Chapter 1
Comparison of Accelerometer Selection Algorithms for a Modal Filter Application

Tyler F. Schoenherr

Abstract A modal filter is a useful tool to characterize the full-field dynamic response of a structure. It provides the amplitudes of the inherent modes of the structure, which can be used to understand its full-field response. As useful as the modal filter is, the execution of the technique is challenging. Forming a modal filter requires representative modes observed at appropriate measurement locations. Choosing those locations is critical to reducing fitting and over-fitting errors in the modal filter process. One means of choosing the accelerometer locations recently developed is to minimize the Modal Projection Error. This paper examines using the Modal Projection Error and two other means of choosing measurement locations in order to compare their efficacy with respect to the modal filter. This comparison of the different methods are evaluated using an experimental data set and a novel means of comparing the effectiveness of the modal filter.

Keywords Modal · Projection · Error · Filter · Accelerometer

Introduction

One challenging aspect in dynamic environment characterization is the ability to measure the full-field response of a structure. The most direct means of acquiring the response of a structure is to take direct discrete measurements of acceleration using accelerometers. Although data acquisition systems and accelerometers have greatly improved in the past decades, measuring the approximate full-field response of a continuous structure is impractical in most applications.

Strain is caused by relative motion within a structure. Knowledge of the full-field deflection of the structure at any moment of time is imperative if strain and damage is to be quantified in a dynamic environment. Direct discrete measurements are not adequate on their own to calculate the full-field strain in the structure. Acquiring the full-field response requires additional knowledge of the structure in addition to response measurements.

A modal filter is the combination of the structural dynamic properties of a structure in the form of mode shapes and discrete measurements. The mode shapes are curve fit to the measured data through a least-squares process. By combining the mode shapes and measured data, the full-field response of a structure can be calculated.

There are errors generated during the curve fitting process when combining measured acceleration and mode shapes to calculate the full-field response. Such errors have unique challenges in providing meaningful modal filter results and are described in this paper as fitting and over-fitting errors. Intelligent selections of where the discrete measurements are during the dynamic environment can mitigate some of the curve fitting errors.

Several methods with different modal-based objective functions have been previously proposed to improve the modal filter calculation. This paper attempts to compare the efficacy of three existing degree of freedom selection methods: Effec-

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tive Independence [1], condition number, and Modal Projection Error [2]. All three of them are examined in this paper to determine if there are benefits of one method over the others and to evaluate how robust the calculations are to changes and errors in the mode shape basis set.

Theory

The focus of this paper is on the modal filter process. The modal filter is rooted in modal superposition theory that states that a structural response is comprised of a linear combination of the structure's mode shapes

$$\ddot{x} \approx \boldsymbol{\phi}\ddot{q}, \tag{1}$$

where $\ddot{x}$ is the vector of physical accelerations at a set of degrees-of-freedom, $\boldsymbol{\phi}$ is the mode shape matrix at the same set of degrees-of-freedom and $\ddot{q}$ is the modal acceleration or amplitude for each corresponding mode. The linear combination of modes in Eq 1 is only an approximation of the acceleration due to modal truncation.

The modal filter is the inverse of Eq. 1 such that the modal accelerations can be calculated with knowledge of the structure's mode shapes and physical accelerations. In order to transform the physical accelerations into modal accelerations, the physical accelerations are projected on to the mode shapes by using the Moore-Penrose pseudo-inverse of ϕ,

$$\boldsymbol{\phi}^{+}\ddot{x} \approx \ddot{q}, \tag{2}$$

where the $^{+}$ superscript denotes the pseudo-inverse of a matrix. The pseudo-inverse in the modal filter is a least squares fitting process for over-determined systems [3]. Even though this paper focuses on the structural dynamics aspect of this process, the mode shape matrix can also be referred to as a basis set of vectors as the vectors do not need to be structural mode shapes [4]. For the remainder of this paper, this matrix is referred to as a basis set.

The least squares fitting process introduces errors in the calculation of the modal coordinates. If the mode shapes in Eq. 2 do not span the space of the physical accelerations, there is a fitting error that is minimized in a least squares sense and the modal accelerations will not combine with the mode shapes to perfectly reproduce the physical accelerations. The second error is an over-fitting of the shapes to the data. This error occurs when two or more shapes linearly combine to fit the physical acceleration that is not fit by the other shapes. This error can end up being detrimental as these subset of shapes will greatly increase in amplitude to either fit noise or fitting error in the basis set.

