Chapter 11
Analyzing Rotating Bladed Disk Dynamics with Digital Image Correlation and Downsampling

Serena Occhipinti, Alessandra Cesaretti, Paolo Neri, Christian M. Firrone, and Daniele Botto

Abstract During operating conditions, turbomachinery blades are exposed to excitation forces that increase the risk of crack nucleation and reduce their fatigue life if the frequency of the forces matches the natural frequency of the blade. Therefore, it is essential to accurately measure the dynamics, in terms of vibration amplitude and resonance frequency, of rotating bladed discs. Conventional contact-based measurement methods have several drawbacks: they alter the dynamic behavior of the bladed disk by adding mass, have a limited lifetime, and face complex data transmission issues. This has led to a growing interest in non-contact measurement methods. In this context, this study presents an affordably, accurate, full-field, and high-resolution measurement technique. The proposed method employs low-speed, high-resolution cameras combined with down-sampling to capture images of a rotating bladed disk. These images are analyzed with the Digital Image Correlation (DIC) technique to identify the operational deflection shape of the bladed disk. This method was tested on a rotating bladed disk with simplified geometry. The method has been shown to work very well with only just one image per revolution, thus reducing the problems associated with large rigid body motions.

Keywords Turbomachinery blades · Dynamic measurement · Optical technique · DIC, · Down-sampling.

Introduction

Vibrations in turbomachinery can lead to crack formation, significantly shortening the fatigue life of blades. Consequently, effective blade health monitoring is essential to prevent unexpected failures. In synchronous vibrations, the frequency of the external force is an integer multiple of the rotational speed. When the rotational speed of the disk is such that the corresponding frequency of the exciting force approaches a natural frequency, the risk of dangerous resonance phenomena increases, making it crucial for designers to have an accurate knowledge of the disc dynamics.

Historically, strain gauges have been used to monitor rotor blade vibrations and remain the most reliable method of measurement. However, they require complex wireless systems or slip rings to transmit data, and their mass and stiffness interfere with the system dynamics. Furthermore, strain gauges have a limited service life and provide only localized data, related to discrete points on the bladed disk.

As a result, non-contact measurement techniques are becoming increasingly popular. One promising approach is the Scanning Laser Doppler Vibrometer (SLDV) combined with an optical derotator. This system offers a non-invasive way to accurately capture vibration data from specific points with high temporal resolution. The optical derotator ensures the laser beam accurately follows the rigid rotation of the disk by using a controlled optical rotation unit synchronized with the rotation frequency, effectively eliminating the effects of rigid body motion [1]. Despite its advantages, such as precise and non-contact measurements, the high cost of these devices is a significant limitation. In [2], an experimental vibration analysis of a fan was conducted using a custom-built derotator paired with an SLDV. Similarly, [3] describes a method for tracking specific points on rotating structures using a Laser Doppler Vibrometer and a system of plane mirrors—one mounted on the

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rotating structure and the other stationary in front of it. However, these methods require complex hardware configurations, where even slight misalignments can lead to measurement inaccuracies, and the data collected is still limited to a small number of points.

In this context, Blade Tip Timing (BTT) technology has emerged as a valid alternative. BTT measures blade vibrations by analysing the arrival times of blade tips as they pass in front of fixed sensors. By comparing the actual arrival time with the expected time for the rigid body rotation, the system can derive vibration data. BTT systems then use advanced identification methods to process this data, allowing the extraction of modal parameters that define the dynamic behaviour of the vibrating blades. While in most conventional configurations the sensors are positioned radially inward, in the direction of the tips of the blades [4], different probe configurations have been explored to measure different displacement components [5]. However, BTT measurements are inherently one-dimensional and are limited to specific points, typically the blade tips.

