Chapter 4
Toward Generalized MIMO Random Vibration Specifications

Ryan Schultz and Garrett Nelson

Abstract For decades, single-axis vibration specifications have been derived using generalized approaches that may misrepresent the exact test article dynamics and detailed motion that would be experienced in a particular environment. These approaches were developed to represent a conservative envelope of responses which accounts for multiple sources of variability including unit to unit differences and variations in the service environment loading. Testing to generalized specifications has many benefits, including broad applicability to different test units and the ability to encompass multiple environments with one specification. However, this generalization comes at the cost of true replication of field environment responses in the laboratory. Rather than perfect field replication, the intent of most tests is to subject the test article to vibration that is similar to expected field environments and scaled to a conservatively high level in order to sufficiently stress the item.

Despite the growing popularity of multiple-input/multiple-output (MIMO) vibration testing, most MIMO tests utilize replication-style specifications that lack the ability to account for uncertainties often desired for acceptance, qualification, or production testing as is done for their single-axis counterparts. This is likely for two reasons. First, because MIMO testing can accurately replicate complicated environments, accuracy is made a primary goal of the test. Second, there simply are not many available methods to create or utilize generalized specifications for MIMO tests. This paper presents a method to create a generalized MIMO random vibration specification by incorporating an ensemble of responses and statistical conservatism in much the same way as traditional, single-axis specifications. However, the resulting specification should not be directly imported into a MIMO controller for use. A second step is presented which maps the generalized specification to inputs to the MIMO controller, capturing all the detailed, narrowband, test article-specific information required for a good laboratory test that still accounts for additional sources of variability.

Keywords MIMO · Random vibration · Vibration testing · Test specification · Multi-axis

Introduction

In MIMO vibration testing, a control system determines a set of shaker drives that will minimize response error at several response locations on the device under test (DUT). With a good test design and setup, MIMO control can accurately match the acceleration response at various locations or degrees of freedom (DOFs), potentially exactly mimicking the complicated response patterns that the DUT may see in a service environment. While a perfectly accurate test is appealing, it is often difficult or impractical to achieve for several reasons:

- There may be too few shakers (inputs) or shakers may be in poor locations
- There may be too few accelerometers (outputs) or accelerometers may be in poor locations
- The test specification may be incompatible with the DUT dynamics
- There may not be enough relevant field data available to make a MIMO-compatible specification

The first two issues can often be remedied with good test design methods. The last two issues are the real problems that make MIMO testing difficult to apply to many systems. Specifications must be compatible with DUT dynamics for the test to be controllable and for the test to be run efficiently. For example, if the specification has a peak response at 100 Hz, but the DUT has modes at 50 and 200 Hz, then it will likely be difficult to drive a peak at 100 Hz while suppressing motion at 50 and 200 Hz. Each difference in spectral shape can require significant drive energy and makes the test difficult to run, especially at high test levels, as the shakers need to force the DUT to move in ways that it does not naturally want to.

Ryan Schultz · Garrett Nelson
Sandia National Laboratories, P.O. Box 5800 – MS0346, Albuquerque, NM, 87185
e-mail: rschult@sandia.gov; gdnelso@sandia.gov

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Having enough field data to support MIMO specifications is another challenge. For the field data to be useful in traditional MIMO test specification derivation, the field data needs to be derived from a very similar system (similar frequencies, mode shapes, and damping as the DUT), have similar load paths as the MIMO test (shaker locations), and have similar boundary conditions. There are very few field tests that were designed and performed with MIMO testing in mind, and as a result, most of the available field test data will not be very useful for deriving MIMO specifications.

Another gap with traditional MIMO specification derivation methods is that they rely on a single data set (one test system, one environment). While this may allow the response of that particular test to be perfectly replicated in the laboratory, that is not always the goal of vibration testing. Often, the tests are not replication tests (where the field response is matched perfectly) but are qualification tests, where the goal is to subject the DUT to a conservatively high vibration environment that encompasses or represents a set or ensemble of environments, test articles, or test conditions. Statistics are computed on an ensemble of field data to estimate the maximum predicted environment (MPE) [1, 2], which is then used in test specifications to ensure the DUT can survive "worst case" conditions expected during a field environment.

What is needed for MIMO vibration specifications are techniques that can maximize the usefulness of the available field data and utilize multiple data sets together to estimate statistics. These techniques should:

- Allow multiple data sets to be combined, even if the gauge locations differ from test-to-test (e.g., allow for spatial or zone averaging) [1].
- Be robust to changes in system dynamics, allowing for unit-to-unit or test-to-test shifts in modes without overly penalizing levels or variance estimates used in MPEs (e.g., allow for frequency averaging).
- Result in specifications that are testable by typical MIMO control systems (e.g., can be translated or mapped to narrowband power spectral densities (PSDs)).
- Allow for a small number of specifications (individual PSDs) to meet the intent of the test. A small number of frequency-averaged PSDs is easy to communicate, document, and compare against test results.

The benefits of simplification and general applicability are highlighted in Figure 1 and Figure 2 which show traditional, replication-style MIMO specifications (Figure 1) vs. the proposed, generalized MIMO specifications (Figure 2). In traditional MIMO specifications, there is a unique narrowband PSD for each control DOF. With tests often using 20 or more control DOFs, the result is many, many specification PSDs. In contrast, with a generalized specification framework, specifications can be defined at zones and there can be far fewer zones than control DOFs. Those specifications can be defined as frequency-averaged quantities, further reducing the number of specification PSDs and increasing the applicability of those specifications to other variations of the system. It may even be possible to reduce the generalized MIMO specification PSDs for a system down to a familiar breakpoint table which can be reported in test documents as the size of the specification goes down to: N_{zones} x $M_{frequency\ bands}$ where $N_{zones} << N_{control\ DOFs}$ and $M_{frequency\ bands} << M_{frequency\ lines}$. Take for example a case with 20 control DOFs, 2000 frequency lines, 5 zones and 10 frequency bands. The traditional MIMO specification

