Chapter 5
A Method for Response Replication at Component-Level in MIMO Random Testing

Enrico Proner and Emiliano Mucchi

Abstract This study focuses on Multiple-Input Multiple-Output (MIMO) vibration control tests, designed to accurately mimic a structure's operational vibration environment. In MIMO random testing, the Cross Spectral Density (CSD) between each pair of control channel plays a key role in the outcome of the test. In fact, the same Power Spectral Densities (PSDs) but with different CSDs may determine a different response on the Device Under Test (DUT) and different drives required to conduct the test. In this scenario, this paper proposes a methodology that aim to replicate a random vibration environment at the component level with minimal excitation. The approach pre-defines the phase and correlation of the reference Spectral Density Matrix (SDM) that maximize the response of the DUT. Then, the reference SDM is completed with the minimum PSD values ensuring that the structure's response in the laboratory aligns with the target vibration environment. This method enhances the consistency of laboratory tests with actual operational environments while minimizing the required energy, ultimately preserving the excitation system's integrity. The paper provides both a theoretical and an experimental analysis of the proposed methodology. The experiments are carried on specifically designed specimen excited by a multi-axis vibration environment, provided by a three-axis shaker.

Keywords MIMO · Random testing · Fatigue · Drive reduction · Environmental testing

Introduction

In the case of MIMO random control tests, the control target is a reference Spectral Density Matrix (SDM) comprising of Power Spectral Densities (PSDs), the diagonal elements, and Cross Spectral Densities (CSDs), out of diagonal elements [1]. The PSDs provide information about the energy of each control channels, the CSDs determine the correlation between different control channels, defined by means of phase and coherence [2]. Typically, only the PSD terms of the reference SDM are known. This is due to the legacy of single-axis testing procedures, which only require the PSD of the control channel, along side with the difficulty and impracticality of combining coherence and phase information from different operational conditions. In the recent past, several researches have focused on formalizing new methodologies to complete the reference SDM, exploiting the phase and coherence between the control channels to obtain more efficient tests [3, 4, 5, 6]. Moreover, the CSDs can have a significant effect on the DUT response [7, 8, 9]. At the authors knowledge, the available drive minimization techniques focus on minimizing the drives required to reach the test specifications, without considering the effects on the fatigue damage inflicted to DUT. Moreover, if the control location is on the DUT, the physical constraints of the DUT may not allow to achieve the minimum drives condition of the selected drive minimization technique. In this scenario, this paper proposes a novel MIMO random target generation technique, named *Minimum PSDs Method (MPM)*, which aims to minimize the energy required by the exciters to conduct the test while while preserving the damage inflicted to the DUT. The *MPM* predefines the phase and coherence between the control channels that maximize the response on the DUT for any given set of excitation PSDs. Then, the reference SDM is completed with the PSD values that generate the target vibration on the tested article. Because of the chosen phase and coherence the obtained PSDs are the minimum PSDs that generate the target vibration environment, dealing the same damage to component with minimal energy.

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The proposed Minimum PSDs Method

This section provides the mathematical implementation of a novel MIMO random target generation technique, namely *Minimum PSDs Method* (*MPM*). The concept of the method is to define the minimum PSDs of the reference SDM that allows to replicate a set of response PSDs on the DUT. The minimum PSDs can be obtained by using the Extreme Dynamic Response Method (EDRM) [7] to predefine the phase and coherence between the control channels. In fact, the EDRM aims to maximize the trace P of the response of the DUT, which, in the case of l control channels, can be written as follows:

$$P = \Sigma_{j=1}^{l} S_{yy,jj}^{ref} W_{jj} + 2\Sigma_{j=1}^{l-1}\Sigma_{k=j+1}^{l}\sqrt{\gamma_{jk}^2 S_{yy,jj}^{ref} S_{yy,kk}^{ref}}|W_{jk}|cos\left(\phi_{jk}-\xi_{jk}\right) \tag{1}$$

