Chapter 12
Estimating Component Level Environments Using Next Assembly Measurements

Marcus Behling, Matthew S. Allen, Randall L. Mayes, and Washington J. DeLima

Abstract A significant obstacle to properly recreating vibration-induced damage in qualification tests is the lack of availability of flight environment data. Typically, data from a previous flight with similar hardware is used to create the specification for the new flight. In addition, data is usually only available on the next assembly and not on the components that are being qualified, though in-flight component damage is more accurately reproduced when the environment is available on the component itself. Hence, this study seeks to estimate the flight environment on the device under test (DUT) using environment data only on the next assembly as only this data is typically available. This is performed using the System Equivalent Reduction Expansion Process (SEREP). The accuracy of the response expansion is shown to be sensitive to the amount of next assembly used to model the flight system's dynamics as well as the condition number of the mode shape matrix. Full flight system dynamics are shown to not generally be observable with accelerometers on the next assembly only, so some error is expected in DUT response estimation with only next assembly measurements. The condition number of the mode shape matrix is shown to improve significantly when a few measurements are available on the DUT, so accurate full-field DUT responses can be estimated if the next assembly is well instrumented and if there are a few measurement channels on the DUT. Hence, one can significantly improve damage simulation if sparse DUT environment measurements are available.

Introduction

Traditional single axis vibration tests generally result in improper simulation of operational vibration induced damage [1][2][3][4]. These tests are usually assumed to cause more damage than would occur in the actual flight due to enveloping of the environment and an impedance mismatch between the test fixture and in-flight next assembly [5]. Off-axis motion, e.g., due to rotational shaker dynamics, can also artificially increase single-axis test severity [6]. Impedance Matched Multi-Axis Tests (IMMATs) were developed to fix some of the problems with traditional tests [7]. They use multiple shakers to simultaneously excite multi-axial motion and part of the next assembly is included in the lab test to match the flight boundary condition better. These tests have shown the potential for improved environment reconstruction, particularly when there are more control accelerometer channels than shakers and when these accelerometers are well distributed across the DUT [8].

However, environment data is typically not available on the DUT, and if it is, there are usually not enough accelerometers to completely capture the dynamics of the component. More often, rather, environment data is only available on the next assembly from a past flight. The usual practice is to mount the component on a large, single-axis shaker system and apply the

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Alexandra Karlicek et al. (eds.), *Dynamic Environments Testing, Vol. 7*, Conference Proceedings
of the Society for Experimental Mechanics Series, https://doi.org/10.13052/97887-438-0152-8_12

next assembly motion one axis at a time to the base of the component. The authors proposed a method that seeks to improve on this, which is called the Transmission Simulator IMMAT Framework (TS-IMMAT). This method uses multiple shakers to cause the base of the component to exhibit the same motion that it did during flight. If the base of the component is rigid then this entails using at least six shakers to control to six degrees of freedom, although additional shakers can be used if the base is flexible. While this can improve environment reconstruction relative to a traditional single-axis test, past experience has shown that these tests do not generally reproduce the environment on the component as accurately as an IMMAT test [9]. Hence, if one could use the response on the next assembly to estimate the response on the DUT, then one could perform an IMMAT test rather than a TS-IMMAT test, and reproduce the response on the DUT more faithfully, reducing the costs associated with failed parts due to over-test or in field failures due to under-test. Additionally, some tests require component level data, e.g., the intrinsic connection excitation (ICE) framework [10] and fixture neutralization (FINE) methods [11]. In general, controlling component level data compensates for boundary condition mismatches between the flight and lab setups and is useful [12].

The goal of this study is to see if accurate DUT response reconstruction is possible given next assembly measurements only. Specifically, we hope to answer: 1) How much of the next assembly should be included in the expansion model to accurately reconstruct the response on the DUT? 2) How accurate can one expect the reconstructed DUT responses to be if only next assembly measurements are available? 3) If next assembly measurements are insufficient, what is the minimum number of accelerometers that can be placed on the DUT to improve response estimation at unknown locations? These questions are studied by analyzing the Finite Element Model (FEM) case study shown in Figure 1, where the DUT is a stool intended to represent a structure on which a circuit board might rest, and the next assembly is a wedding cake-like structure. In this study, it is assumed that the next assembly geometry does not change from when the environment is measured (or simulated, in this case), i.e., the same system that data was measured on is the current system of interest. In practice, data may only be available from a previous flight, so the next assembly geometry may change. In this case, the methods presented in [13] might be applied to estimate the environment on the new next assembly, and the methods presented in this paper could then be applied to estimate the DUT environment.

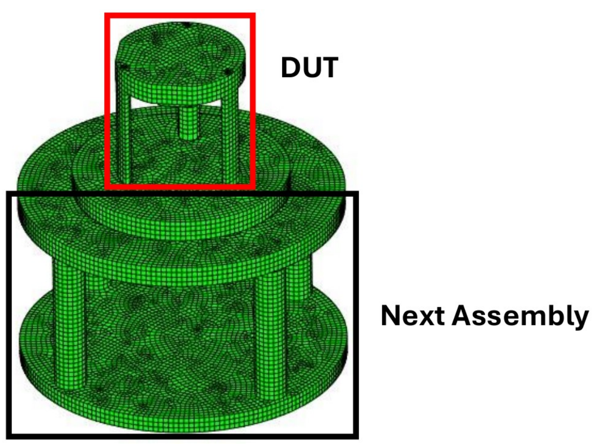

Fig. 1 Flight Setup Case Study

Environment Estimation Methods

The first efforts to accurately estimate flight environments in the absence of measured data seem to have been at NASA in the 60’s. One study presented two methods, one for components directly excited by acoustic or aerodynamic pressures, and another for those primarily excited by transmission through other parts of the structure. These methods involved scaling previously measured responses using ratios of forcing function magnitude and the relative masses of the original and new component [14]. Another study divided environment estimation methods into two categories: gross prediction techniques, in which broad correlations between forcing function and response magnitude are derived from previous flights, and custom prediction techniques, which involve more detailed modeling of the forcing function and structure of interest [15]. Statistical energy methods were also proposed as a way to predict environments at higher frequencies [16]. In short, early methods generally modified previous flight data to estimate new environments, though they were hindered by a lack of computational power and the inability to create accurate models of system hardware. These methods might accurately estimate response magnitude, though it would be difficult to apply them and obtain a physically meaningful environment at multiple locations on a DUT.