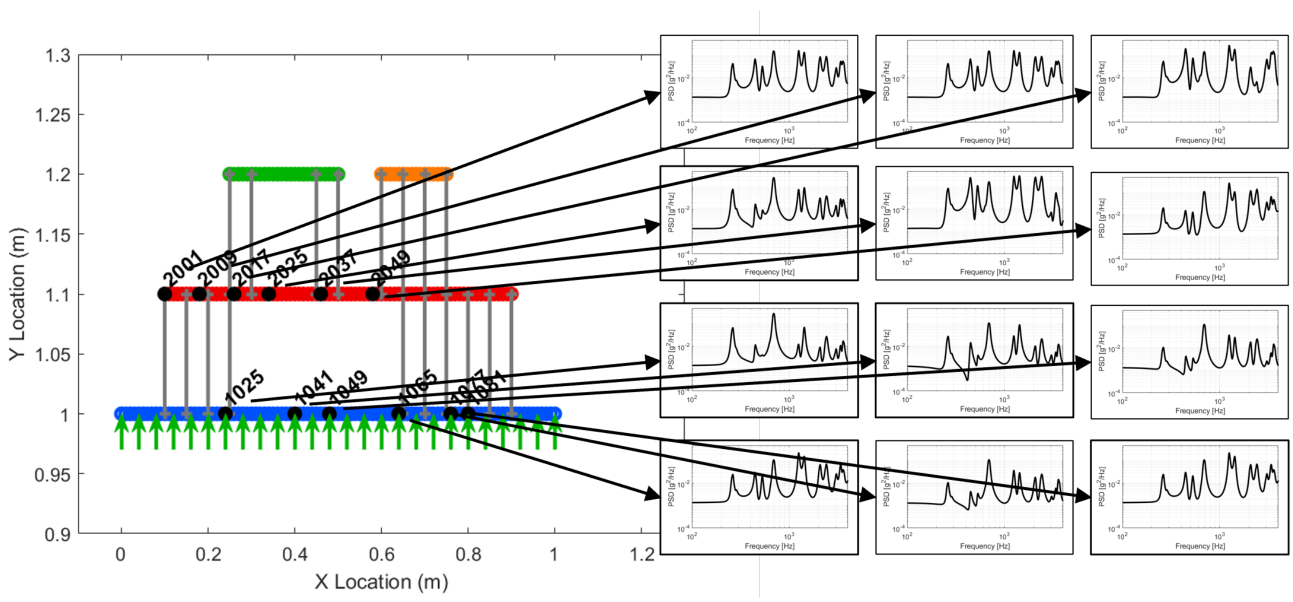


Fig. 1 Traditional MIMO specification example where each DOF has a unique specification PSD.

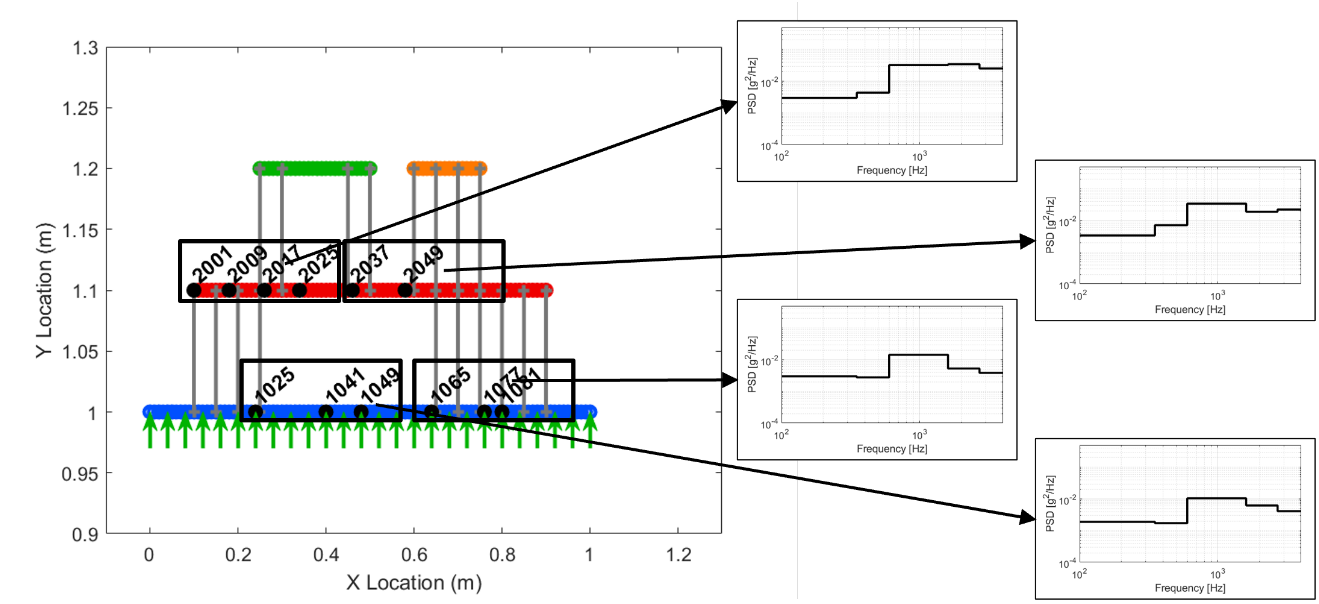


Fig. 2 Generalized MIMO specification example where DOFs are averaged into zones and each zone has a frequency-averaged specification PSD.

would entail 40,000 unique PSD values, ignoring cross-terms, (20 x 2000) whereas the generalized MIMO specification would only require 50 unique PSD values (5 x 10).