where

$$S_{yy,jk}^{ref} = |S_{yy,jk}^{ref}|e^{i\phi_{jk}} = \sqrt{\gamma_{jk}^2 S_{yy,jj}^{ref} S_{yy,kk}^{ref}}e^{i\phi_{jk}} \tag{2}$$

and

$$\mathbf{W} = \mathbf{F^H F} \tag{3}$$

in which $\mathbf{S_{yy}^{ref}}$ is the reference SDM, γ_{jk}^2 is the magnitude square coherence between the j^{th} and k^{th} channel, $\mathbf{F}$ is the transfer function matrix between the control and response point, ϕ_{jk} and ξ_{jk} are the phase between the j^{th} and k^{th} terms of the reference SDM and of the $\mathbf{W}$ matrix, respectively. .

For any given set of PSDs, Equation (1) is maximum when the coherences and the cosines are equal to 1. Therefore, to maximize the trace of the response SDM the following condition must be satisfied:

$$P\ is\ maximum\ \Leftrightarrow \begin{cases} \gamma_{jk}^2 = 1 \\ \phi_{jk} = \xi_{jk} \end{cases} \quad \forall j,k = 1:l, j \neq k \tag{4}$$

Furthermore, the *phase pivoting algorithm* can be used to ensure that the chosen phases determine a physically realizable target [6]. In particular, Equation (4) is applied to the $(l-1)$ terms with the biggest $|W_{jk}|$. The remaining terms are retrieved by respecting the existing phase dependencies ($\phi_{jk} = \phi_{ik} - \phi_{ij}$). Once the EDRM is used to retrieve the phase and coherence of the reference SDM, the l reference PSDs required to obtain the m responses can be calculated by numerically solving the following set of equations:

$$\begin{cases} S_{xx,11} = \Sigma_{j=1}^{l} S_{yy,jj}^{ref} G_{jj}^1 + 2\Sigma_{j=1}^{l-1}\Sigma_{k=j+1}^{l}\sqrt{\gamma_{jk}^2 S_{yy,jj}^{ref} S_{yy,kk}^{ref}} G_{jk}^1 cos\left(\phi_{jk}-\alpha_{jk}\right) \\ \dots \\ S_{xx,mm} = \Sigma_{j=1}^{l} S_{yy,jj}^{ref} G_{jj}^m + 2\Sigma_{j=1}^{l-1}\Sigma_{k=j+1}^{l}\sqrt{\gamma_{jk}^2 S_{yy,jj}^{ref} S_{yy,kk}^{ref}} G_{jk}^m cos\left(\phi_{jk}-\alpha_{jk}\right) \end{cases} \tag{5}$$

where $G^m = F(:,m)^H F(m,:))$ and α_{jk} is the phase angle of $G(j,k)$. In Equation (5) matrix G^m can be retrieved during the system identification phase of the MIMO random test, the coherences γ_{jk} and phases ϕ_{jk} are determined using Equation(4), $S_{xx,mm}$ are the target response PSDs on the DUT, and the only unknown terms are the reference PSDs $S_{yy,jj}^{ref}$. The solution of Equation (5) allows to find the minimum reference PSDs that determine the DUT target response. As a consequence, the MPM guarantee the reduction of the drives required to conduct the test while maintaining the response on the DUT constant.

Experimental Verification

The aim of experimental activity is to showcase the applicability and potential of the proposed procedure. The experiments are carried out on a specifically designed specimen, consisting of an cantilever aluminum beam. The main objective of this test case is to demonstrate the ability of the *MPM* to mimic the fatigue damage of a target vibration environment using the least amount of energy possible. The target vibration environment is obtained from a test conducted with flat uncorrelated PSDs as excitation. Furthermore, the responses and the drive PSDs obtained with the target vibration environment and the MPM are compared to those obtained by conducting a test with the same specifications as the target vibration environment but correlated using the *Minimum Drives Method (MDM)*, which is a state of the art drive reduction technique. An additional test is performed by using the *MDM* to directly control the response of specimen to match the target vibration environment. The tests are conducted using the three-axial electrodynamic shaker available at the University of Ferrara. The data acquisition hardware and vibration control software used are the SCADAS Mobile SCM202V (V8 input and DAC4 output modules) and MIMO Random of Simcenter Testlab, respectively.