More recent methods benefit from improvements in computational power and the ability to model systems accurately via modal testing and FEM. Various response expansion methods have been devised, and some of these are summarized in [17]. Perhaps the most well-known is the System Equivalent Reduction Expansion Process (SEREP), in which the system modal responses are obtained from a sparse set of measurements and then expanded back out to estimate responses at desired degrees of freedom [18]. Another is the pseudo-force method, in which a set of forces is applied in simulation to the system of interest, the FRF relating these forces to the known responses is inverted, and the forces are applied to the system to obtain responses at the desired degrees of freedom [19]. Chebyshev polynomials have also been used as the basis functions that responses are projected onto, though this may not extend well to complex geometries [20]. Each of these methods can be used to obtain accurate responses at various locations on a DUT if the basis functions being projected onto – e.g., mode shapes, frequency response functions, or Chebyshev polynomials – span the DUT's response space and are well conditioned.

Conditions for Accurate Response Estimation

The response expansion method of choice in this study is SEREP. SEREP was introduced in [18], although the idea of using the modes of a structure as a basis to estimate or control the motion or stresses in a component is well known and used in many areas without reference to the formalisms put forth in SEREP (see, e.g. [21]). The SEREP theory is briefly rewritten here in the context of dynamic environment testing specifically. In flight, the system response is a linear combination of some set of flight mode shapes times their response amplitudes as shown in Equation (1),

$$\mathbf{x}_m^{\mathrm{DUT+NA+V}}(\omega) = \mathbf{\Phi}_m^{\mathrm{DUT+NA+V}} \mathbf{q}^{\mathrm{DUT+NA+V}}(\omega) \tag{1}$$

where the superscript indicates that the DUT and Next Assembly (NA) are installed in the flight Vehicle (V), and the subscript indicates that the response is observed at a set of points m. Note that the set of points m may include points on the next assembly, denoted p as well as points on the DUT, denoted d, so $m = p \cup d$ (or, in Matlab notation, $m = [p; d]$). A full FEM of the flight vehicle is often not available, and if it is, it is typically not detailed enough to accurately represent the dynamics of the DUT and its next assembly locally. Instead, we assume that a FEM is available of the DUT attached to the Next Assembly but with free boundary conditions. We also assume that the mode shapes of this assembly accurately span the response space of the DUT and Next Assembly in flight as in Equation (2),

$$\mathbf{x}_m^{\mathrm{DUT+NA+V}}(\omega) \cong \mathbf{\Phi}_m^{\mathrm{DUT+NA}} \mathbf{q}^{\mathrm{DUT+NA}}(\omega). \tag{2}$$

There is certainly no guarantee that the mode shapes of the DUT and Next Assembly in the free configuration will span the space of the motions observed when they are assembled into the vehicle. There may be dynamic interaction between the Next Assembly and the rest of the vehicle, and the connection to the vehicle may load the interface, altering the response. Past studies have sought to understand how much of the flight system should be included in the Next Assembly [22][23]. There is no general answer, so best judgment must be employed to include the portions of the vehicle adjacent to the DUT that significantly influence its dynamics and to exclude insignificant portions.

Assuming a set of accelerometers was installed on the next assembly in flight, SEREP can be applied to a set of measured responses to estimate the modal responses of the DUT + Next Assembly configuration using the Moore-Penrose pseudoinverse (denoted $()^+$ below). For this calculation, we assume that a set of measurement points p are available and that these measurements are only available on the Next Assembly. Hence p would be a subset of the available measurement points m considered in Equation (2),

$$\mathbf{q}^{\mathrm{DUT+NA}}(\omega) = (\mathbf{\Phi}_{\mathrm{p}}^{\mathrm{DUT+NA}})^+ \mathbf{x}_{\mathrm{p}}^{\mathrm{DUT+NA+V}}(\omega). \tag{3}$$

If the modal responses are accurately estimated, then the physical responses of the DUT, denoted points d can be estimated using the mode shapes at the desired DUT degrees of freedom as in Equation (4),

$$\mathbf{x}_{\mathrm{d}}^{\mathrm{DUT+NA+V}}(\omega) \cong \mathbf{\Phi}_{\mathrm{d}}^{\mathrm{DUT+NA}} \mathbf{q}^{\mathrm{DUT+NA}}(\omega). \tag{4}$$

As previously mentioned, accurate response reconstruction partially depends on satisfying Eq. (2), i.e., selecting the next assembly so that the modes of the DUT + Next Assembly in the free configuration (i.e. when not installed on the vehicle) span the space of their response in flight. In addition, the DUT + Next Assembly modes must be observable with the given accelerometer set p, and the mode shape matrix, $\mathbf{\Phi}_{\mathrm{p}}^{\mathrm{DUT+NA}}$, must be well-conditioned. When the condition number of this matrix is high, any noise in the flight environment will cause significant error in estimating the modal responses, $\mathbf{q}^{\mathrm{DUT+NA}}(\omega)$, resulting in error in estimating the DUT's flight environment, $\mathbf{x}_{\mathrm{d}}^{\mathrm{DUT+NA+V}}(\omega)$. To accurately estimate the response using this approach one must have at least as many accelerometers as there are active system modes and they must be placed to effectively distinguish those modes.