The over-fitting error is caused when two or more vectors in the basis set can closely reproduce any other vector in the basis set. This allows the least-squares fitting process to inflate all these vectors to fit measurement noise or fitting errors. If the vectors cannot combine to fit errors, then the resultant error from the projection will be a fitting error. It is advantageous to have the error be from the fit as it provides evidence that is easier to interpret and is quantified by the Modal Projection Error (MPE) [5].

One way to reduce the overfitting error is to select a set of degrees of freedom to measure that makes the basis vectors unable to combine to form another basis vector. Many methods of selecting degrees of freedom for ultimate usage in a modal filter application exist. Three of those methods are examined in this paper. The first method is a minimization of the MPE. The modal projection error is a quantity of how well a set of vectors can linearly combine to create a given vector calculated

$$\text{MPE} = \Psi_n^2 = 1 - \bar{\phi}_{Tn}^{+}\boldsymbol{\phi}_S\boldsymbol{\phi}_S^{+}\bar{\phi}_{Tn}, \tag{3}$$

where Ψ_n is the modal projection error of the n^{th} target vector, $\boldsymbol{\phi}_{Tn}$. The set of vectors, $\boldsymbol{\phi}_S$, is fit to the target vector.

The other two objective functions are the maximizing of the Effective Independence (EFI) of the basis set [1] and the minimization of the condition number of the basis set. These quantities do not directly compute the basis vectors ability to linearly combine to recreate a separate vector in the basis set, but these methods have been used in other studies of modal filter applications with success. In order to optimize the MPE, EFI, or condition number, one degree of freedom is removed at a time and the objective function is recalculated. This process is repeated over and over in a Greedy algorithm [6] until a desired number of degrees of freedom are selected.

Test Hardware and Calibration to the Model

In order to demonstrate the efficacy of different instrumentation selection methods, experimental data is ideal. Use of experimental data is ideal because it contains realistic errors in the basis set from the finite element model (FEM) shapes in their ability to perfectly fit the measured experimental data.

The hardware used to generate data for this paper is the frame-plate-wing (FPW) test bed [7]. This hardware is chosen because development of an adequate model is quick and a large experimental data set already exists. A large channel count test provides channel reduction options for the optimization methods. In addition, a large channel count also provides more validation for the basis set of mode shapes. The FPW was tested in the laboratory in a free boundary condition and impacted with a modal hammer. A picture of the test setup can be seen in Figure 1.

