evaluate the efficiency of the system, such as: the thrust force $T(t) = -d\int_{-L/2}^{L/2} \Delta P(x,t)[\alpha(t) + w'(x,t)]dx$, the input power expended $P(t) = d\int_{-L/2}^{L/2} \Delta P(x,t)[\dot{h}(t) + (L/2 + x)\dot{\alpha}(t) + \dot{w}(x,t)]dx$ and so the Froude efficiency $\eta = \frac{\overline{T}U}{\overline{P}}$, where the overline represents the mean value. Another important parameter is the peak-to-peak amplitude of the flexible panels' trailing edge $\chi = 2\max\left[h(t) + L\alpha(t) + \frac{w(L/2,t)}{1-\alpha(t)^2/2}\right]$, which is an output of the system that depends on the balance between inertial, elastic and external fluid forces. Through this, it is then possible to calculate the Strouhal number, $St = f\chi/U$, used to evaluate the optimality of the biomimetic propulsion. The global optimal efficiency for the two-dimensional elastic plate then occurs when two conditions are simultaneously satisfied: (i) the oscillation frequency of the panel produces an amplitude of motion that yields a Strouhal number in the optimal range $0.25 < St < 0.35$ and (ii) this frequency of motion is tuned to the structural resonant frequency of the panel. The propulsive thrust parameter in dimensionless form became $C_T = \frac{\bar{T}}{\frac{1}{2}\rho U^2 dL}$ and will be reported in the design maps as function of the inertia and stiffness dimensionless parameters, S_d and R for a specific operating condition (i.e. U, f_α, A_α).

Experimental Setup

An experimental set-up has been designed to measure the propulsive force exerted by the fins, (Figures 1-2). It features a closed-loop circulation channel with a width of 536 mm and a water free surface height of 650 mm. A support structure is installed to house the actuators, transmission, and propeller, isolating the system from channel vibrations that could interfere with the experiment and data extraction. The pumps control the free stream velocity up to 1 m/s. The actuation system consists of a brushless SmartMotor (1.45 Nm continuous torque, 3.39 Nm peak, encoder resolution 8.000 counts/rev, support for DeviceNet, PROFIBUS, and CANopen communication protocols) connected to a planetary gearbox and a 6-axis ME load cell for measuring propeller performance (50 N, 5 Nm). An elastic joint minimizes misalignment effects between the gearbox and load cell. The drive shaft connects to the channel's bottom via a flanged bushing, with fins secured by hexagon socket set screws to prevent damage and ensure proper placement.

Characterization of Propulsion System's Material Properties

After preliminary numerical simulations, the optimal geometric configuration for the propulsion of a fish-like AUV with a length of about 1.5 m, designed to operate in a cruising speed range from 0.1 to 0.9 m/s, is selected. The propulsion system, represented by the fin, has a rectangular geometry, as shown in (Figure 3 - a), with dimensions equal to $L = 510\,\text{mm}$, $d = 150\,\text{mm}$, $h_b = 1, 2, 3\,\text{mm}$. The materials chosen for the fin include Aluminium (AL), for the metallic configuration, and plastic materials as HDPE and PMMA, selected to cover a range of stiffness in line with that of biomimetic systems, i.e. $R << 1$ and $10^{-3} < S_d < 10^3$. Figure 3 - b) shows an example of real fin (AL 3 mm).

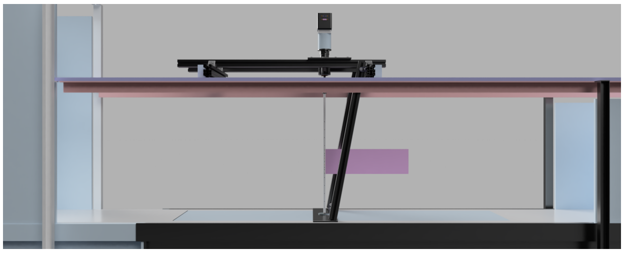


Fig. 1 Experimental setup in hydraulic channel

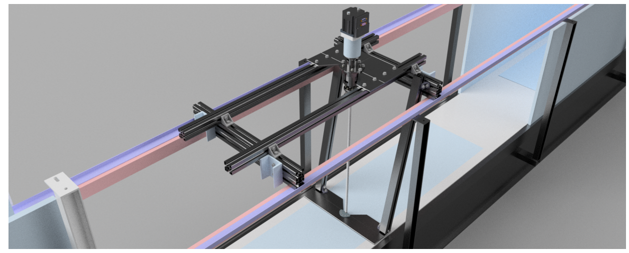


Fig. 2 Rotational handling system

a) *b*)

Fig. 3 Rectangular fin configurations a) rendering and dimension b) real fin (AL 3 mm)

To identify the mechanical properties of fin materials, a preliminary density measurement was made by weighing each fin, the results of which are shown in Table 1. Next, an experimental analysis was conducted to determine the first natural frequency under dry conditions for each fin with a thickness of 3 mm in free-free condition. The obtained experimental frequencies were compared with those computed by modal analysis conducted by the finite element method (FEM) using Ansys software. The results showed excellent agreement between the experimental and simulated frequencies, with errors of less than 2 % (Table 2).