In general, though, there may be modes of the combined DUT and Next Assembly system that are not observable with accelerometers on the next assembly only. When this is the case, some environment data is required on the DUT, so some of the sensors d must be added to Equation (3). To determine how many accelerometer channels are required on the DUT to fully observe all system modes, the Singular Value Decomposition (SVD) can be applied to the mode shape matrix, $\mathbf{\Phi}_{\mathrm{p\cup d}}^{\mathrm{DUT+NA}}$, as in Equation (5),

$$\mathbf{\Phi}_{\mathrm{p\cup d}}^{\mathrm{DUT+NA}} = \mathbf{U\Sigma V^T}, \tag{5}$$

where $\mathbf{\Sigma}$ is an $\mathrm{n_{accels}}$ x $\mathrm{n_{modes}}$ diagonal matrix with singular values on the diagonal in order of descending magnitude. A normalized singular value vector, $\mathbf{\Sigma}_{\mathrm{norm}}$, can be calculated by dividing each singular value by the largest singular value as in Equation (6),

$$\mathbf{\Sigma}_{\mathrm{norm}} = \left\{ \begin{array}{cccc} \sigma_1/\sigma_1 & \sigma_2/\sigma_1 & \ldots & \sigma_{\mathrm{n_{modes}}}/\sigma_1 \end{array} \right\}^{\mathrm{T}}. \tag{6}$$

If there is some maximum acceptable condition number, $\mathrm{C_n}$, of the mode shape matrix $\mathbf{\Phi}_{\mathrm{p\cup d}}^{\mathrm{DUT+NA}}$, which is inverted in the projection process, then the number of entries in $\mathbf{\Sigma}_{\mathrm{norm}}$ greater than the reciprocal of $\mathrm{C_n}$ is the number of modes that are observable with the next assembly accelerometer set, $\mathrm{n_{mo}}$, and the number of entries less than the reciprocal of $\mathrm{C_n}$ is the number of modes that are not observable with the next assembly accelerometer set, $\mathrm{n_{mu}}$. At least $\mathrm{n_{mu}}$ accelerometer channels must be measured in flight on the DUT, therefore, to observe all modes of the DUT. In practice, it is likely that a few more channels are required than this minimum to reduce the condition number of the mode shape matrix to an acceptable value.

In the following sections, this theory is applied to the system in Fig. 1 for three separate cases. In the first case, a small portion of the next assembly is included in the projection model, and the environment is assumed to only be known on the next assembly. In the second case, the only available measurements are still on the next assembly, though the full in-flight next assembly is included in the projection model. Finally, in the third case a few accelerometer channels are included on the DUT with the full next assembly also included. These case studies are summarized in Table 1.

One final note is that tests are practically performed using Power Spectral Densities (PSDs) rather than linear spectra; these two can be related according to,

$$\mathbf{S}_{\mathrm{XX}}(\omega) = \mathrm{E}\left(\mathbf{x}(\omega)\,\mathbf{x}(\omega)^*\right), \tag{7}$$

where "E" is the expected value operator, or the average over many measurements.

Table 1 Definition of Cases Studied

Case	Assembly Considered	Measurements Available
Case 1	DUT+small portion of the next assembly	on next assembly only
Case 2	DUT+full in-flight next assembly	on next assembly only
Case 3	DUT+full in-flight next assembly	on next assembly and DUT

Results

A flight environment was simulated on the flight vehicle setup in Fig. 1 by applying a 1 $\mathrm{N^2/Hz}$ force from 10-2000 Hz at each of the locations shown in Figure 2. An FRF relating the forces at these locations to the responses at the desired locations was synthesized using all 13 of the system's modes below 2000 Hz. A damping ratio of 0.01 was assumed for each of the elastic modes. This environment is used for each of the three cases presented here.

Case 1: Small portion of next assembly, no DUT data

In the first case, only the top plate of the next assembly in Fig. 1 is included in the model. Hence, while the flight assembly includes all of the components shown in Figure 1, the lab assembly includes only the components shown in Figure 3. This represents a case where the full flight vehicle model is not available, so a portion of the supporting structure is included to represent the local dynamics of the vehicle. There are 9 modes of this system below 2000 Hz that were included in the

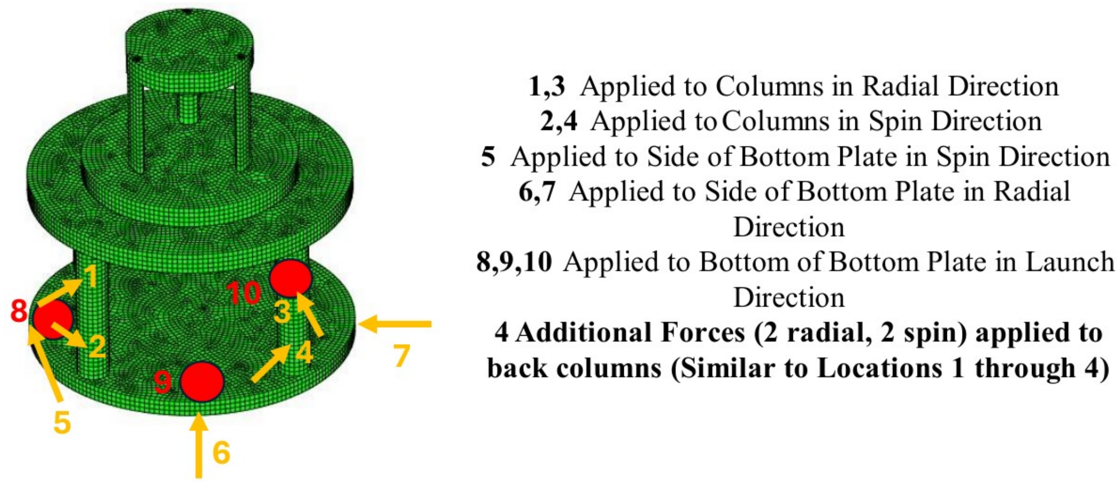

Fig. 2 Force Locations Used to Simulate Flight Environment

model, and a damping ratio of 0.01 was again assumed for these modes. The accelerometer locations that are projected onto (labeled 1-5), as well as the reference accelerometer 6 on the DUT that we hope to recreate the response of, are shown in Figure 3. All accelerometers are triaxial, measuring the response in a global xyz coordinate system where the z axis is parallel to the columns and will be referred to as the launch direction while x and y are in the plane of the plate and will be referred to as the radial and spin directions. Hence, the mode shape matrix in Eq. (2) was 15×9. A small amount of noise is added to the flight environment to make this analysis more realistic and to show the effects of a high mode shape matrix condition number.