The specimen chosen for the tests is a cantilever aluminum beam with rectangular section and U-shaped notches near the fixed end. A lumped mass of 0.47 kg is placed on the free-end of the beam, to tune its natural frequencies in a convenient frequency range. The detailed geometry of the beam is depicted in Figure 1.

Figure 2 shows the specimen mounted on the shaker table and the sensors used during the tests. Two accelerometer are used. The *tab* accelerometer is placed near the fixed end of the specimen and the *tip* accelerometer is placed on the lumped mass. The *tip* accelerometer is the location where the vibration environment is measured to obtain the PSDs to be replicated using the *MPM*. Additionally, a strain gauge rosette is bonded near the notches, allowing to study the fatigue damage inflicted to the specimen. The accelerometers are ICP (Integrated Circuit Piezoelectric) triaxial accelerometers by *PCB Piezotronics*, model 356B21. The strain gauge used is a 3/120-RY93 by HBM, which allows to measure a multi-axis strain state. In order to conduct the fatigue analysis, the obtained stress state is converted into an equivalent uni-axial stress state using the Von Mises PSD method.

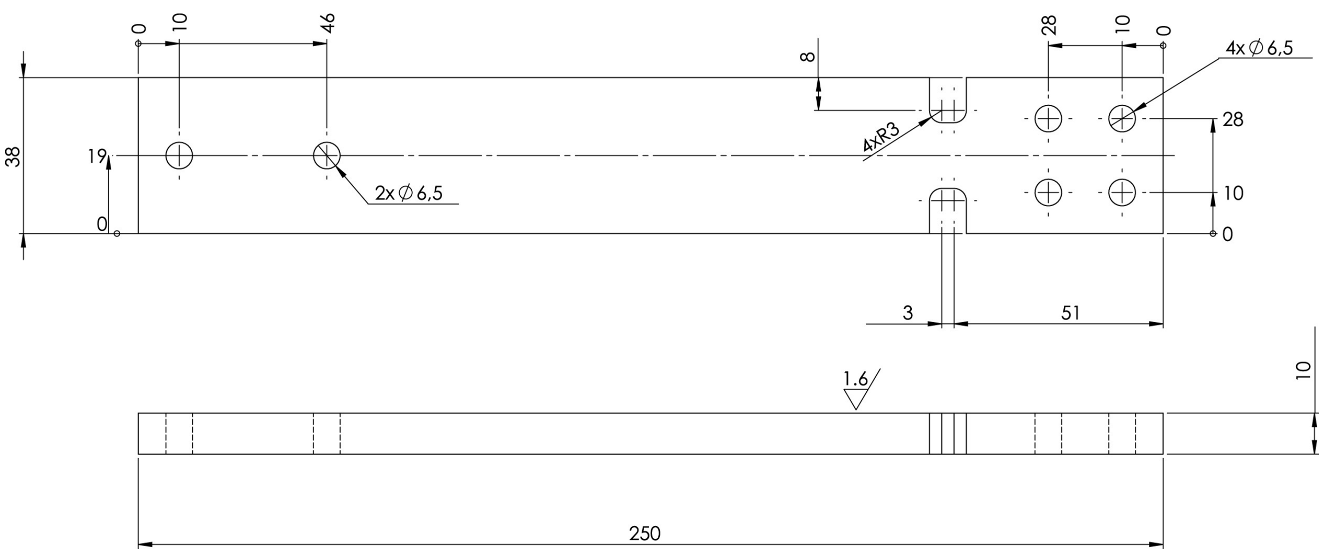

Fig. 1 Detailed geometry of the specimen used.