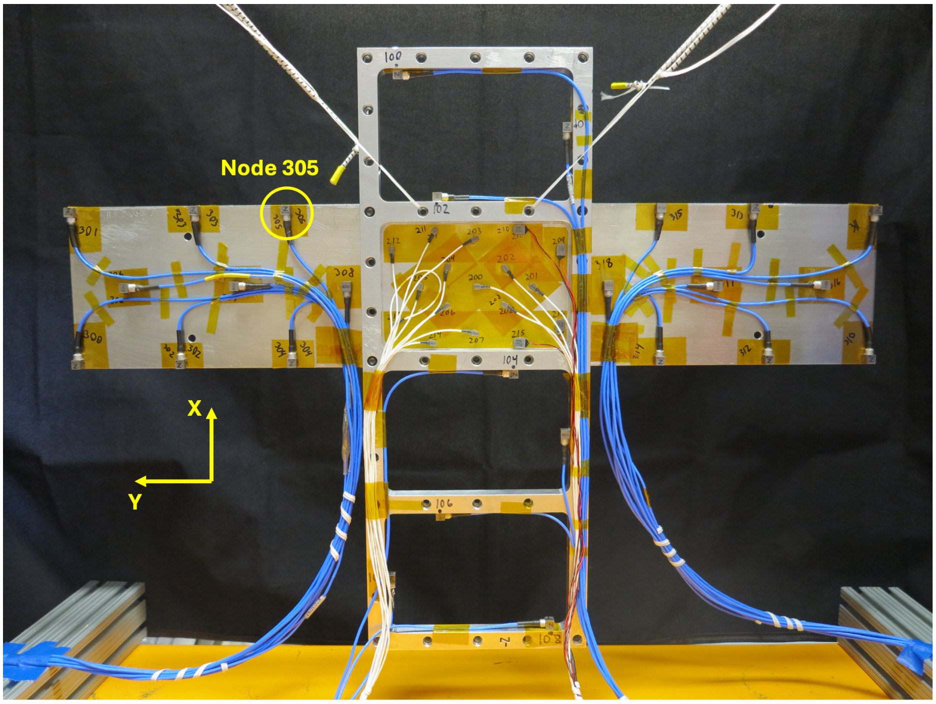

Fig. 1 Photograph of the experimental setup and node locations for the Frame Plate Wing structure

The FEM is compared to experimentally derived mode shapes and natural frequencies. The Modal Assurance Criteria (MAC) with associated natural frequency comparisons can be seen in Figure 2. From the comparison in Figure 2, it can be seen that the model does a good job fitting to the experimental shapes below 800 Hz even though the model natural frequencies are 7-10% higher than the experimentally measured natural frequencies. Above 800 Hz, the FEM shapes begin to represent the physical hardware less. Having modes that match well and match poorly is leveraged in this paper to demonstrate how well the modal filter works with a basis set expected to fit the data well under 800Hz and a basis set with that will incur fitting error under 1800Hz.

Experimental Results

In order to fit a basis set to test data, test data is needed. The response data used in this study is a transient set of data caused by a modal hammer strike of the FPW structure at node 305 in the direction normal to the flat surface of the wing as shown in Figure 1. For all modal filters implemented in this study, the modes are generated by the FEM of the hardware.

The test data is ideal for comparing different means of selecting accelerometer degrees of freedom because the test is highly instrumented at 95 channels. This channel count provides a large pool from which the different algorithms can select degrees of freedom.

Evaluating the efficacy and accuracy of the modal filter is challenging. The output of the modal filter is modal accelerations if the data measured is from accelerometers. Since there is no direct measurement for modal acceleration, success is inferred. With imperfect mode shapes, the modal filter will always be a least squares fit to the test data. Only modes and accelerations measurements from a finite element model would provide a perfect fit and produce exact modal accelerations. Using experimental data, there is always some error in the estimations of the modal accelerations.

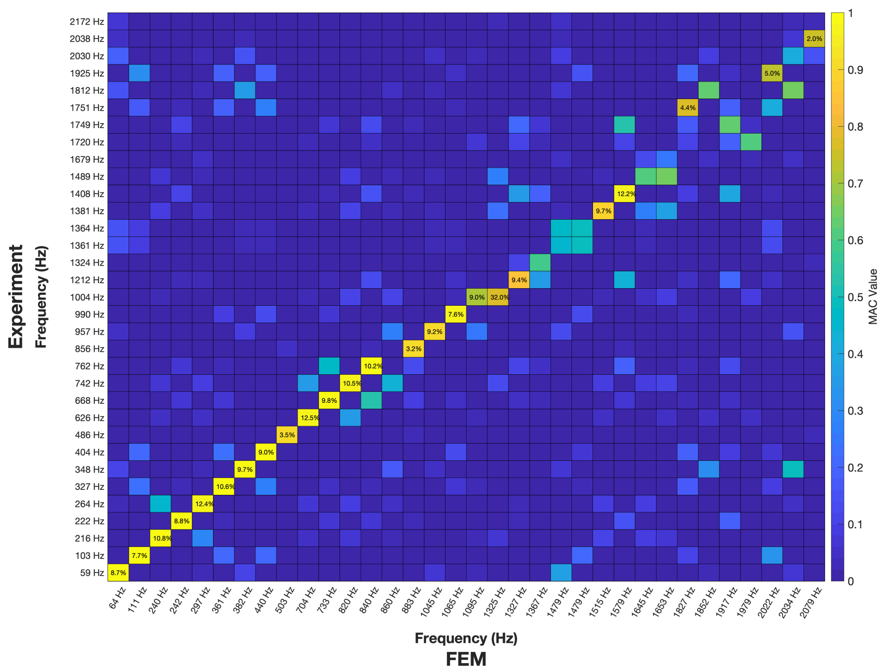

Fig. 2 Modal Assurance Criteria and associated natural frequency error of the Finite Element Model when compared to experimental modal results