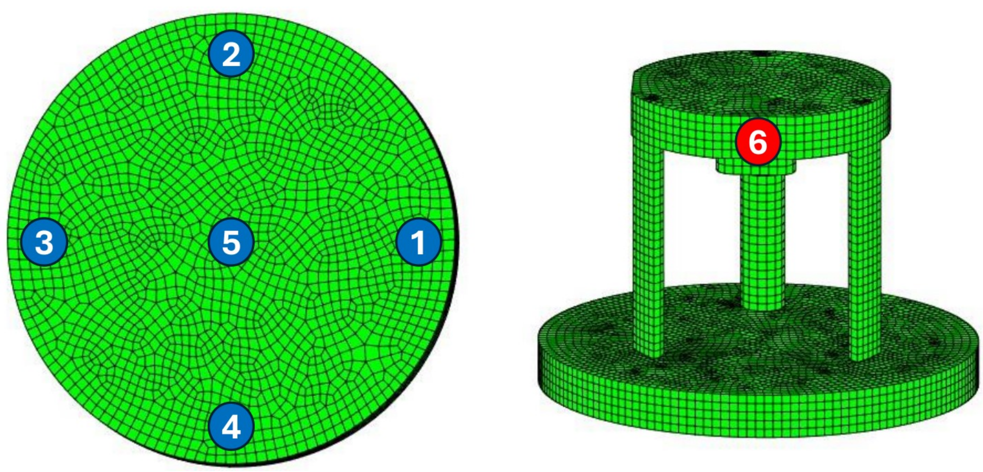

Fig. 3 Case 1 Assembly and Triaxial Accelerometer Locations (all on the plate) used in SEREP (Left) and Reference Triaxial Accelerometer on DUT (Right)

The stool environment estimated obtained from SEREP is compared to the actual flight environment in Figure 4, and there is clearly significant error in estimating the DUT responses. In this case, the estimated environment exceeds the magnitude of the actual environment at most frequency lines, so a test performed controlling to the estimated environment would result in high levels of over-testing. Conservatism is necessary in design and testing, though this level of conservatism seems extreme, and the actual test level would likely be increased due to enveloping, adjustments for increased reliability, and so on [24]. So, even though the environment is generally conservative and even has similar natural frequencies to the actual environment, this estimate is considered insufficient.

One reason this estimate is so poor is that the mode shapes of the Stool-Plate configuration do not quite span the in-flight response space of the DUT. As previously mentioned, this configuration has 9 modes below 2000 Hz, while the flight configuration has 13. One of these flight modes, shown in Figure 5, is a bubble mode of the plate that is accentuated by

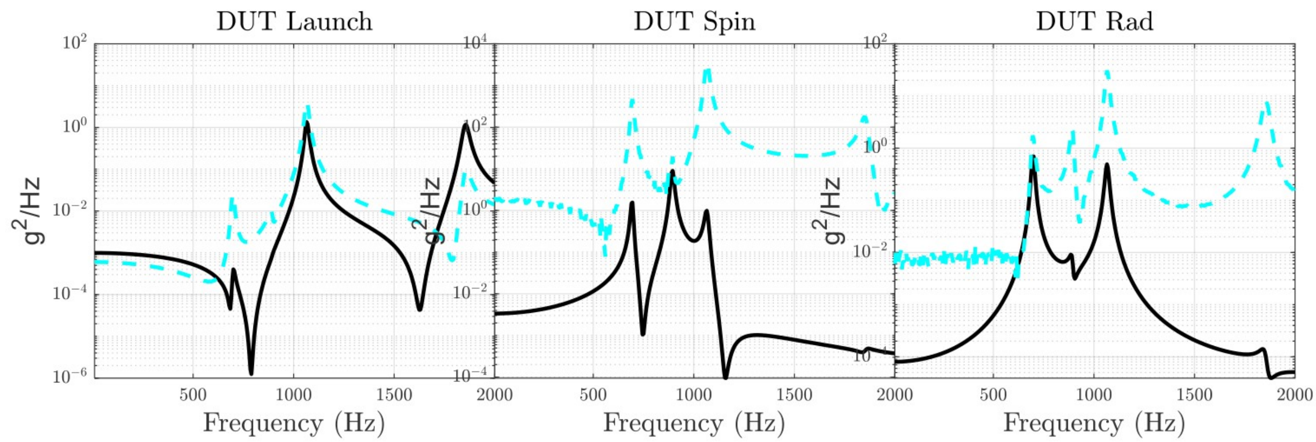

Fig. 4 Estimated DUT Response PSDs estimated by SEREP from noisy measurements for Case 1 (Cyan) vs. Actual Stool Response PSDs (Black)

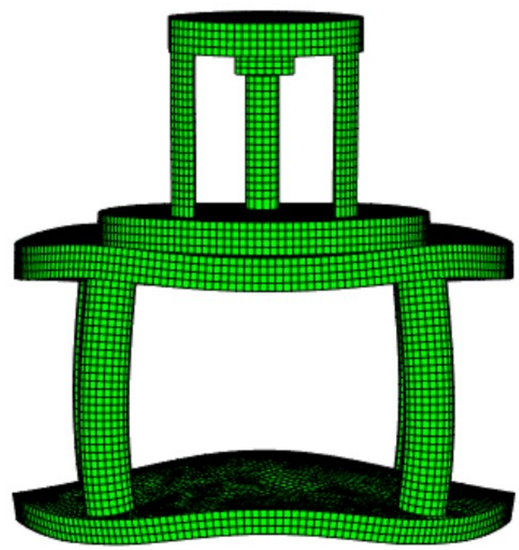

Fig. 5 Flight Configuration Mode #13 at 1854 Hz

the columns, so it is not observed in the stool-plate configuration. This modal response is not properly replicated in the projection process, causing increased error in the DUT environment estimation.

Another reason for the inaccurate response estimation is that the modes of the Case 1 Stool-Plate configuration are not fully observable with the set of accelerometers used in the expansion process. Two of the three elastic modes of this configuration are shown in Figure 6; the third is a bending mode similar to the right side of Fig. 6 but in the out-of-page direction. In these modes, the stool experiences significant elastic deformation, while the plate remains mostly rigid. To the plate accelerometers, these modes are basically indistinguishable from the rigid body modes of the system. As a result, the responses of these modes cannot be accurately estimated from the measurements on the base plate, resulting in significant error in estimating their responses and hence the environment on the DUT.