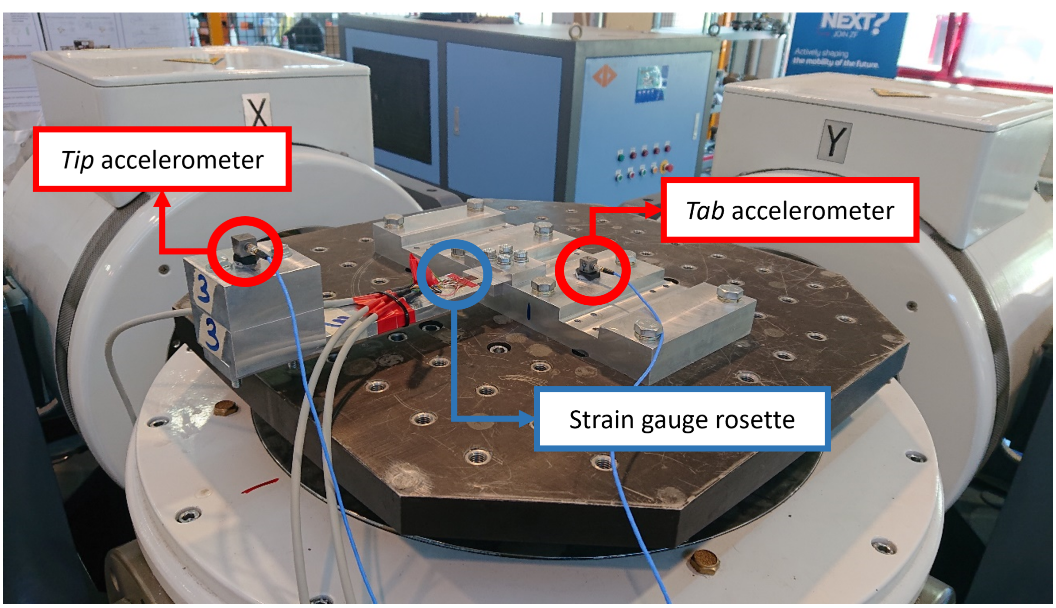

Fig. 2 Specimen mounted on the shaker table and sensors used.

The test specification used to generate the target vibration environment consists of flat uncorrelated profiles of 1 gRMS in the frequency range $[20-600]\ Hz$, replicated simultaneously on each direction the *tab* accelerometer.

Figure 3 depicts the comparison of the response PSDs measured the *tab* accelerometer. The three subplots on the left show the acceleration PSDs along the three orthogonal direction measured by the *response* accelerometer. These include the target PSD (blue solid line), the PSD obtained using the *MPM* (orange dashed lines), the PSD obtained in the test performed controlling the *tab* accelerometer and using the *MDM* (yellow dotted-dashed line) and the PSD obtained when controlling the *tip* accelerometer and using the *MDM* (purple dotted line) . On the right side of Figure 3, the bar plot reports the acceleration RMS value measured by the *tip* accelerometer for each direction.

It can be observed that test performed using the *MPM* and the test performed using *MDM* and controlling the *tip* accelerometer (labeled *MDM tip*) both provide a good match with the *target* measurement PSDs. Furthermore, the acceleration RMS obtained by the *MPM* and the *MDM tip* tests is close to the *target* RMS for each direction. In particular, the target acceleration RMS are $8.57g$, $20.01g$ and $5.30g$, the *MPM* test achieved an acceleration RMS of $8.89g$, $20.1g$ and $5.05g$, the *MDM tip* test achieved an acceleration RMS of $8.77g$, $19.71g$ and $5.43g$ along the X, Y and Z axis, respectively. On the other hand, the *MDM tab* test, performed controlling the *tab* accelerometer by using the *MDM* and with the same excitation PSDs of the *target*, determined different results. In particular, the response of the specimen is lower than the response obtained during the *target* measurement. The difference is significant especially along the Y and Z direction where the acceleration RMS determined during the *MDM tab* test is $12.73g$ and $3.77g$, respectively. In fact, changing the correlation of the refer-