It is of interest to alter parameters of the modal filter that have the largest effect on the calculation of the modal accelerations. When analyzing the modal accelerations output by the modal filter, four parameters are considered:

- Method of selecting the degrees of freedom in the filter
- Number of degrees of freedom selected
- Number of modes in the basis set
- Accuracy of the basis set

The changes in modal accelerations when altering the aforementioned parameters are compared to ‘truth’ modal accelerations. The truth modal accelerations are calculated from using all the degrees of freedom and modes 1 through 32 with modes 20 and 25 omitted in the modal filter. Modes 20 and 25 are omitted because they individually closely resemble other modes in the basis set. Using this set of shapes will be referred to as the full set of shapes.

The frequency content of the truth modal accelerations calculated for modes 1 through 18 can be seen in Figure 3. Because the input force is an impulse from a hammer strike, the rigid body modal acceleration for modes 1 through 6 should be flat at low frequencies with translation in the Z direction (mode 3), rotation about X (mode 4), and rotation about Y (mode 5) being of largest magnitude. These rigid body modes should be highest in magnitude of the rigid body modes due to the location and direction of the force vector. The modal acceleration of the elastic modes 7 through 15 present themselves as single degree-of-freedom oscillators which is expected from modal analysis theory. The modal accelerations in Figure 3 provide evidence that the modal filter worked well enough to use these modal accelerations as ‘truth’ data.

To increase our confidence in the truth modal accelerations calculated from a modal filter, the modal accelerations are multiplied by the basis set to calculate the response at the measured DOFs . The set of measured DOFs in Figure 4 are not included in the modal filter process so they are considered a blind check of the validity of the modal acceleration calculation. Comparing the response of the measured DOFs to the resynthesized DOFs below 600 Hz, the comparison provides evidence of the modal accelerations being an excellent representation of the response of the system.

To compare the efficacy of the three different DOF selection methods, four modal filters are applied to the test data with varying modal filter parameters. A summary of the modal parameters can be found in Table 1. The first filter uses only modes 1 through 19 and 20 degrees of freedom. Modes 1 through 19 are chosen because they all have high MAC values when compared to the experimental modal data.

The second run also uses modes 1 through 19 in the modal filter, but uses 34 degrees of freedom in the modal filter. The third run uses the full set of shapes and 31 degrees of freedom. The ratio in the third run basis set provides one more degree of freedom than basis vectors. The final run uses the full set of shapes and 45 degrees of freedom.