The overall goal in this work is generalized MIMO specifications: if MIMO specifications can be generated with those features, then they can likely be more broadly applied to more test articles, encompass more environments, and make use of existing field data sets. This paper presents an approach to enable generalized MIMO random vibration specifications by mapping from the generalized specifications to the specific PSDs required as control inputs to the MIMO controller. First the approach is discussed, including some gaps and assumptions. Next, an example problem is shown to demonstrate how this can be useful. Finally, results and conclusions are presented along with recommendations for future work.

Proposed Method For Generalized MIMO Specifications

There are two phases of the problem to consider: first, creating generalized MIMO specifications and then second, being able to use those generalized MIMO specifications in a test. The intent is to create a set of generalized specifications which may be derived from a various versions of a fielded system and various sets of field vibration data, and then to be able to use that for a specific DUT, which may have different dynamics. A translation or mapping is required to go from the generalized specification (which may be a small set octave-averaged, zone-averaged PSDs) to a set of MIMO controller PSDs (which may be a larger set of narrowband PSDs which reflect DUT-specific dynamics). Note that the generalization methods described in section 2.1 should not be used without the mapping methods described in section 2.2, they are intended to be used together to create the most representative test conditions.

Creating Generalized MIMO Specifications

The first step in the proposed method is to create a generalized MIMO PSD by averaging over gauges, frequencies, and data sets. Averaging over gauges allows the number, location, and direction of gauges to vary somewhat test to test while still being reasonable to compile together. This softens the problem by saying the average response of a set of gauges in a region or zone of the DUT should match an averaged specification. The same zone-averaged specifications could be applicable to multiple variants of a DUT even if the gauge layouts change somewhat test to test. It is useful to note that many single-axis specifications are exactly this, where there is just one zone (the base/input to the system), and multiple sensors (all mounting locations) are averaged together to represent this zone.

Averaging over frequencies allows the dynamics to vary between field test to field test and field test to lab test without unnecessarily forcing the dynamics of the field system onto the lab DUT. The intent here is to allow the DUT to vibrate as it naturally would in service (and not as some other field-test system vibrated in service). Frequency-averaging can be accomplished by using octave averaging or any band-wise frequency averaging scheme. Note that the term octave averaging is used in this paper, even if the frequency bands are not necessarily octave spaced. Ideally, the frequency bands would encompass dynamic features of interest and allow those features to change in frequency somewhat without moving to an adjacent band. This softens the problem by saying the average response across a range of frequencies in a given octal band should match rather than saying every single frequency line should match field vs. lab. Note that there will be a balance of generalization vs. accuracy in choosing frequency bands. Too narrow of a band means that features can cross band edges and are no longer mappable from field to lab. Too wide of a band means that too many features will be in the band and may average together and make it difficult to enforce desired motion on the lab DUT.

Specifications can be made conservative by computing statistics (mean and variance) over multiple data sets and using those statistics to estimate the MPE. It is important to note that the statistics may be very different if computed with narrowband PSDs vs. octave-averaged PSDs (i.e., the variance in octave-averaged PSDs is much lower than narrowband PSDs due to frequency shifting). As a result, for MIMO applications it is recommended for statistics to be computed on octave-averaged quantities.

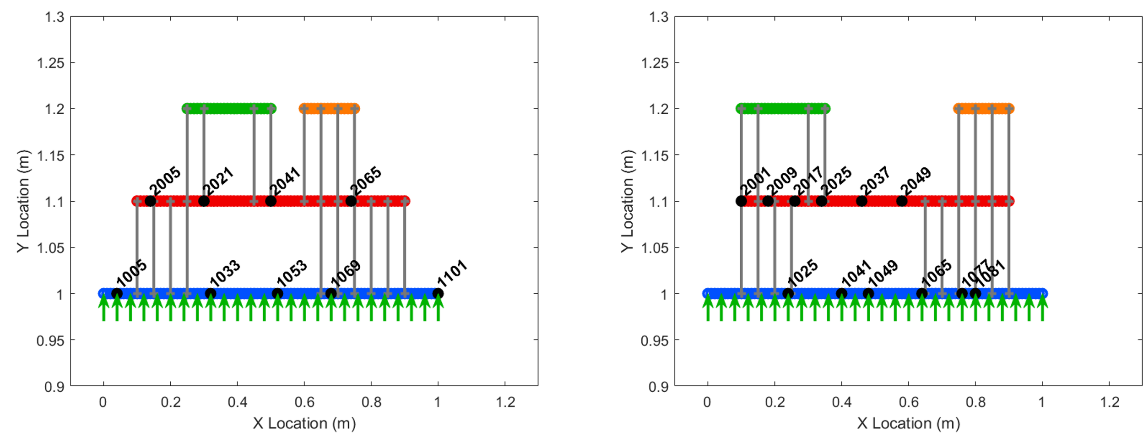
An example process for creating generalized MIMO random vibration specifications would be:

1. Compile the set of relevant field test data. Any test on a similar system to the DUT in a similar environment should be included to maximize the use of available field test data.
2. Determine frequency bands for octave averaging. These bands should encompass dynamic features in the field systems and field data and the lab DUT. Ideally, bands would have similar features (i.e., modes) for both the field and lab systems. Bands can be determined by examining field test PSDs and lab test system identification or frequency response function (FRF) data.
3. Determine groups of gauges to put into zones for spatial averaging. These should be groups with similar locations on similar components or hardware features, but the gauge locations within each zone can be different from field test-to-field test or field test-to-lab test. The goal is to have the response be similar across gauges in the zone. Appropriate zones can be determined by inspection (knowledge of the system construction) or by examining modal or field test data.
4. Apply octave averaging to the field test data, resulting in, for example, $1/3^{rd}$ octave PSDs for each field test at each DOF.
5. Apply zone averaging to the field test data, resulting in, for example, a $1/3^{rd}$ octave PSD for each zone.
6. Compute the mean and variance from the set of zone-averaged, frequency-averaged PSDs across all field test data sets. Do typical due diligence in understanding the distribution, identifying outliers, etc.
7. Use the mean and variance estimates to estimate the MPE for a given level and confidence interval, for example P95/50. Add additional scaling to account for deterministic factors such as known system-to-system differences. The resulting scaled P95/50 level averaged PSDs would be the MIMO specifications.