The advancement of video-based measurement techniques for dynamic analysis has introduced the use of optical cameras to monitor rotor vibrations. Stereo Digital Image Correlation (Stereo-DIC) enables the measurement of multiple points across a structure's surface. This is accomplished by applying a speckle pattern to the surface, which allows DIC to track points on object surface. In this way, the stereo-DIC is able to provide full-field three-dimensional surface displacements. While DIC is a well-established technique for dynamic experimental analysis of non-rotating structures, applying it to rotating systems presents significant challenges. Traditional DIC algorithms struggle to track points accurately across a rapidly changing field of view, leading to inaccurate displacement estimates when dealing with large displacements and rotations. A common approach to overcome this limitation involves using an incremental procedure [6]. This method analyzes images of objects undergoing large displacements by updating the reference image whenever the displacement between frames exceeds the measurable limit. However, when the reference update occurs, an error is introduced into the calculated displacement [7]. Thus, this procedure applied on rotating structure, where high displacement are due to the rigid body rotation, results in frequent updating of the reference image and to non-negligible noise accumulation. Moreover, the incremental procedure requires small displacements between consecutive frames, necessitating the use of high-speed cameras to capture the blade rotation with precise time resolution. High-speed cameras may initially appear to be the most suitable solution for obtaining measurements at higher frequencies, but they result in low resolution and high setup costs. For this reason, methods have been developed to use DIC with cameras at low frame rate to measure the dynamics of structures at frequencies above the Nyquist-Shannon limit. For example, using the down-sampling methods presented in [8, 9], it is possible to correctly measure the frequency content of steady state signals while avoiding aliasing even if the sampling rate does not comply with the Nyquist-Shannon sampling theorem.

This paper explores the capability of the Digital Image Correlation (DIC) technique as a measurement method for analyzing full-field, high-frequency synchronous vibrations of a bladed disk using a cost-effective, low-speed stereo-camera system. The study employs a down-sampling technique to prevent aliasing errors when acquiring samples at frequencies below the Nyquist limit. The presented methodology was tested on a dummy bladed disk mounted on a spinning test rig. The bladed disk has a simplified flat geometry and the vibration modes are mainly out-of-plane bending.

Measurement Methodology

The disk was tested in a spinning rig, where a set of permanent magnets, evenly spaced around the casing, induced synchronous vibrations in relation to the rotor speed. Synchronous vibration is a common phenomenon in rotating machinery blades, where the blade vibration frequencies are integer multiples of the rotational frequency.

During each revolution, two cameras captured a single image of the rotating disk at a specified angular reference position, triggered by an optical probe. These images were then processed using the Stereo-Digital Image Correlation (Stereo-DIC) technique, enabling the calculation of the three-dimensional full-field displacement of the disk's surface as it passed through the reference position. However, since the sampling frequency matched the rotational frequency, the displacement data were downsampled, according to the sampling theorem. To extract meaningful information from the original downsampled data, appropriate identification and reconstruction algorithms were required. Among the various algorithms available for this task [10–13], the post-processing was performed by using a fitting method called Single Parameter Technique (SPT) [14].

Stereo-Digital Image Correlation

The stereo-Digital Image Correlation (stereo-DIC) algorithm operates by processing images taken from different perspectives using DIC techniques. This process tracks surface displacement and deformation by analyzing how the position and shape of specific patterns change over time. In this study, the open-source Matlab program Ncorr [15] was used to implement the DIC algorithm. Another essential aspect of a stereo-DIC setup is the calibration of the stereo-camera system. The calibration process for each camera in a stereo pair establishes the transformation needed to map 2D image points from the camera sensors to their corresponding 3D coordinates in the global system, based on the pinhole optical model [16]. For this work, the calibration was carried out using the MATLAB Stereo Camera Calibrator App [17]. Once the positions of image subsets are identified in both camera frames and the stereo-camera model is defined, triangulation is used to compute the 3D positions of each point over time. This allows the calculation of spatial displacement and deformation fields.