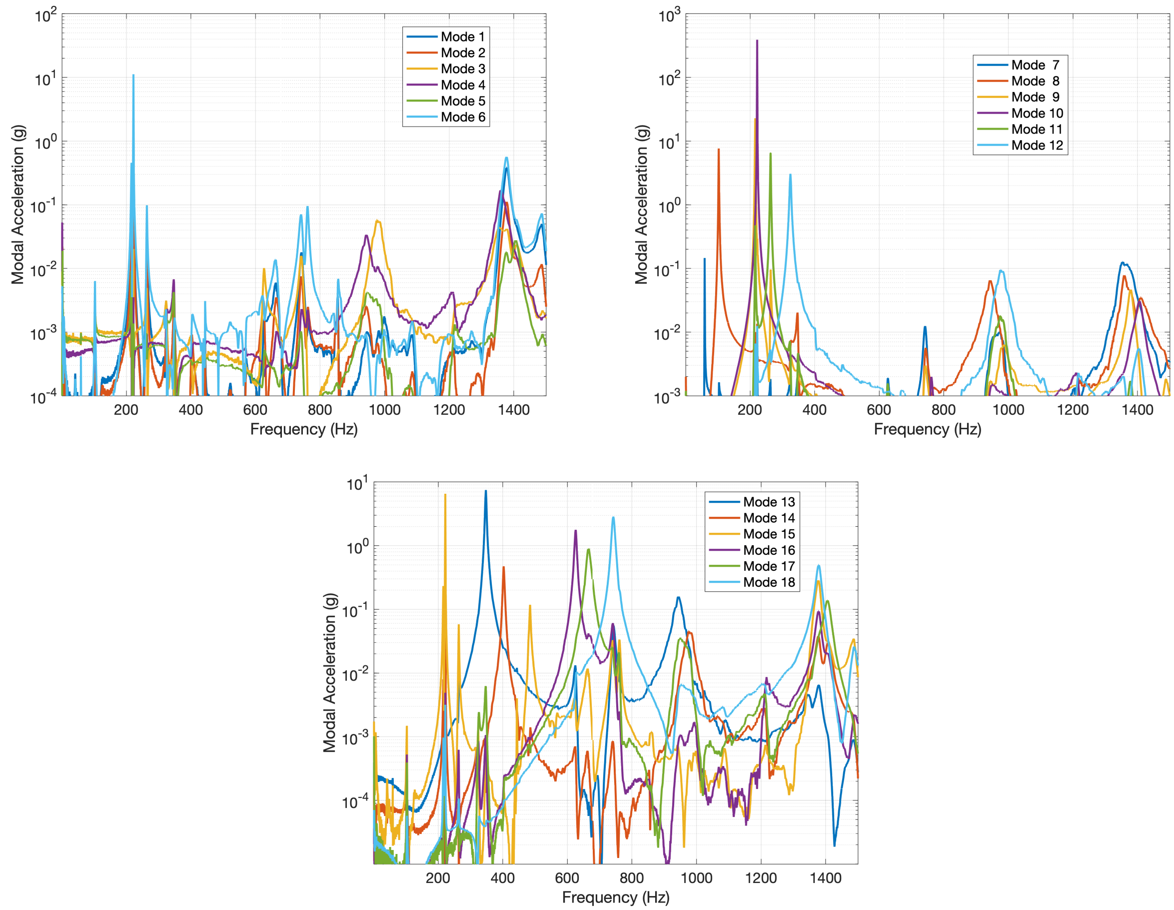

Fig. 3 Modal accelerations of modes 1 through 18 taken to be the 'truth' values

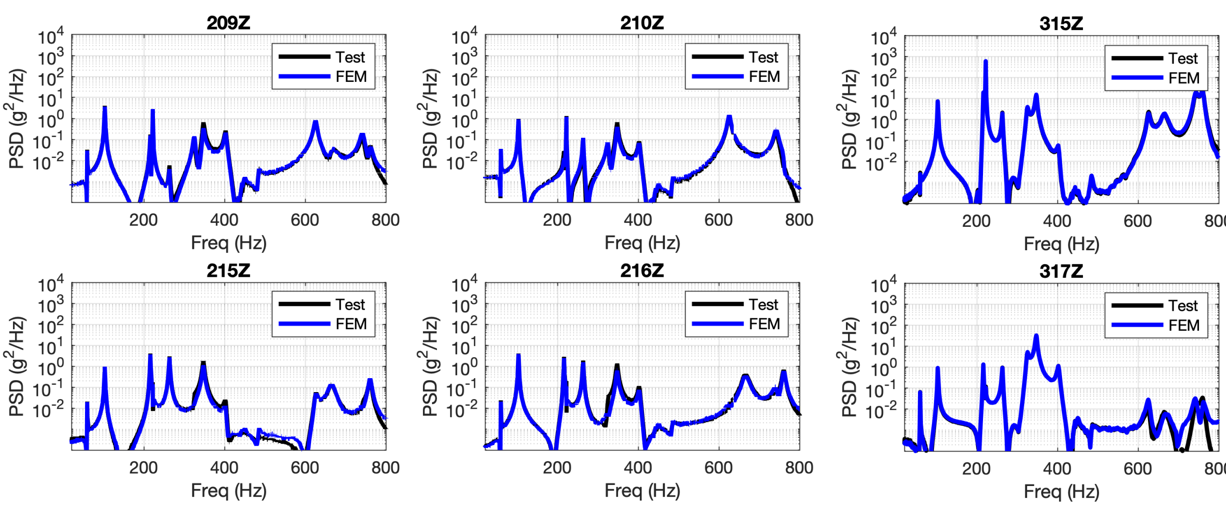

Fig. 4 Selection of degrees of freedoms not included in the modal filter with the modal filter having 19 modes and 20 degrees of freedom in the filter

For all four runs, the different basis sets are used to calculate the modal accelerations. For all comparisons, the peak of the modal acceleration of the Power Spectrum Density is compared to the modal accelerations from the truth calculation.