Table 1 Material mechanical properties of 3 mm fins

Material	ρ_s $[kg/m^3]$	ν	E $[GPa]$
AL	2633.62	0.33	65.89
PMMA	1191.60	0.42	2.35
HDPE	933.42	0.42	1.96

Knowing all the geometric characteristics, it was possible to calculate the Young's modulus for each material analyzed, using the appropriate relationship $E = \frac{\rho_b A_b}{I}\left(\frac{2 f_1 \pi L^2}{\lambda_1^2}\right)^2$, with the inertia modulus $I = \frac{d h_b^3}{12}$ and λ_1 the first free-free beam wavenumber. The results are shown in Table 1 where the standard value for the Poisson's coefficient ν was considered.

Experimental Identification of Dry and WET Fin's Frequency

The experimental setup involves clamping a plate inside a rod. The rod mimics an elastic support rather than a perfect clamp, decreasing the constraint stiffness and fin's natural frequencies and so necessitating a realistic modeling approach. To accurately determine the elasticity of real constraint, an experimental modal analysis was conducted to identify natural frequencies of the fin in air, using an accelerometer to measure the response of the structure when it is struck at specific grid points. These results are shown in Table 3 first column. Then, the fins were modeled with Ansys software, applying a 1-cm elastic support to the clamp side, with a shell element discretization of 5 mm. The tuning of the modeled stiffness

Table 2 First natural frequency in free-free condition of 3 mm fins: Experimental vs FEM

Material	**AL**	**PMMA**	**HDPE**
Experimental results $[Hz]$	59.3	22.2	17.2
Numerical results $[Hz]$	60.101	22.36	17.2
Percentage error $[\%]$	-1.344	-0.716	0.397

in the FEM was then done until good agreement was obtained in the comparison between the numerical FEM analysis and the experimental one. From this comparison the elastic constraint identified was a distributed stiffness of 1010 N/m^3, which provides good agreement between the experimental and numerical frequencies, all reported in Table 3.

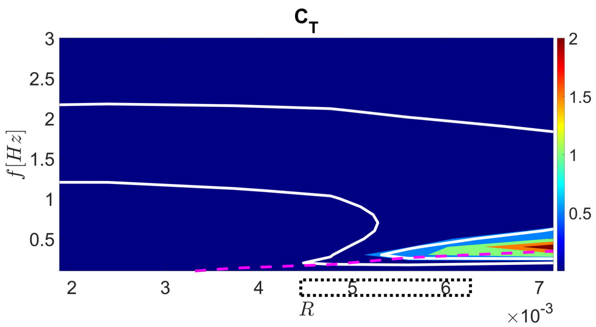
Table 3 Dry and wet fins natural frequencies in clamped configuration

	Dry 1^{st} **Frequency** $[Hz]$		**Wet** 1^{st} **Frequency** $[Hz]$
Plate Thickness	**Exp**	**FEM**	**FEM**
HDPE			
1 mm	-	0.96	0.134
2 mm	1.65	1.90	0.357
3 mm	2.46	2.80	0.576
PMMA			
1 mm	1.12	1.10	0.175
2 mm	2.07	2.16	0.469
3 mm	2.85	3.15	0.766
AL			
1 mm	3.16	3.17	0.749
2 mm	5.32	5.40	1.768
3 mm	6.14	6.14	2.33

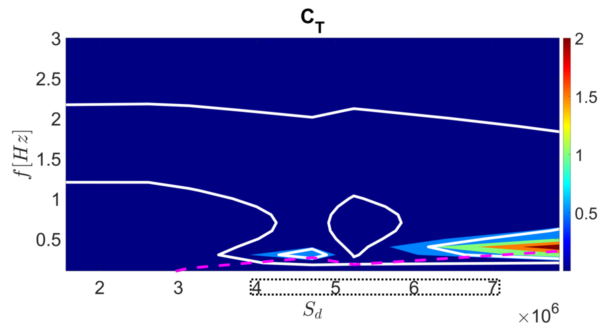
To get the desired wet frequencies, fundamental parameter to evaluate the propulsion efficiency, a FEM analysis was carried on in Nastran software adding both the elastic support and the water body around the plate. The result can be seen in Table 3 whose values will be compared in the future with those measured experimentally.