Single Parameter Technique (SPT)

The Single Parameter Technique (SPT) assumes that each point on the disk behaves like a single-degree-of-freedom (SDOF) vibrational system, with damping and subject to a constant-amplitude harmonic excitation force. The system is described by the following differential equation:

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = \frac{F_0}{m}\cos(\omega t + \varphi_0) \tag{1}$$

where x is the point displacement; ζ is the damping ratio; ω_n is the natural angular frequency of blade; F_0 is the amplitude of excitation force; m is the mass; ω is the angular frequency of excitation force; φ_0 is the initial phase. The excitation frequency ω is directly related to the rotational speed of the disk Ω:

$$\omega = \Omega \cdot EO \tag{2}$$

where EO is the engine order. Thus, the steady state solution for the vibration equation is:

$$x(t) = X(\omega)\cos(\Omega t \cdot EO - \phi(\omega) + \varphi_0) \tag{3}$$

where,

$$\begin{gathered} X(\omega) = \frac{\delta}{\sqrt{\left[1-(\omega/\omega_n)^2\right]^2 + (2\zeta\cdot(\omega/\omega_n))^2}} \\ \tan(\phi(\omega)) = \frac{2\zeta\cdot(\omega/\omega_n)}{1-(\omega/\omega_n)^2} \end{gathered} \tag{4}$$

Here, δ represents the static displacement of the blade tip under the action of the force F_0. Thus, assuming a certain response order, corresponding to the selected EO, for each point downsampled response the function $x(t)$ is fitted to the data in order to compute model parameters.

Test Rig

Figure 1 illustrates the test rig. The tested dummy disk is made of aluminum and features a simple design with 12 blades. Each blade is shaped like a cantilever beam, measuring 150 mm in length and 25 mm in width, with a thickness of 5 mm. The disk has an outer diameter of 400 mm. At the tip of each blade, cylindrical magnets are glued into housings drilled for this purpose. The dummy disk was tested in a laboratory spinning rig. The test rig has a vertical axis and the rotating shaft is positioned under the floor. The disk is fixed between two flanges at the top of the shaft. The excitation system of the rotation setup employs cylindrical permanent magnets that interact with the magnets mounted on the disk during its rotation. This interaction generates impulsive repulsion forces, as the magnets on the rotating disk pass by the fixed magnets, creating periodic excitation. Two coaxial stationary disks were designed to lock these magnets in fixed angular positions around the rotation axis. The number of magnets used in a the test has to match the main engine order (EO) corresponding to the excitation force being simulated. A modal shape associated with a particular nodal diameter number (ND) can be excited by different values of EO. For a disk with N blades, the EO the engine orders capable of exciting a nodal diameter ND can be calculated as:

$$EO = k \cdot N \pm ND, \quad \forall k = 0, 1, 2, 3, \ldots \tag{5}$$

As expressed in Eq. 2, the excitation frequency is related to both the engine order, EO, and the rotational frequency, Ω. Based on Eq. 5, 16 magnets were equally spaced around the test rig to excite the nodal diameter $ND = 4$ of the bladed disk.

Given the limitations of high-speed cameras in terms of cost and sensor resolution, the disk's motion was measured using a low-speed 3D-DIC system, as shown in Fig. 1 This system consists of two Optomotive Spinosaurus cameras, each equipped with a 23 MP Fujinon lens featuring a 12 mm focal length. These cameras are equipped with high-resolution sensors capable of capturing 7.1 MP images at a maximum frame rate of 200 fps and with a minimum shutter time of 2 µs. To prevent motion blur, high-intensity illumination was required to compensate for the reduction in image brightness due to the short shutter time. For this purpose, two 200 W Stratus LED modules were used to illuminate the blade surface effectively. The data acquisition for both cameras was synchronized using a common trigger signal. To achieve this, an optical probe was mounted on the non-rotating part of the rig, pointing towards the upper flange that secures the disc. A reflective patch was attached to this flange, which rotates along with the disk. Each time the reflective patch passed in front of the probe, the optical probe generated a trigger signal, ensuring that both cameras were synchronized with each other and with the shaft's rotation.