Table 1 Summary of the four modal filter parameters compared

Run #	Modes Included (m)	# of DOFs in Filter
run 1	1:19	m+1
run 2	1:19	m+15
run 3	1:19, 21:24, 26:32	m+15
run 4	1:19, 21:24, 26:32	m+1

The percent difference to the truth modal peak acceleration is documented and is shown in Figure 5 for comparison. It is important to acknowledge that the real modal acceleration value is unknown, but observing how much the modal acceleration changes when the degrees of freedom of the basis set changes is the focus of this research.

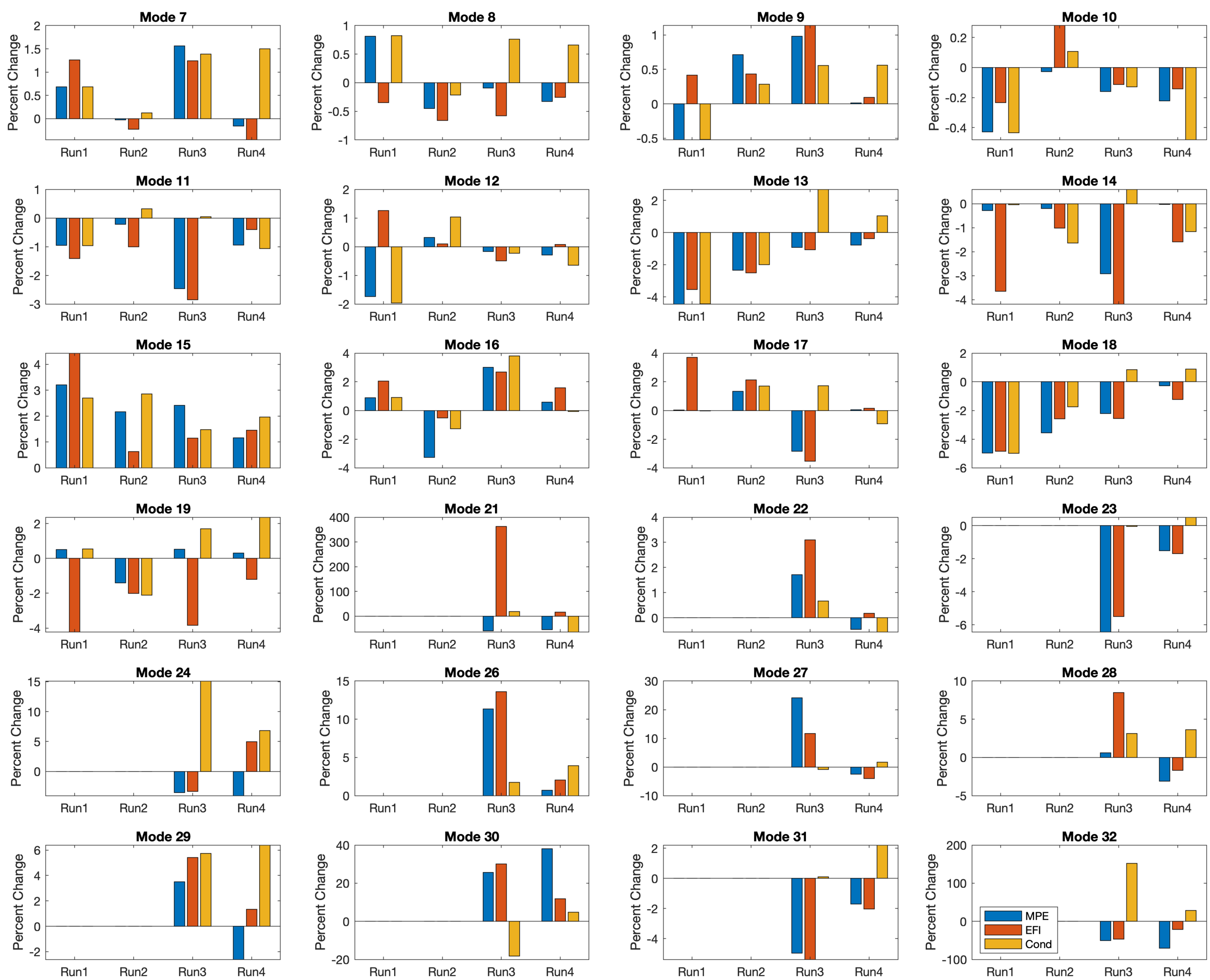

Fig. 5 Error of the modal acceleration at its resonance for each of the FEM modes used in the inverse Run 1: 19 Modes with 20 DOFs in inverse Run 2: 19 Modes with 34 DOFs in inverse Run 3: 30 Modes with 45 DOFs in inverse Run 4: 30 Modes with 31 DOFs in inverse