Numerical and Experimental Results

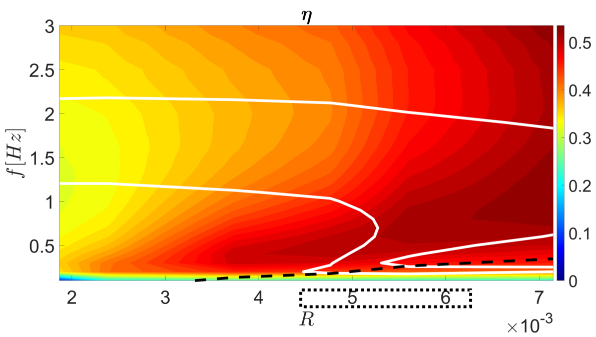
After having characterized the experimental system, the dynamic simulations of all fins under analysis were conducted when the flow velocity is $U = 0.1m/s$, the maximum amplitude of the imposed pitching motion is $A_\alpha = 5°$, in a range of oscillation frequency under 3 Hz chosen so as to ensure the visualization of the first wet natural frequency. Figures 4a and 4b show, for the analyzed operating conditions, the maps of the propulsive thrust parameter, as a function of frequency and of the inertia and stiffness dimensionless parameters, S_d and R , respectively. Figures 4c and 4d show the corresponding maps for efficiency. On each map the conditions for which an optimal Strohual number is obtained (solid white lines) and those for which the first resonance of the system is reached (dashed lines) are shown. This way of reporting numerical simulations allows one to quickly visualize the geometric conditions (combinations of S_d and R) such that optimal biomimetic propulsion is achieved, i.e when the resonance condition met the optimal Strohual condition (intersection of two lines). The maps in Figure 4 show the results for the plastic configurations, while those in Figure 5 of the metallic one. Under the same operating conditions, it can be seen that the rigid configurations will never guarantee propulsion optimality and for equal pitching condition the elastic configurations are still characterized by higher propulsive efficiency. The optimal range of S_d and R identified corresponds to the configurations PMMA 2 mm and HDPE 3 mm. The differences between the two optimal configurations are minimal: the 2 mm PMMA corresponds to $\eta = 0.44$ and a $C_T = 0.22$, while the 3 mm HDPE $\eta = 0.42$ and $C_T = 0.27$. Finally, 3 mm HDPE was considered as the best configuration because even with just a slightly lower yield it provides a larger C_T thrust gain. The experimental campaign was then started by analyzing this configuration at the experimentally identified wet resonance $f_\alpha = 0.2Hz$. It is important to underline that the experimental tests are preliminary and will require more attention on the individual setup subsystems, the error committed and the effect of the free surface.



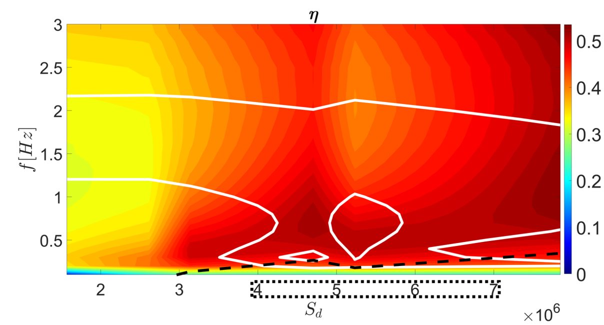
(a) Thrust dimensionless coefficient variation in inertial range