Results

First, the dummy disk was modeled in Ansys APDL to determine the rotational velocity range that excites the first modal family. The nodes were constrained to replicate the actual boundary conditions. For this modal family, the frequency range

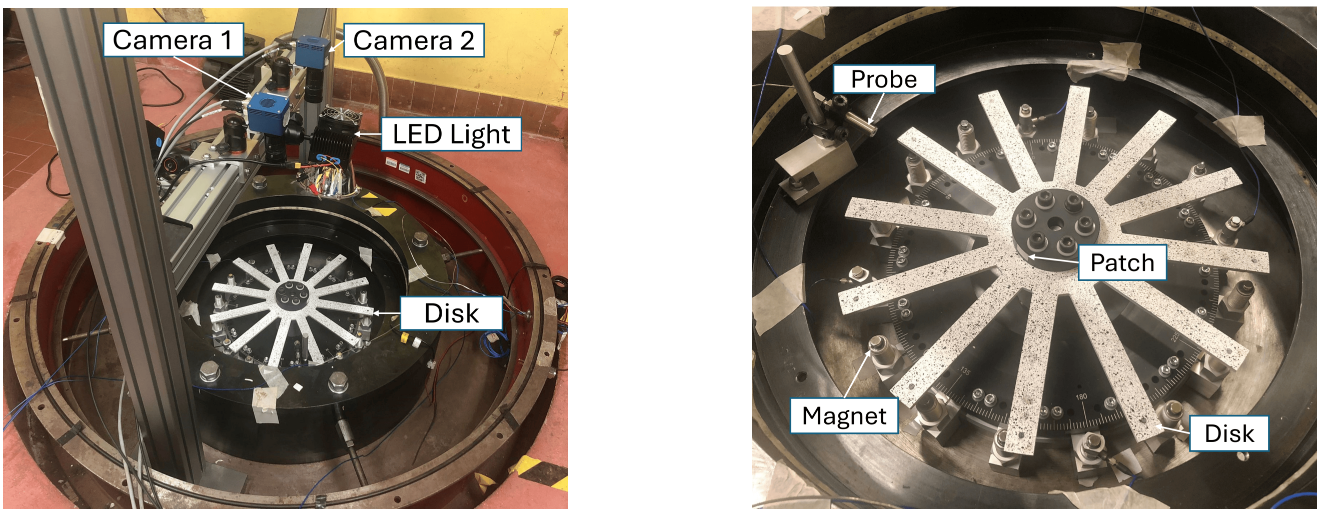

Fig. 1 Test rig setup (left) and close-up view of the disk (right).