There are three notable observations of the percent differences of the different selection methods. The first observation is that the largest error occurs for the Effective Independence selection method for mode 21 as the magnitude changes by a factor of 3. However, this result is misleading because the magnitude of the modal acceleration for mode 21 is small enough that it is fitting only electronic noise and not structure response. Small changes to the basis set affecting the modal acceleration calculation is expected when the modal acceleration is in the noise floor.

The second observation is that the methods of selecting DOFs did not appear to have an effect on the stability of the modal filter. Each picking method has modes that caused the result to be the most consistent and the least consistent. Not shown in Figure 5 is the result when a random set of DOFs are chosen. When selecting a random set of DOFs for the basis set identical to run1, the inverse had an average different of around 800%. It is noteworthy that the DOFs that are selected by the three techniques are different from each other. Figure 6 shows the DOFs selected for run 1 by the three methods.

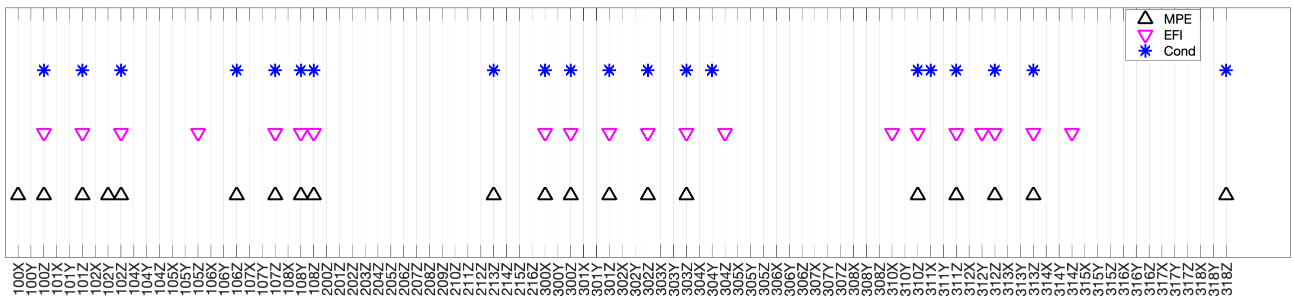

Fig. 6 Degrees of freedom chosen by 3 separate algorithms

The third observation is that the modal accelerations calculated are fairly robust to changes in the basis set given that the basis vectors do a good job of projecting to the measured data. Changes to number of modes included or number of DOFs included generally only altered the magnitude of the modal acceleration by less than 5%. This level of robustness is encouraging for usage of modal accelerations as quantities for characterizing the full-field response in dynamic environments characterization.

Conclusion

This research intended to provide insight into the benefits of using either the EFI, condition number, or MPE DOF selection method over another with respect to a modal filter calculation. In using a relatively accurate model to provide the basis set to fit experimental acceleration measurements, the results show that there is very little benefit of using one selection method over another. However, it is important to use one of the methods as random selection resulted in serious degradation of the modal filter.

The manner in assessing the success of the modal filter in this paper is novel in comparison to other papers on modal filter techniques. This paper uses the modal accelerations to compare techniques instead of the measured and resynthesized acceleration data. Direct comparison of the modal accelerations instead of a subset of measured accelerations provides a full-field response comparison that otherwise is implicitly estimated by comparing selected degrees of freedom.

A benefit to comparing modal accelerations directly is that changes in the magnitude of the modal acceleration is explicit instead of implied when comparing physical coordinates. Additionally, examination of the modal accelerations while altering the parameters of the modal filter provides some information regarding its stability. This is important because an unstable modal filter would be an indication of a poor modal filter. In addition, it appears that most of the modal accelerations change by less than 5%. This conclusion provides a confidence level of using modal accelerations in environment response characterization.

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