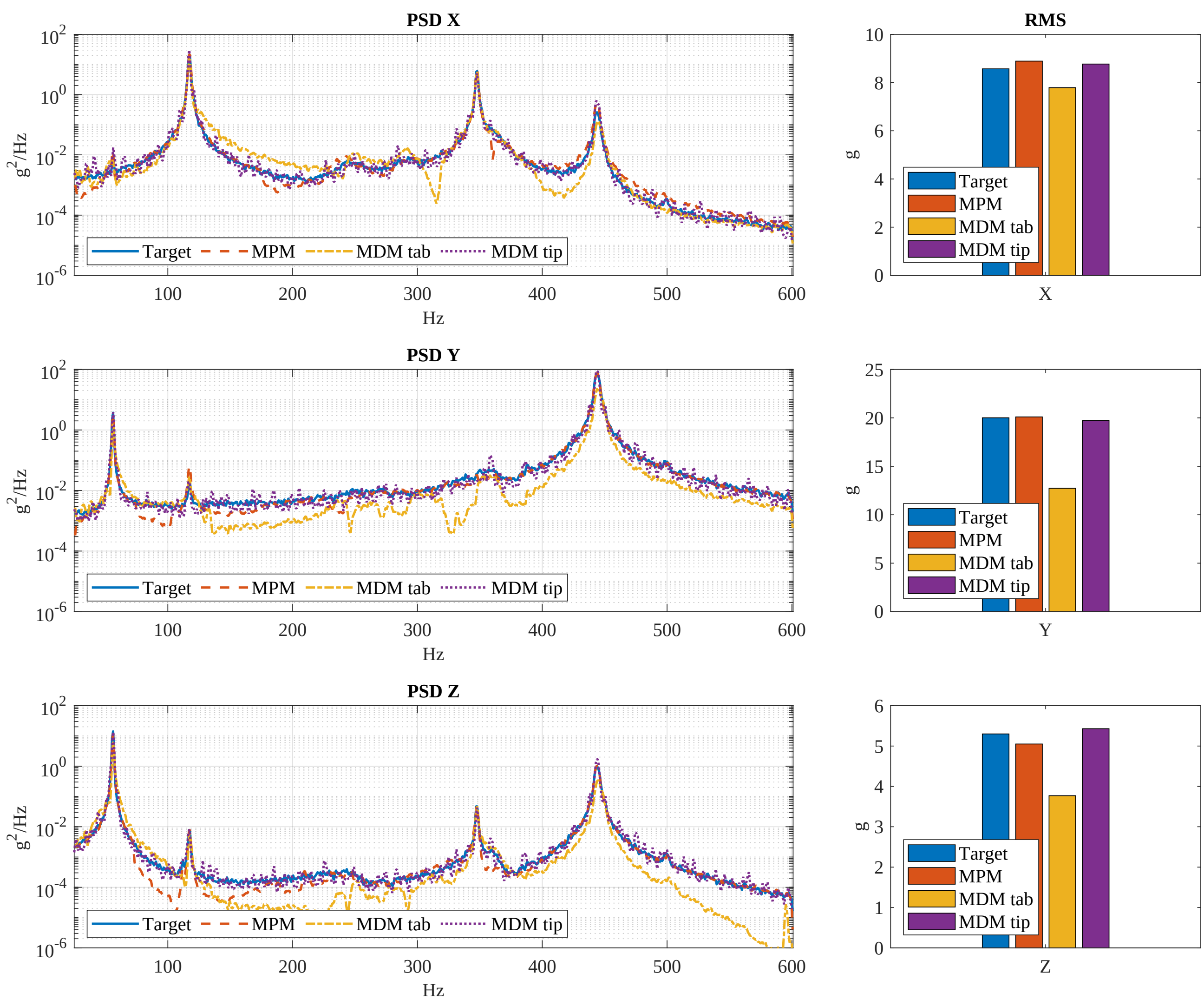

Fig. 3 Acceleration PSDs measured by the *tip* accelerometer during the tests and their respective RMS value