To further demonstrate the inability of the plate accelerometers to fully observe the stool-plate system modes, the modal assurance criterion (MAC) was applied to the stool-plate mode shapes using the Case 1 accelerometer locations. A MAC value of 1 indicates that mode shapes are identical, while a MAC of 0 indicates that modes are orthogonal. Ideally, cross-MAC values between different modes should be very low, e.g., below 0.1, as this indicates that the set of sensors being used can properly distinguish between each mode and properly identify modal responses from environment data. As seen in Figure 7, though, the off-diagonal MAC terms are very high for the elastic modes. Specifically, the torsion mode is very similar to rigid body rotation about the z-axis, the first bending mode is similar to y-translation and x-rotation, and the

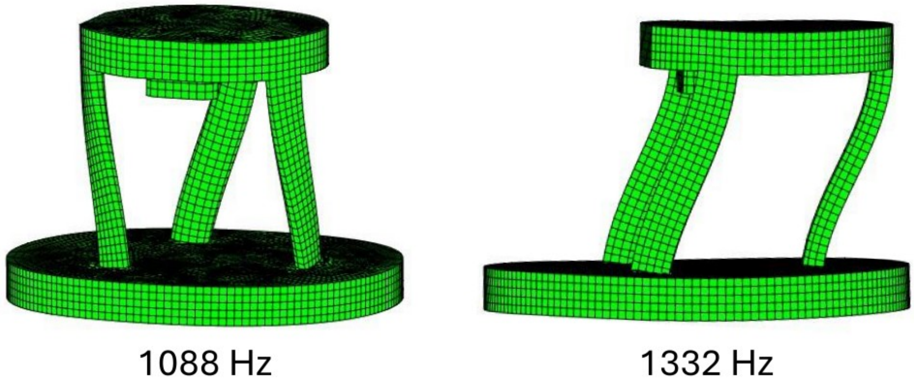

Fig. 6 Torsion and Bending Modes of the Case 1 Stool-Plate System

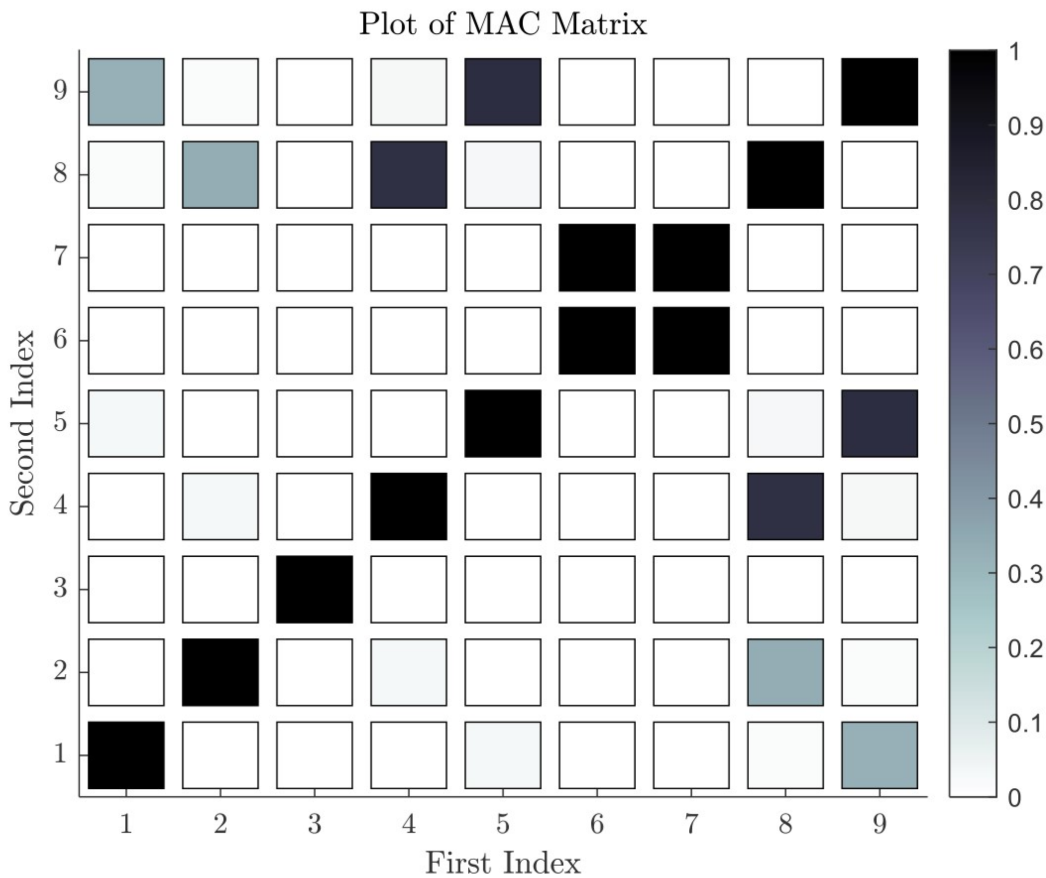

Fig. 7 MAC for Stool-Plate Configuration

second bending mode is similar to x-translation and y-rotation. This confirms that the elastic modes of the stool cannot be distinguished from the system's rigid body modes given the set of plate accelerometers that was used in Case 1 and that this is a cause of the error observed here.

Case 2 – Full next assembly, no DUT data

Since the portion of the next assembly included in the previous section did not accurately capture the motion of the flight assembly, the full-flight next assembly is used in the SEREP process in this case, although measurements are still assumed to not have been obtained on the DUT. The simulated accelerometer locations are shown in Figure 8. Since the number of channels (54) is much greater than the number of system modes (13), and since the accelerometers are well distributed across on the next assembly, we assume that the next assembly's motion is properly observed. The DUT reference accelerometer is the same as in the previous case.