(b) Thrust dimensionless coefficient variation in stiffness range

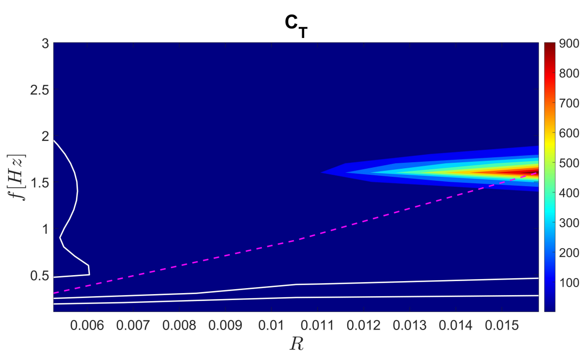


(c) Efficiency variation in inertial range

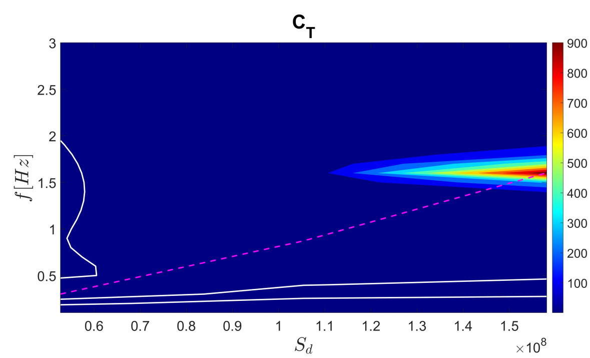


(d) Efficiency variation in stiffness range

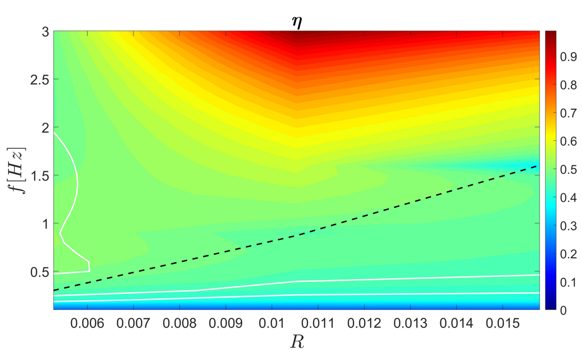
Fig. 4 Efficiency variation in stiffness range



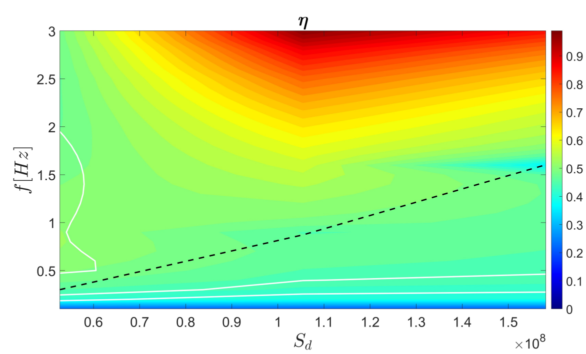
(a) Thrust dimensionless coefficient variation in inertial range



(b) Thrust dimensionless coefficient variation in stiffness range



(c) Efficiency variation in inertial range



(d) Efficiency variation in stiffness range

Fig. 5 Rigid fins: propulsive map in 0-3 Hz range

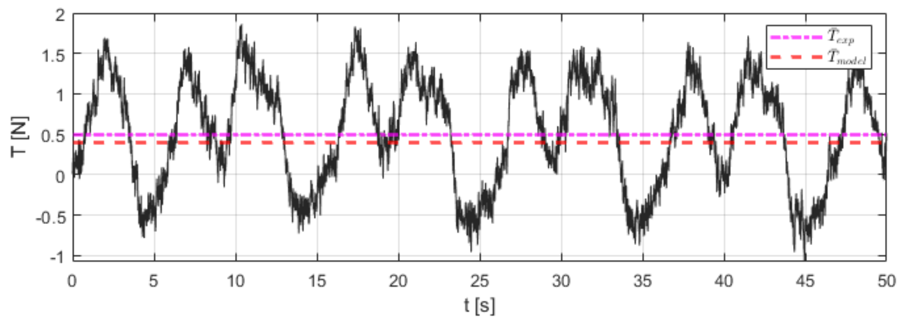


Fig. 6 Experimental evaluation of Thrust in time for the best configuration

From the measured thrust trend (Figure 6) an average value of approximately $\bar{T} \approx 0.5N$ (pink line) is estimated with a relative $C_T = 1.18$. Under the same conditions, the mathematical model identifies the first wet frequency at about $0.3Hz$. This discrepancy caused by the limitations of the developed approximate model, however, ensures good agreement in terms of qualitative comparison for propulsive force since $\bar{T} \approx 0.4N$ (red line) giving $C_T = 1.06$. The result shows a good starting point for the experimental results and a necessary correction of the model in use in order to estimate appropriate values for the coupling terms (added mass, hydrodynamic damping) that are critical for the wet frequency estimation.

Final Remarks

This article presents an analysis of the propulsive efficiency of elastic fins used as innovative propulsion systems for large fish-like robots. A semi-analytical coupled model, previously presented and validated by the authors, is utilized as a design tool to help choose the best fin configuration based on the conditions in which the robot will operate. A preliminary experimental campaign was conducted to identify the elastic materials properties, their natural frequencies through modal analysis, and frequency analysis to characterize the various subsystems involved in the experimental setup. This was designed in a hydraulic channel to simulate the oscillation of the fin in water. These experimental tests, although preliminary, are fundamental for an initial numerical assessment of the propulsive efficiency, which will be subsequently compared with the actual experimental results. The experimental campaign is currently in its initial stage and will be completed in the near future, considering a wider range of operating conditions, materials, and fin geometries.

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