ence SDM can significantly affect the response intensity on the DUT. Therefore, not adjusting the reference PSDs may cause significant deviation from the target vibration environment. These results highlight how the *MPM* is capable of replicating the target vibration environment. Furthermore, Figure 4 presents the comparison between the drives required to conduct each test. The plots on the left show the drive PSDs, the bar plot on the right illustrates the difference in voltage RMS value in dB with respect to the drives required by the *target* measurement. It can be noticed that the *MPM* achieved the greatest drive reduction for each one of the three drives. In particular, the *MPM* achieved a $1.46dB$, $0.89dB$ and $1.02dB$ reduction for drive 1, 2 and 3, respectively. On the other hand, the *MDM tab*, achieved a reduction of $0.23dB$, $0.11dB$ and $0.25dB$ for drive 1, 2 and 3, respectively. It should also be noticed that the *MDM tip* test determined a significant increase in the energy required to conduct the test. This is caused by the physical constraints of the system. In fact, when controlling the response of the specimen, the control target must abide to the dynamic behaviour of the specimen. In particular, at the resonance frequencies the response of the DUT is driven by its natural mode shape and the phase and coherence of the response are independent of the phase and coherence of the multi-axis excitation. Figure 5 shows the comparison between phase of the CSDs obtained during the *MDM tip* test (red crosses) and the phase required to achieve the minimum drives according to the *MDM* when controlling the *tip* accelerometer (green circles) at each natural frequency. The phase obtained during the test is determined by the dynamic behaviour of the specimen and, as a consequence, does not always match the phase required by the *MDM*. This phase mismatch between the control and target phase determines an increase in the required drives. Moreover, at the