This process is illustrated in Figure 3 and Figure 4 which shows two variations of a system, System A and System B, and the associated field data. Suppose field data only exists for System A but a lab MIMO test needs to be performed on System B. System B has somewhat different dynamics and instrumentation from System A. Those differences are highlighted in Figure 5a where the response of each system is overlaid and shifts in dynamics in the ensemble of data are distinctly visible. Simply controlling the lab test of System B to responses from System A would result in an inaccurate and inappropriate test – forcing System B to response exactly as System A had in a field environment and not the way System B would have if it had been in the same environment. Instead, the System A field data can be averaged over zones of like-DOFs (shown in the black boxes in Figure 3) and averaged over frequency bands (shown in the averaged PSD in Figure 5b). Those averaged quantities are then transferrable to the lab test of System B.

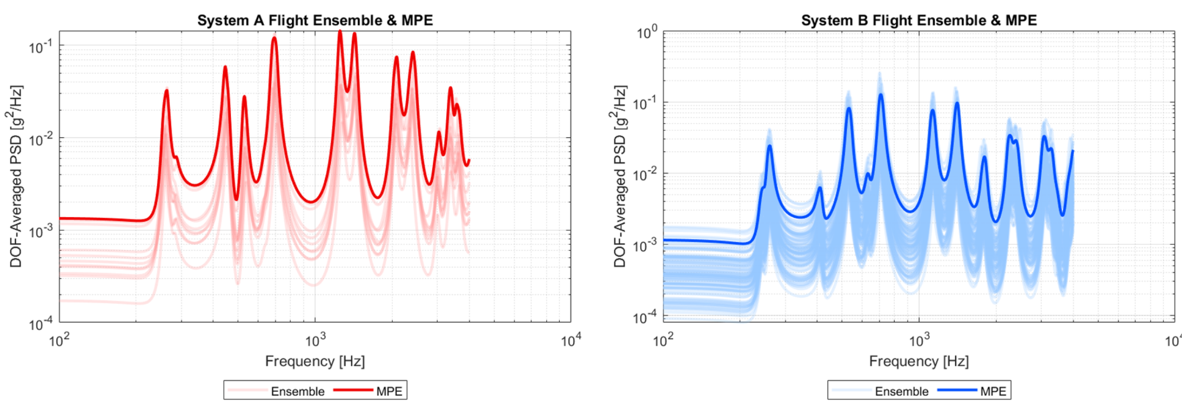
Converting The Generalized MIMO Specifications To Testable PSDs

The averaged or generalized MIMO specifications are not directly usable, meaning you cannot simply import them into a controller and run a meaningful test. This is for a few reasons. First, controllers need narrowband PSDs as specification inputs, so the octave-averaged specification needs to be converted to narrowband. Second, the specification is in terms of zone-averaged PSDs, so it needs to be converted to the DOFs in the DUT. And third, we want to incorporate the dynamics



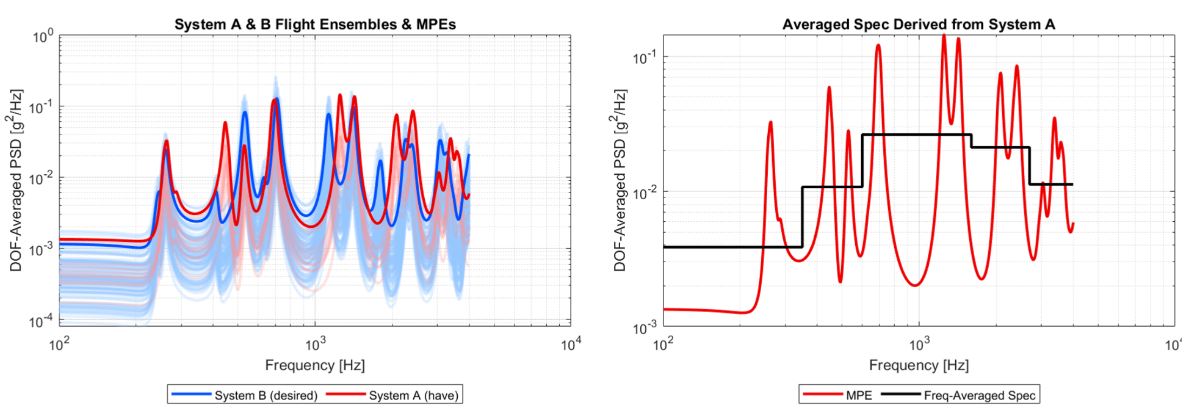
(a) (b)

Fig. 3 Two configurations of an example multi-component dynamic system with DOF grouped into zones as shown in black boxes. (a): System A. (b): System B.



(a) (b)

Fig. 4 Example field response and MPE for System A and B. (a) Ensemble of field data and a P95/50 MPE for System A. (b) Ensemble of field data and a P95/50 MPE for System B.



(a) (b)

Fig. 5 Example field response, MPE, and specification for System A and B. (a) Comparison of System A and B field data to show dynamic differences. (b) Example System A MPE and octave-averaged PSD.

of the DUT as much as possible, so it is helpful to use some DUT test response as a basis for converting from the specification octave PSDs to the narrowband PSDs needed for the controller. If available, DUT field response would be the most representative, but DUT lab system identification response can work in many cases as well.