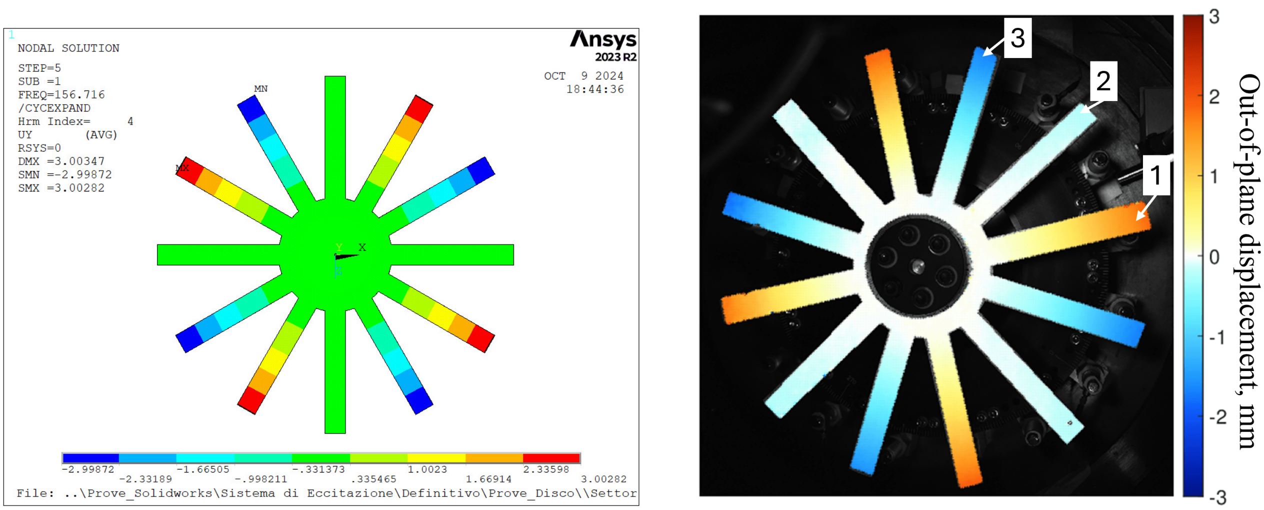

Fig. 2 Out-of-plane bending mode for the dummy disk at ND=4 in the first modal family: numerical simulation (left) and experimental results (right). The numbers on the right figure indicate the points for which the displacement is plotted over time in Fig. 3.

of interest was identified between 148 Hz and 158 Hz. Specifically, the mode with nodal diameter $ND = 4$, shown in the left side of Fig. 2, was found at 154 Hz. Accordingly to Eq. 2 the excitation force was estimated to be around 580 rpm.

Therefore, a linearly increasing speed profile was applied to the disk, ramping up from 0 rpm to 600 rpm over a duration of 11 minutes. Cameras capture the disk motion with a sampling frequency, imposed by the external optical probe, and equal to the rotation frequency. Thus, the maximum sampling frequency is approximately 10 Hz. The color map on the right side of Fig. 2 illustrates the displacement measured by DIC as the reflective patch passed in front of the optical probe at a rotational speed of 567 rpm, highlighting that the mode shape with a nodal diameter of 4 was effectively excited.

The downsampled out-of-plane displacement signal in three points located at the tips of three blades are illustrated in blue Fig. 3. Due to the limitations of having downsampled data, it is not possible to directly compute the resonance frequencies and damping ratio. Consequently, the Single Parameter Technique (SPT) was applied, with the red curves representing the results of the fitting procedure. The parameters obtained for these three points are summarized in Table 1. SPT was performed separately for each of the 27,000 DIC points on the disk surface, yielding a mean resonance frequency of $151.42 Hz$ with a standard deviation of $0.07 Hz$, and a mean damping ratio of 2% with a standard deviation of 0.06%.