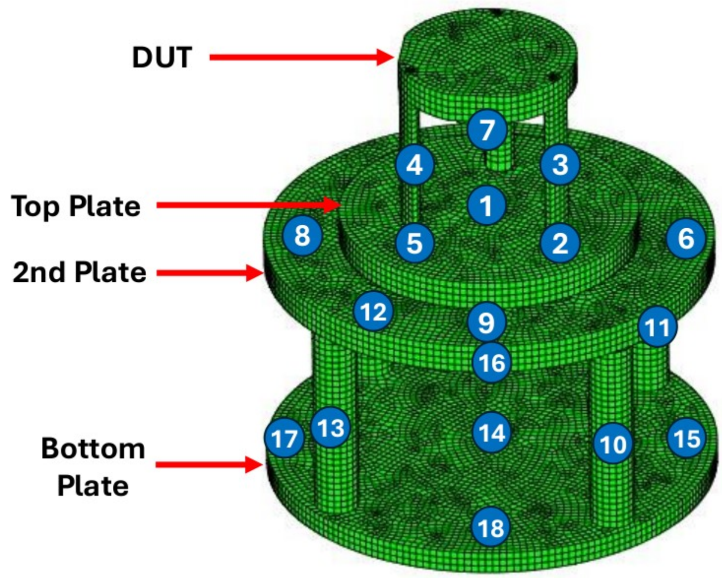

Fig. 8 Triaxial Accelerometer Locations Used in SEREP on Flight Configuration. 1-5 are on Top Plate, 6-9 are on 2^{nd} Plate, 10-13 are on the Columns, and 14-18 are on the Bottom Plate.

As seen in Figure 9, the response reconstruction on the DUT is slightly improved over the first case due to the increased portion of the next assembly included, although significant error remains in the spin and radial directions. This suggests that the system's modes are not fully observable with Case 2 accelerometer set, which are on the next assembly only, and that the mode shape matrix must be ill-conditioned in this case.

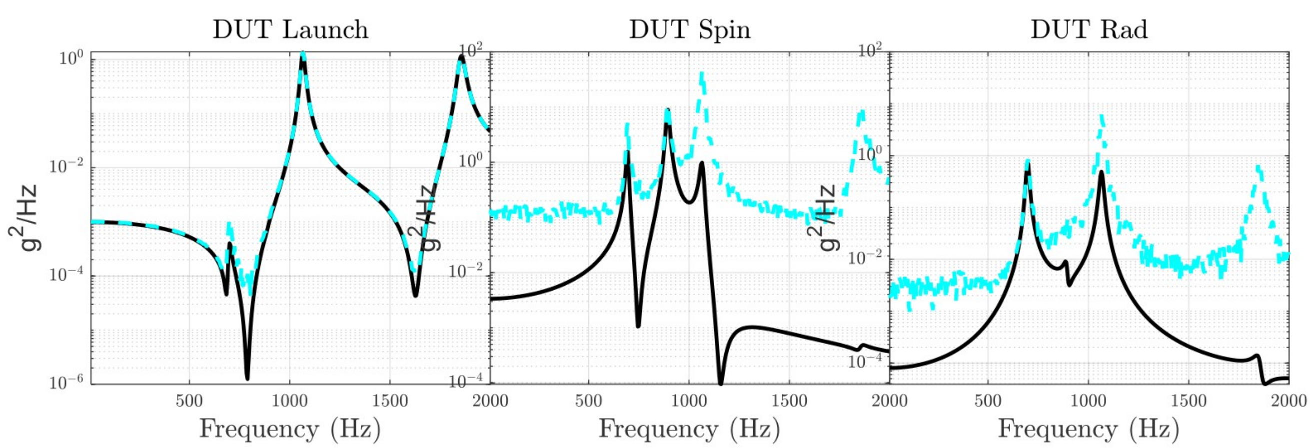

Fig. 9 Case 2 DUT Response Reconstruction Using SEREP with Noise in Measurements

The mode shapes of the flight system, shown in Figure 10, help explain these results. The 13^{th} mode shape is not included in this figure, as it was shown previously in Fig. 5, and the 8^{th} and 12^{th} mode shapes are not shown because they are similar to the 7^{th} and 10^{th} modes in the orthogonal direction. More of the system's modes are identifiable with next assembly accelerometers in this case because the next assembly deforms along with the DUT. Modes 7, 8, 9, 11, and 13 seem to have unique shapes, although the next assembly's motion in modes 10 and 12 seems to be a combination of torsion and bending, so these modes are not observable with accelerometers on the next assembly only, and their modal responses are not expected to be accurately recreated.

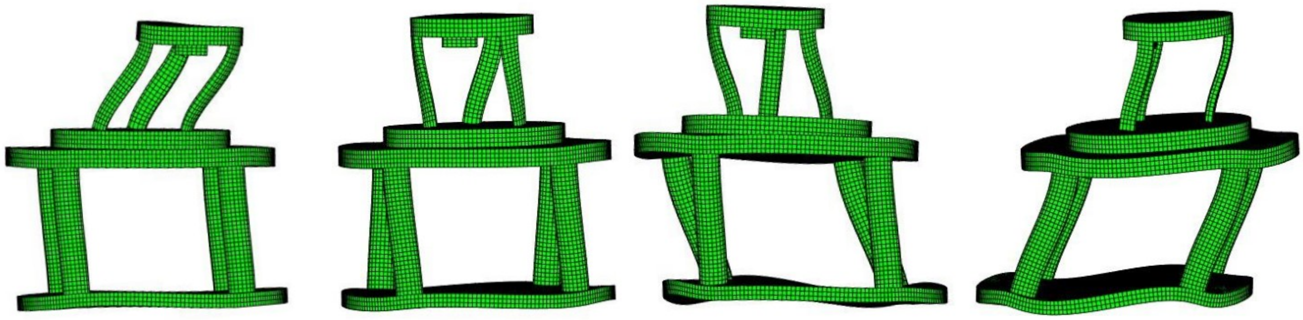

Fig. 10 Flight System Elastic Mode Shapes. Mode #13 is shown in Fig. 5.

These observations are confirmed by taking the SVD of the mode shape matrix using the accelerometer set employed in this case. The matrix $\left[\boldsymbol{\Phi}_{\mathrm{p}}^{\mathrm{DUT+NA+V}}\right]$ was 54×13, and the singular value ratios are shown in Table 2. Any singular value ratio less than 1% corresponds to a condition number of over 100. These seem to be the cause of the response reconstruction error in the studies performed here, and hence this seems to be a reasonable rule of thumb. With accelerometers on the next assembly only, 10 modes are observable, and 3 are not, so at least three accelerometer channels would be required on the DUT to properly observe these modes. This is assuming that there are no other candidate locations on the next assembly that would observe these modes, which seems to be a reasonable assumption given the number and spread of accelerometers that are already included on the next assembly.