Generalized specifications are beneficial in that they are simple, averaged representations of response. However, that simplification requires we “fill in the blanks” with lab DUT-specific information to perform a meaningful test. This can be done by utilizing the DUT system identification test data (PSDs) as a set of basis functions. Those DUT PSDs contain all the DUT-specific frequency, mode shape, and damping information we want to incorporate into the MIMO test.

The basis PSDs allow the generalized PSDs to be mapped to narrowband PSDs with the following process:

1. Create the set of basis PSDs for the lab DUT (a set of $N_{control\ DOFs}$ x $M_{frequency\ lines}$ narrowband PSDs). This most easily obtained from system identification test data on the lab DUT, but may come from other sources.
2. Octave average the basis PSDs to the same frequency bands as the generalized MIMO specification PSDs.
3. Zone-average the octave-averaged basis PSDs, making the averaged basis PSDs comparable to the generalized MIMO specification PSDs (size N_{zones} x $M_{frequency\ bands}$)
4. Compute the difference between the generalized MIMO specification PSDs and average basis PSDs as a set of scale factors (size N_{zones} x $M_{frequency\ bands}$).
5. Interpolate the scale factors to narrowband frequency lines (size N_{zones} x $M_{frequency\ lines}$). This can be done with, for example, spline interpolation on each zone’s PSD scale factors.
6. Apply the narrowband scale factors to the basis PSDs for each DOF in each zone where the DOFs in the same zone are scaled the same. This results in a set of basis PSDs which are scaled to match the generalized specification in terms of location (zone) and frequency band (size $N_{control\ DOFs}$ x $M_{frequency\ lines}$)

Note that the objective of the above process is to scale the narrowband basis PSDs such that they match the zone-averaged and octave-averaged PSDs in the specification. The interpolation of scale factors from octave to narrowband can cause the octave-averaged scaled basis PSDs to be slightly different from the desired specification PSDs, so the process can be repeated for a few iterations and get better agreement.

The mapping from generalized averaged specification PSDs to testable narrowband PSDs is shown in Figure 6, Figure 7, and Figure 8. Figure 6 shows the averaged specification PSD along with the narrowband PSD it was derived from (the System A MPE PSD) in comparison with the System B basis PSD. Note the differences in spectral content between the System A MPE and System B basis PSDs. Also note how the frequency bands capture similar features in both sets. For example, the band around 1000 Hz captures three peaks in both the System A MPE and the System B basis.

Figure 7 shows an example of the octave and narrowband scale factors used to scale the basis PSD to the averaged specification PSD, including each scaling iteration. Figure 8 shows the resulting mapped specification in both narrowband and octave-averaged form compared with the basis PSD and desired octave-averaged specification. Two key observations:

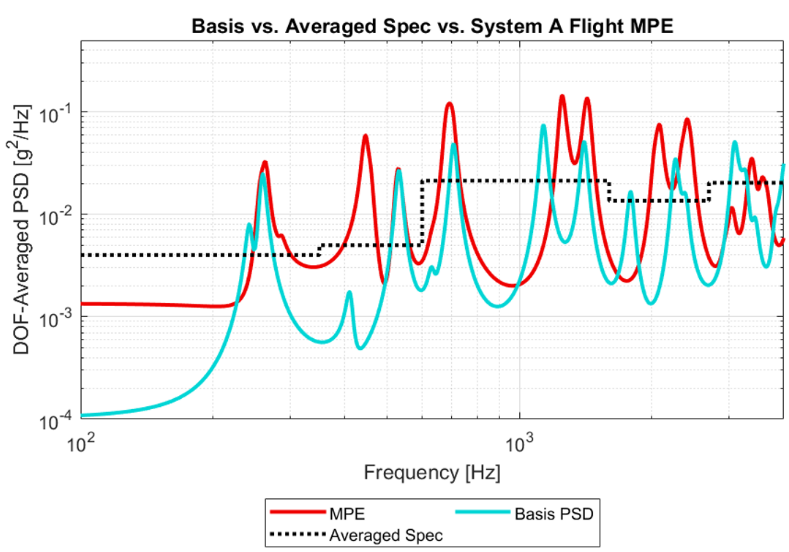


Fig. 6 Comparison of the System A MPE narrowband PSD vs. the System B basis PSD with the octave specification PSD to show band edges .

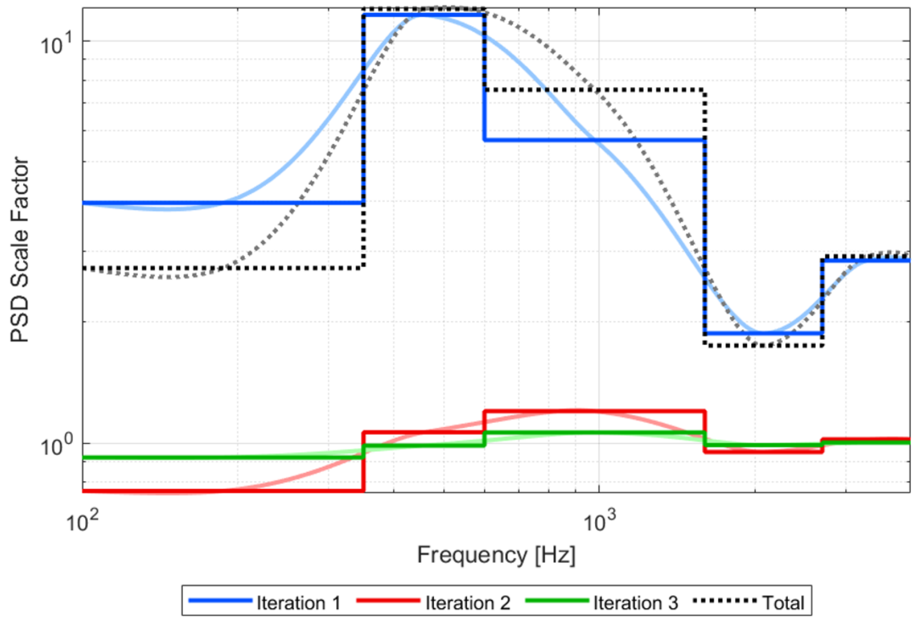


Fig. 7 Scale factors between octave specification and octave-averaged basis and those scale factors interpolated to narrowband frequency lines. Straight lines indicate octave band scale factors and smoothed curves indicate the narrowband scale factors from interpolating the octave scale factors. Black is the total scaling after all iterations. Colors show individual iterations which converge to unity scaling after a few iterations.