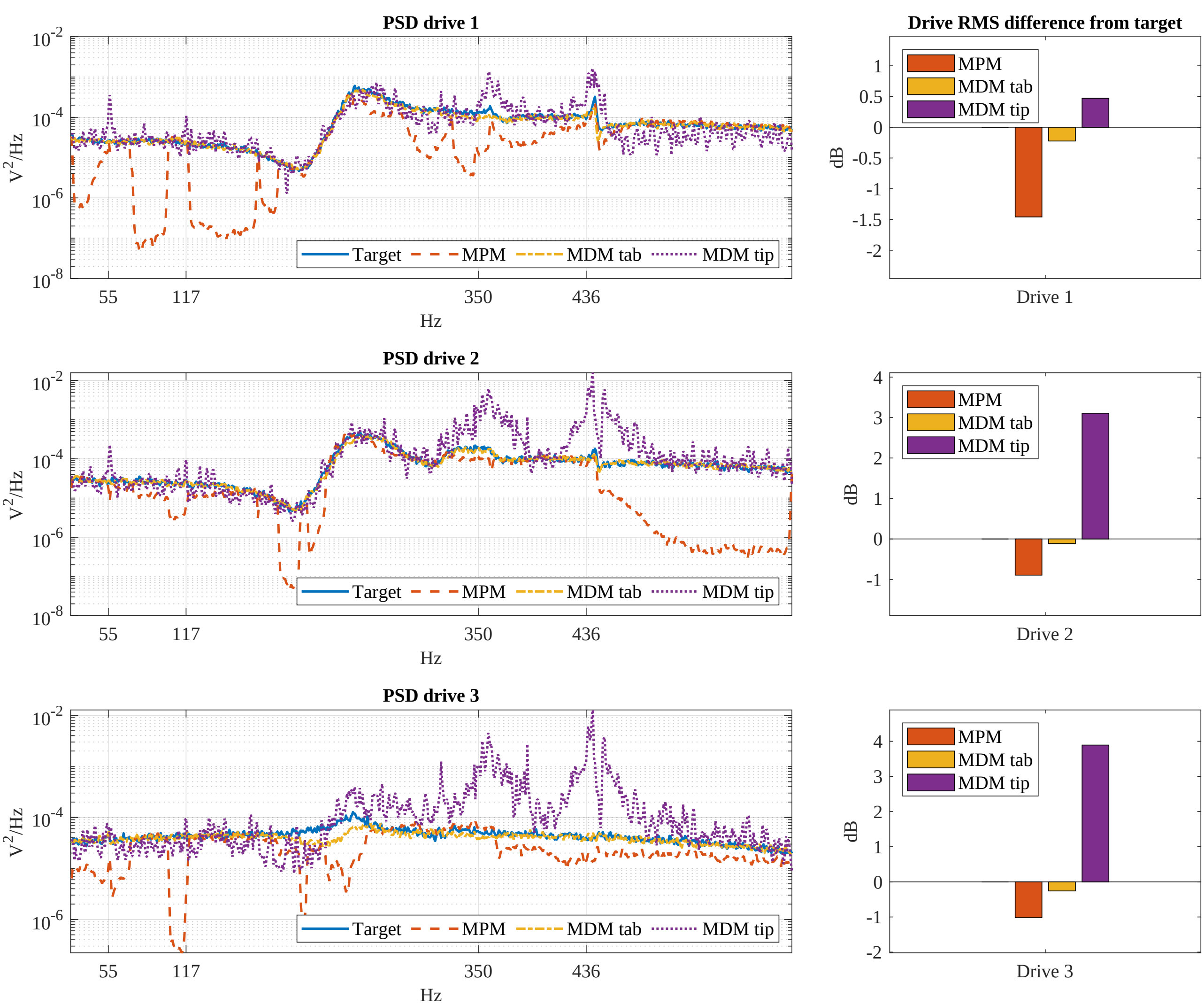

Fig. 4 Comparison of the drives required by the *target measurement*, the *MPM* and the *MDM* when controlling the *tab* and *tab* accelerometer (dB reference: drive 1: 0.23 V, drive 2: 0.22 V, drive 3: 0.16 V).