Table 2 Normalized singular values of mode shape matrix for Case 2 and 3 assembly, comprised of the DUT + Full Next Assembly. These were computed without (Case 2) and with (Case 3) three DUT accelerometer channels.

Singular Value No.	No DUT Channels	3 DUT Channels
1	1	1
2	0.99	0.98
3	0.98	0.94
4	0.67	0.64
5	0.59	0.57
6	0.58	0.57
7	0.57	0.55
8	0.41	0.41
9	0.41	0.40
10	0.17	0.32
11	**6e-3**	0.07
12	**6e-3**	0.05
13	**2e-3**	0.03

Case 3 – Full next assembly with DUT data

Since next assembly accelerometers alone were unable to observe three of the flight system's modes, the environment would need to be known on at least three accelerometer channels on the DUT to properly estimate the environment at the unknown locations. The degrees of freedom shown in Figure 11 were added to the measurement set. The resulting 57×13 matrix $\left[\boldsymbol{\Phi}_{\mathrm{p\cup d}}^{\mathrm{DUT+NA+V}}\right]$ was found to reduce the condition number of the mode shape matrix significantly as shown in Table 2. Hence, with these three channels on the DUT and the previously shown channels on the next assembly, all system modes are observable, and all modal responses should be accurately estimated using SEREP, so the unknown DUT response in Fig. 3 should be estimated with little error.

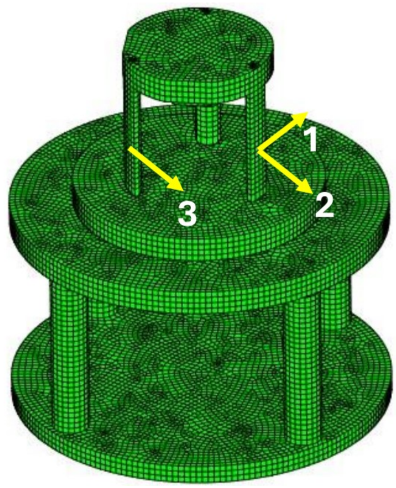

Fig. 11 DUT Accelerometer DOFs to Include in Modal Projection

As shown in Figure 12, this new accelerometer set yielded excellent reconstruction of the unknown DUT accelerometer environment. There is slight error in each direction, likely because the condition number of the mode shape matrix is ∼35, although it is small relative to the overall environment. The estimated environment is, again, conservative, so any error in estimation would not result in undertesting. Though only one accelerometer's response is shown here, we assume that the responses at any location on the DUT would be recreated with similar accuracy since this triaxial accelerometer's response is influenced by all of the modal responses. Hence, accurate replication of its response would require accurate replication of all modal responses, meaning that the DUT's response should be estimated accurately globally.

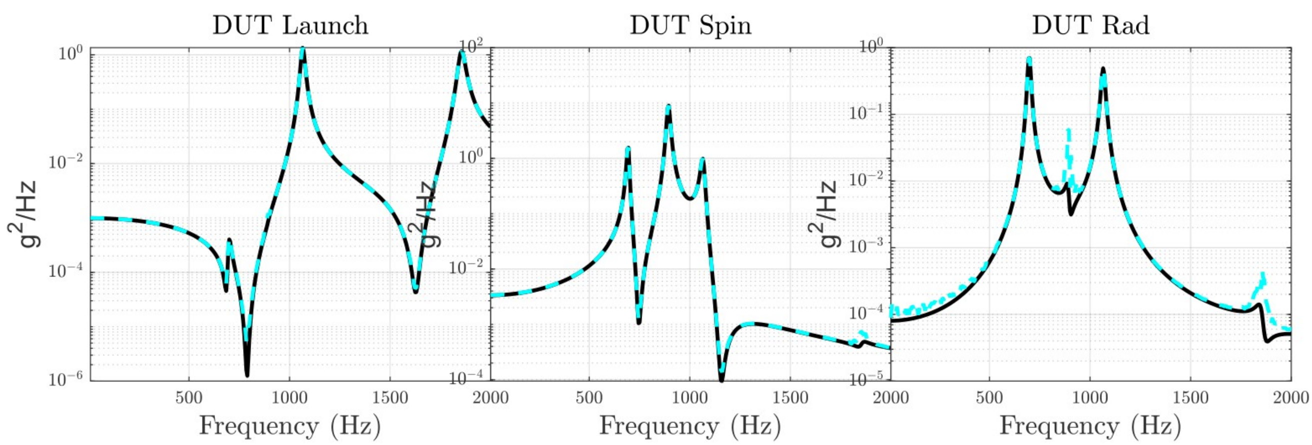

Fig. 12 Case 3 Reconstructed DUT Responses. Case 3 used the full Next Assembly plus three DUT channels.

The Benefit of Data on the DUT

In single axis tests, having component level data available on the DUT allows response limits to be implemented [5], which reduces the amount of overtest. In multi-input/multi-output (MIMO) tests, the DUT accelerometers can be controlled directly, resulting in improved reconstruction of the environment at uncontrolled locations on the DUT relative to controlling to measurements on the next assembly only. To show the benefit of having component level data, a MIMO test is simulated, first controlling the fixture (i.e. next assembly only) and then controlling the DUT responses. The shaker locations in Figure 13 are used for simulating the test. Though there are 13 modes of the system, 10 shakers at the locations in Fig. 13 were found to be sufficient for accurately reproducing the response.

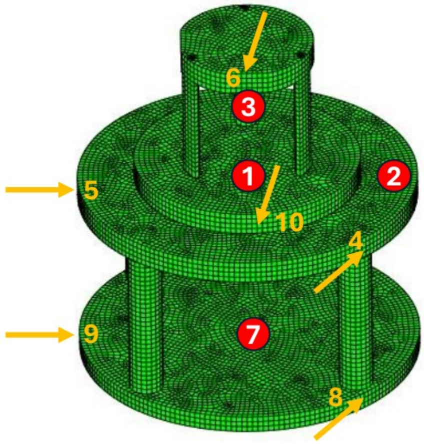

Fig. 13 Shaker Locations for Simulating MIMO Test

The next assembly control accelerometers are the same 54 channels used in Cases 2 and 3 and were shown in Fig. 8. The DUT control accelerometers are shown in Figure 14. In both the next assembly and DUT control cases, the goal is to replicate the environment at the reference DUT accelerometer shown in Fig. 3.