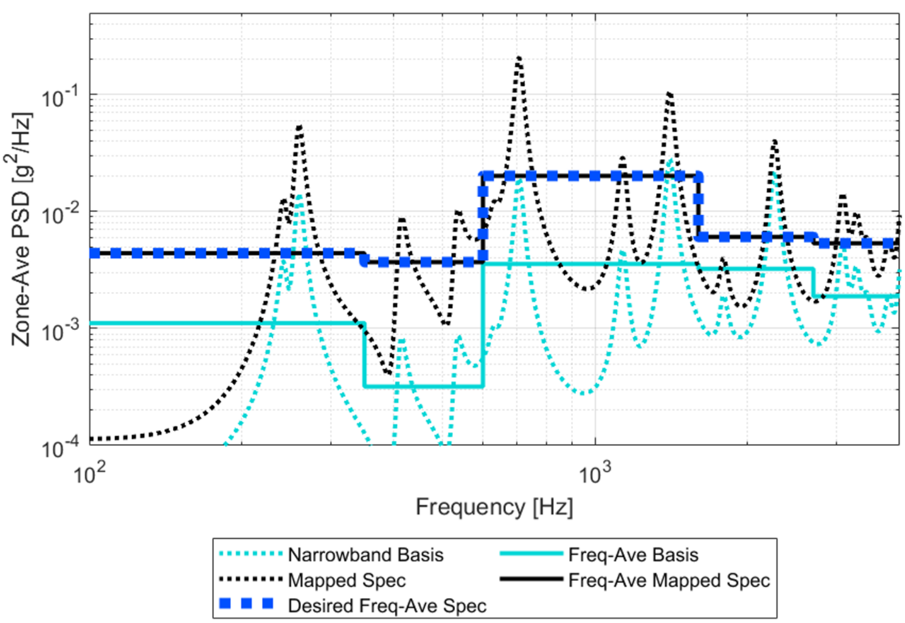


Fig. 8 Basis PSD scaled to match the averaged specification PSD.

first, the mapped specification matches the desired specification at each octave band, meaning the energy is appropriately captured frequency band by frequency band. Second, the details in the System B basis PSD are generally preserved after scaling, indicating that the System B modes will be appropriately excited during MIMO testing. Of course, there is the caveat that the basis PSDs must reasonably match the field PSDs for those modes to be appropriately excited, so some attention should be paid to obtaining useful basis PSDs. Importantly, these mapped specifications will not force System B to vibrate exactly like System A did in flight, rather they will force System B to vibrate approximately like System B would have in flight.

Simulation Demonstration

The generalized MIMO specification and mapping process will now be demonstrated on an example numerical system. Two versions of the system model were created, System A and System B, as shown in Figure 3. They have nominally the same construction with four beams connected via spring elements. The bottom beam has a pressure load applied in the field environment (e.g., the outer surface of a missile in flight). System B has a different configuration of the green and orange components compared with System A, as well as some minor changes to stiffness and damping. The intent is for System A and B to be similar in overall character while having some minor differences in modes that may be expected in real world examples, reflecting the differences due to unit-to-unit variability or different design iterations. A field environment is simulated with a partially-correlated pressure load applied to the bottom beam [3, 4], to simulate a free-flight environment. Multiple field tests are simulated with randomly varying load amplitude to provide an ensemble of responses. The MPE is then computed from the ensembles as shown in Figure 4. Note that in practice, there will not be field data for System B (the DUT), but here it is useful to have that truth data for comparison with the mapped specifications and test results.

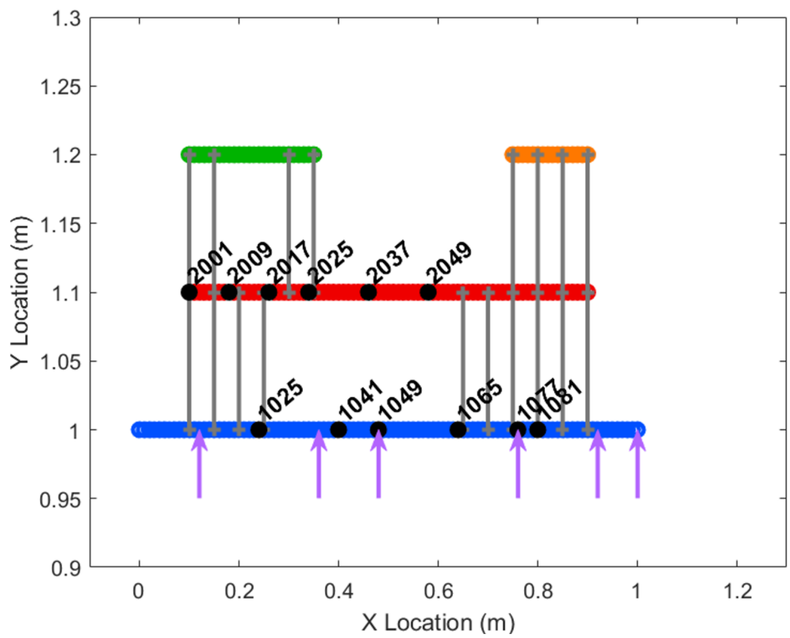


Fig. 9 Six shakers located on System B.