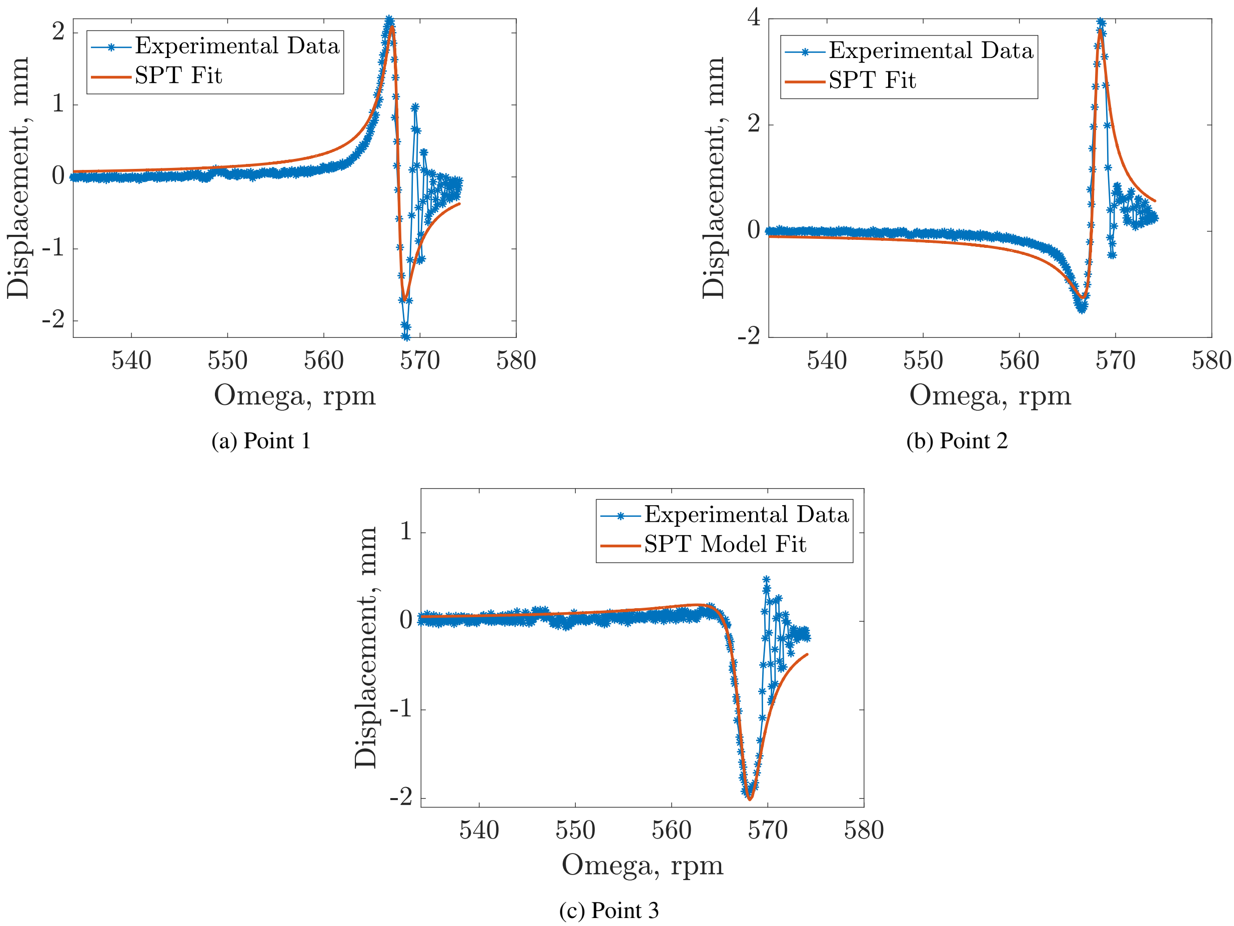

Fig. 3 The blue line depicts the out-of-plane displacement, down-sampled using Digital Image Correlation (DIC), for three points located at the tips of three adjacent blades, as shown in Fig. 2. The red line represents the fitted curve obtained through the Single Parameter Technique (SPT).

Table 1 SPT results for the three points located at the tips of three adjacent blades, as shown in Fig. 2.

	Point 1	Point 2	Point 3
ω_n (rad/s)	951.18	951.58	951.18
ζ	1.1 %	1.4 %	2.7 %
$\delta(\mu m)$	8.7	13.6	11.8
$\varphi_0(rad)$	0.1	2.6	5.3

Conclusion

In this paper, a new methodology for measuring the synchronous vibration of bladed disk was proposed. The methodology was tested on a dummy disk characterized by a simplified geometry. Utilizing a spinning rig equipped with permanent magnets to induce synchronous vibrations, the bending mode with a nodal diameter of 4 was captured by two low-frame rate, high resolution cameras. Thus, the Digital Image Correlation (DIC) technique was employed to obtain detailed full-field displacement data from the disk surface. To avoid aliasing error, the downsampled data computed by DIC were post-processed by the Single Parameter Technique (SPT), allowing for an in-depth analysis of the vibration mode. Overall, the results open the way for further exploration of advanced vibration monitoring techniques that can improve the reliability and safety of industrial applications involving rotating components.

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