The environments reconstructed in the fixture and DUT control cases are compared to the original flight environment in Figure 15. When controlling only to accelerometers on the next assembly, there are noticeable errors in reconstructing the DUT's responses in the spin and radial directions. When the DUT accelerometers are controlled, however, there is almost no error in reconstructing the DUT environment. The control is improved because the DUT's motion is fully observed with the DUT accelerometer set, while the DUT's motion is not completely observed with accelerometers on the next assembly only. Hence, DUT environment data is useful because controlling it in MIMO tests results in improved simulation of vibration-induced damage relative to only having data on the next assembly and controlling the motion of the fixture. However, the reconstruction of the DUT environment is reasonable even with measurements on the next assembly only, suggesting that the shaker control may be less sensitive to noise and accelerometer locations than SEREP is. Hence, this may be a benefit to

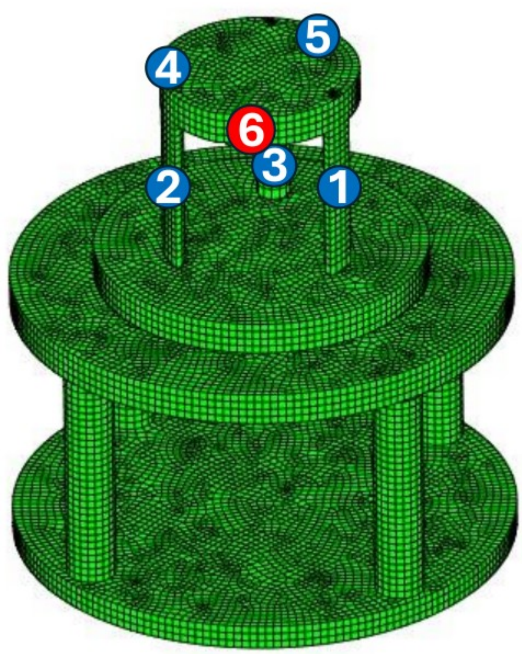

Fig. 14 DUT Control Accelerometer Locations. Accelerometers 1-3 are on the legs of the stool, while 4-6 are on the top plate. Reference Accelerometer in Red.

using the pseudo-forces method [19] instead of SEREP for environment estimation purposes, as the pseudo-force method is essentially the same as the control method used here.

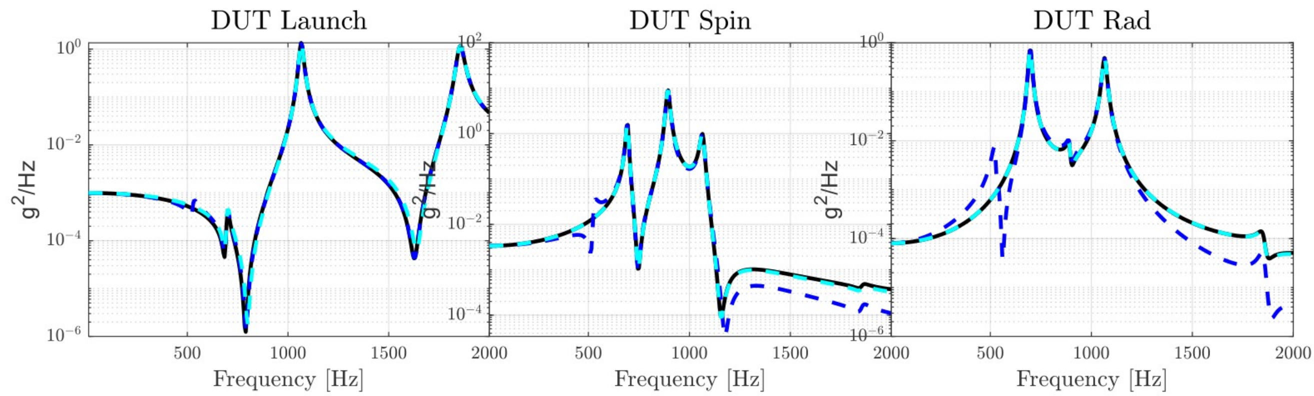

Fig. 15 DUT Environment Reconstruction in two cases. (Dark Blue) Using the shakers in Fig. 13 to control to the 54 next assembly accelerometers in Fig. 8. (Cyan) Using the 10 shakers in Fig. 13 to control to the 5 triaxial DUT accelerometers in Fig. 14. (Black) True Flight Environment

Conclusions

In recent years, methods for accurate vibration qualification have improved significantly, though a major obstacle is that these methods generally require more environment data, which is typically not available. The goal of this paper, therefore, was to understand how accurately component level environments could be estimated given measurements on the operational next assembly only. A MIMO test was simulated showing that DUT response reconstruction is more accurate when environment data is available on the DUT and not just the next assembly. Theoretical conditions for successful environment response reconstruction were presented, and the theory was applied to three case studies.

The first case study showed that error is incurred in response estimation when the amount of next assembly included in the SEREP model does not accurately represent the dynamics of the DUT and next assembly in the flight configuration. The second case study showed that the response reconstruction improved considerably when measurements on the full in flight next assembly were available to use in the test, although noticeable error remained. It was shown that the DUT motion was not fully observable with accelerometers on the next assembly only, and this lack of observability caused error in response reconstruction. The third case study showed that all system modes could be observed if the environment was measured at only a few locations on the DUT. In this case the mode shape matrix was well-conditioned and the environment was estimated accurately.

It is unlikely that one will generally have full-field DUT environment data. This study has shown that it can be accurately estimated, although for the system studied here it was necessary to measure the environment on the next assembly as well as with a few accelerometer channels on the DUT.

Acknowledgments The authors gratefully acknowledge the Department of Energy's Kansas City National Security Campus, operated by Honeywell Federal Manufacturing & Technologies LLC, for funding this work under contract number DE-NA0002839.

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