A MIMO test on System B is simulated by applying six point forces to the bottom beam in the same way that shakers would be set up in an Impedance-Matched Multi-Axis Test (IMMAT) [5, 6, 7]. The number and location of forces (shakers) is important but outside the scope of this paper. In this case, six forces at the locations shown in Figure 9 were sufficient to accurately mimic the response of the simulated field environment.

The basis PSD is obtained by simulating a system identification test on System B, where uncorrelated inputs were applied at the six shaker locations. Next, the basis PSDs are averaged over frequencies and over DOFs per the zones shown in Figure 3, where the field test has 9 DOFs and the lab test has 12 DOFs. Next, the averaged specification, derived from the System A field tests, is mapped to System B test control PSDs by scaling the System B basis PSDs. An example of this scaling is shown in Figure 8. The result of that mapping is a set of 12 narrowband PSDs which comprise the System B MIMO test control PSDs (PSDs that would be provided as specifications to the controller).

The System B MIMO test is simulated using the System B FRFs (from the simulated system identification test) and the mapped test control PSDs. The overall results of that test are shown in Figure 10, where the narrowband and frequency-averaged specification is compared with the test response. The test controlled reasonably close to the specification PSD over all DOFs and zones of DOFs, though perfect control is not always possible due to test setup details (such as shaker locations) or incompatibility of the specifications with the DUT dynamics (such as due to disparate scaling DOF to DOF or zone to zone).

Benefits of this generalized specification and mapping approach are shown in Figure 11 and Figure 12 where the test results are compared with the true System B field response ensemble and MPE. The traditional MIMO specification approach is shown in Figure 11, where the test control PSDs (specifications) are derived directly from the System A flight MPE. As a result, the test response closely mimics System A responses and dynamics, which does not match the true System B flight response. Several peaks are forced to respond at different amplitudes and different frequencies. In contrast, when specification PSDs are generalized and mapped via the System B basis PSDs to get the control PSDs, as shown in Figure 12, the resulting test response is a much closer match to the true System B flight response. Peak frequencies and other spectral features are maintained. Of course, there will be some amplitude error due to the mismatch in response amplitude between System A and B and error in the mapping process, though it should be noted that the amount of error will be problem dependent.

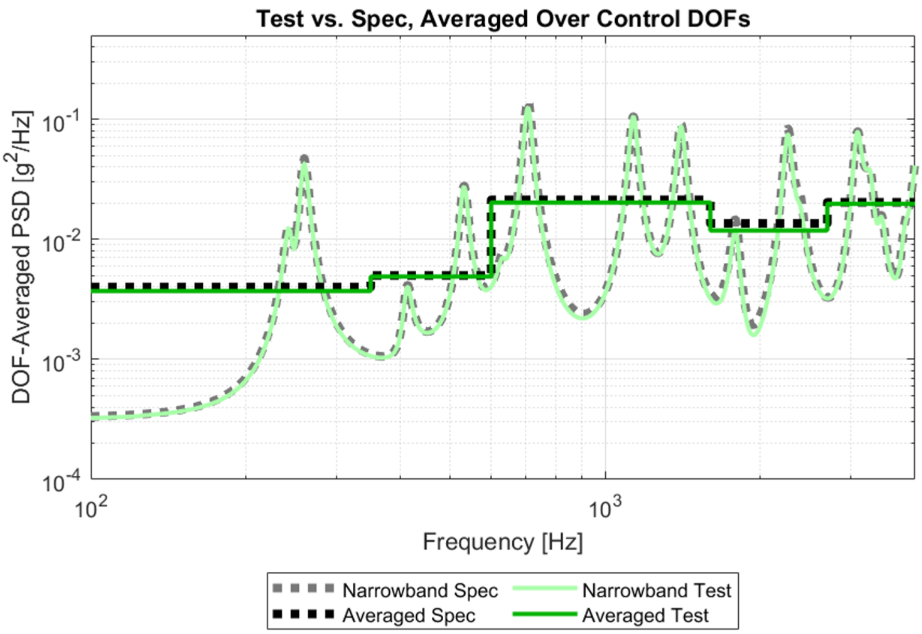


Fig. 10 Simulated MIMO test results shown as the response PSD averaged over all 12 control DOFs.

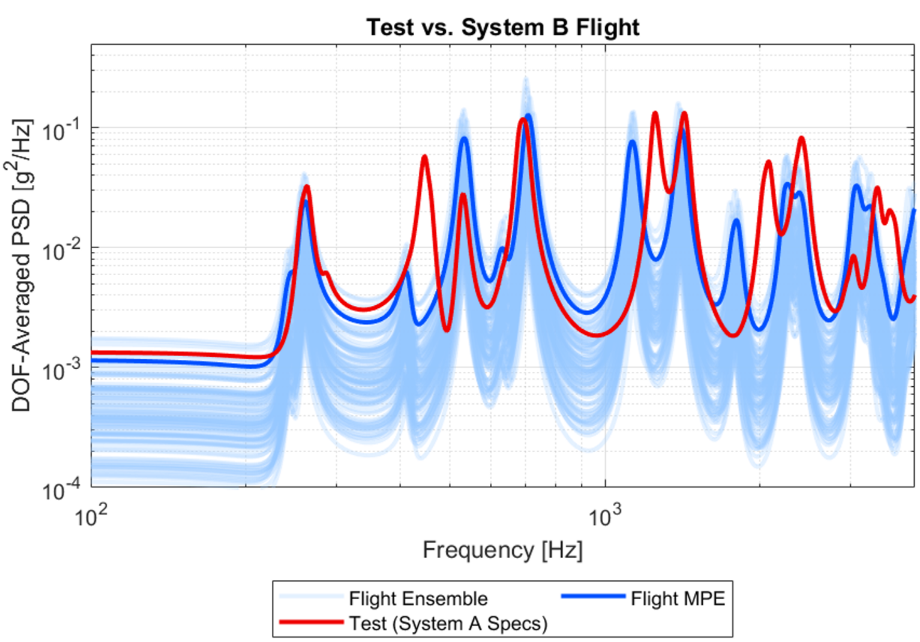


Fig. 11 Simulated MIMO test results compared to the System B field test ensemble and MPE with MIMO specification PSDs taken directly from System A field test data.

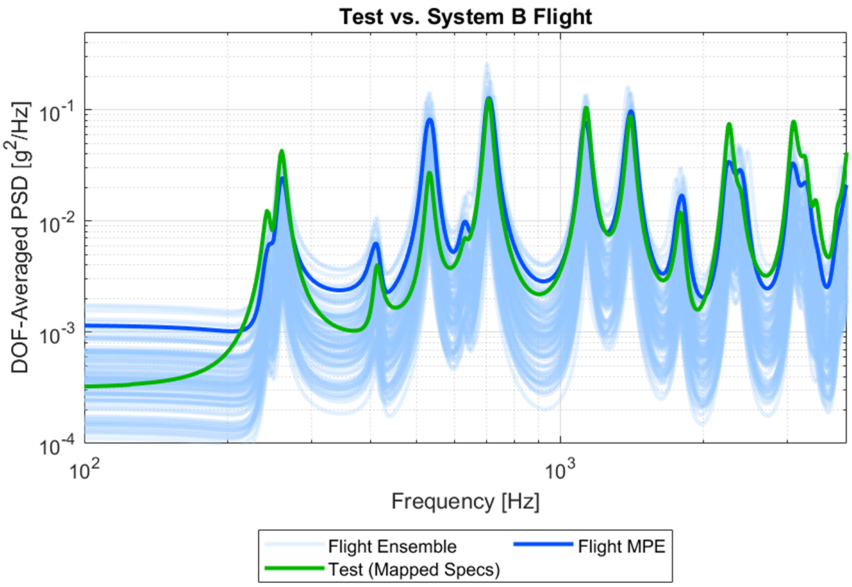


Fig. 12 Simulated MIMO test results compared to the System B field test ensemble and MPE where the generalized MIMO specifications process is used.

Discussion

The presented change to how MIMO specifications can be defined opens up several opportunities for the test engineer to change settings or methods that affect test results. In general, it is better to have more options with MIMO as it gives the test a better chance to run efficiently and accurately compared with a traditional approach where much of the design and nuance is buried in the original field data. That said, several things should be considered before using this process in a test.

The generalized PSDs described in this paper are only auto-power spectral densities and cross-terms are ignored as the concepts of frequency and zone averaging are likely not applicable to coherence and phase. Cross-terms must be incorporated with the control PSDs in order for a MIMO test to be run. These could come from the basis PSD data, or from an example field test, or from some other method [5, 8, 9]. In any case, the test engineer should understand what cross-term approach is appropriate and how it may affect the DUT response.

The choice of the basis PSDs will have a significant impact on the test and every effort should be made to make the basis PSDs as similar as possible to the expected DUT field response. This includes DUT dynamics, boundary conditions, load paths, input correlations, and levels as these factors will all affect DUT response. Naturally, some of these are easier to control or implement than others, but all should be in mind when designing a test. While the example here used uncorrelated inputs to create the basis PSDs, that may not be the right choice for some systems or environments.

Care should be taken when defining DOF zones and frequency bands to ensure similar features are grouped together. If defined poorly, responses could be mapped to the wrong locations, or the wrong spectral features could be amplified or attenuated. A detailed understanding of the system construction, instrumentation configuration, and system dynamics is required to ensure this process works as intended.

The process of mapping from averaged specifications to narrowband controller PSDs is a bit complicated and introduces some assumptions, including the method for interpolating from octave scale factors to narrowband scale factors. That said, it is believed this is better than simply assigning all frequency lines within a band the same PSD value as that will force an unnatural response on the DUT [10]. The test engineer should understand the various assumptions and make appropriate tradeoffs to ensure the intent of the test is met.

Conclusions

A limitation to widespread use of MIMO vibration testing is a lack of specifications to run tests, mostly due to the various requirements needed in a good MIMO test specification. This work aims to soften those requirements while still meeting the intent of the test: to subject the DUT to field-relevant vibration environments while letting the DUT dynamics exist as they would in service. This was accomplished by considering the MIMO specification to be an averaged quantity over both spatial locations and frequency ranges. In this way, DUTs with variations in instrumentation configuration or dynamics can still utilize the same test specifications. This has a few important benefits: it reduces the complexity of the specifications, allowing them to be tabulated or easily plotted, it does not require bespoke specifications for each test or DUT, and it allows multiple disparate field data sets to be combined into one set of specifications, maximizing the usefulness of limited field data.

These generalized specifications were then made usable via a novel mapping process where a set of DUT-specific basis PSDs are used to interpolate the generalized specifications across DOFs and frequency lines, resulting in a set of narrowband PSDs that can be imported as control PSDs in any MIMO controller. Importantly, as this mapping process utilizes DUT-specific basis PSDs, the control PSDs will still maintain the DUT's frequencies and mode shapes, allowing the DUT to vibrate naturally, as it would in service.

Of course, this step toward generalized MIMO specifications comes with drawbacks. Primarily, more effort is required to design the test setup to mimic in-service conditions as closely as possible. As the test setup will affect the basis PSDs, and the basis PSDs affect the control PSDs and therefore the test results, it is imperative that the test setup is designed with this process in mind. Pre-test predictions and sensitivity studies should be performed to verify the configuration and understand design tradeoffs. Similarly, by softening the constraints at the test specification, there is more user interaction and many options to change in the specification mapping process, which necessitates an expert in the loop.

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