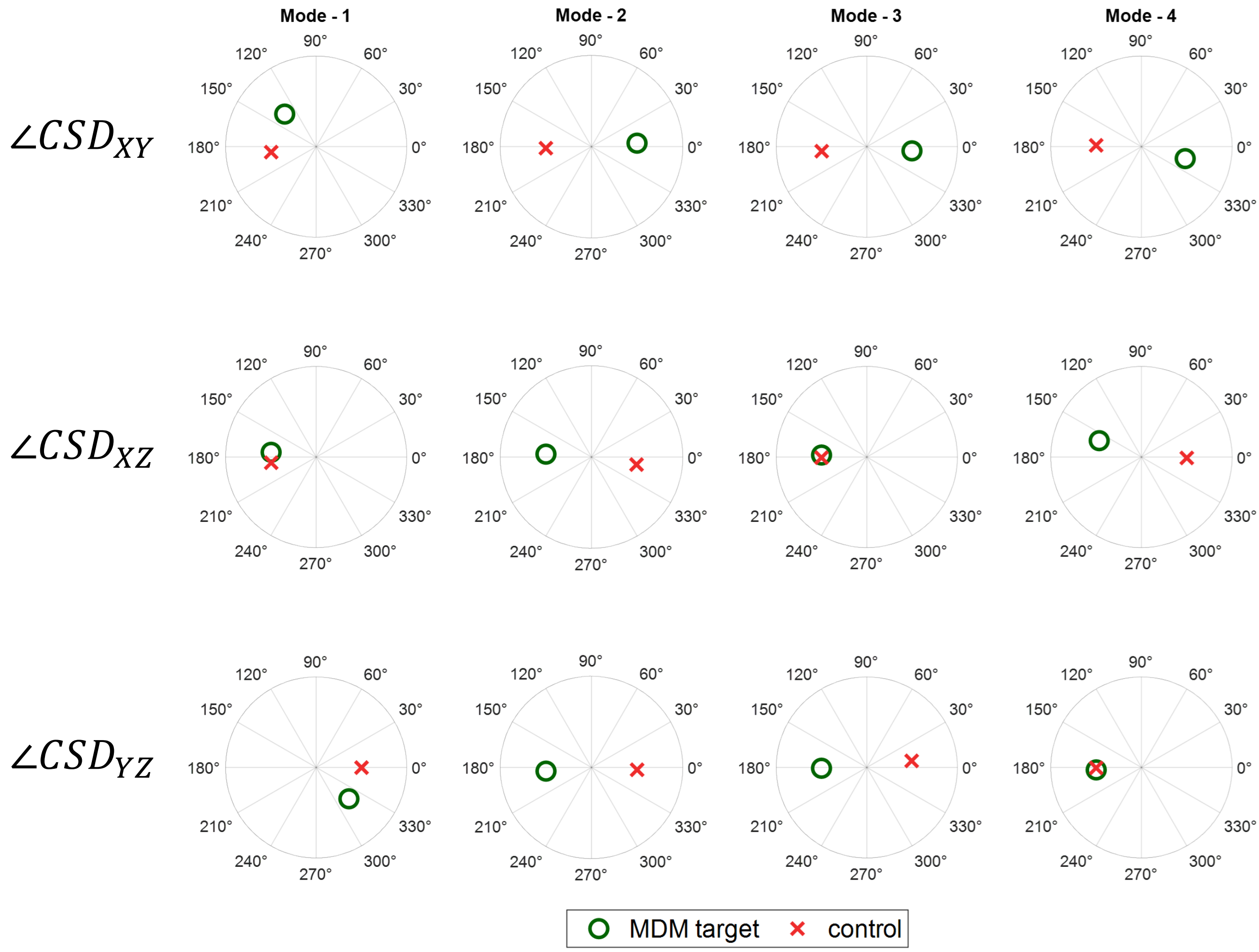

Fig. 5 Phase obtained during the *MDM tip* test on the *tip* accelerometer (red cross) compared to the phase required to achieve the minimum drives (green circles) at each resonant frequency.

second, third and fourth natural frequencies, the phase obtained during the *MDM tip* test are shifted by $180\ deg$ with respect to the *MDM* target. As a consequence, the drives are maximized at these frequencies, as displayed in Figure 4.

The last comparison involves the analysis of the fatigue damage inflicted to the specimen by each test. During each test, the PSDs of the strains were measured using the strain gauge rosette placed at the notch of the specimen. The measured strains are then processed to calculate the Von Mises stress PSD experienced by the specimen. Finally, the Dirlik cycle counting method is used to obtained the amplitude probability distribution for each test and compute the fatigue damage [10]. The left plot of Figure 6 shows the amplitude probability distribution calculated from the Von Mises stress PSD obtained during the *target measurement* (solid blue line), the one obtained with the *MPM* (orange dashed line) and the one obtained with the *MDM* when controlling the *tab* accelerometer (yellow dotted-dashed line) and the the *tip* accelerometer (purple dotted line). The bar plot on the right shows the predicted time to failure (TTF) in hours, derived from each probability distribution. It can be noticed that the *MPM* and the *MDM tip* tests determined a probability distribution very similar to the *target measurement*. On the contrary, the *MDM tab* test determined a completely different amplitude probability distribution. As a consequence the predicted TTF of the *MPM* and the *MDM tip* tests are very close to the predicted TTF of the *target*. On the contrary, the *MDM tab* test determined a significant increase in the predicted TTF. This is to be attributed to the reduced response determined by the *MDM*, as depicted in Figure 3. In particular the predicted TTFs are $9.17h$, $10.46h$, $29.89h$ and $9.06h$ for the *target, MPM, MDM tab* and *MDM tip* test, respectively. In conclusion, the results of Figure 3, 4 and 6 highlight the capabilities of the proposed methodology. The *MPM* proved to be able to generate the same responses and inflict the same damage to the specimen as the *target measurement*, while minimizing the drives required to conduct the test. In particular, in comparison to the state of the art solution *MDM*, the *MPM* required lower drives to achieve the same response as the target

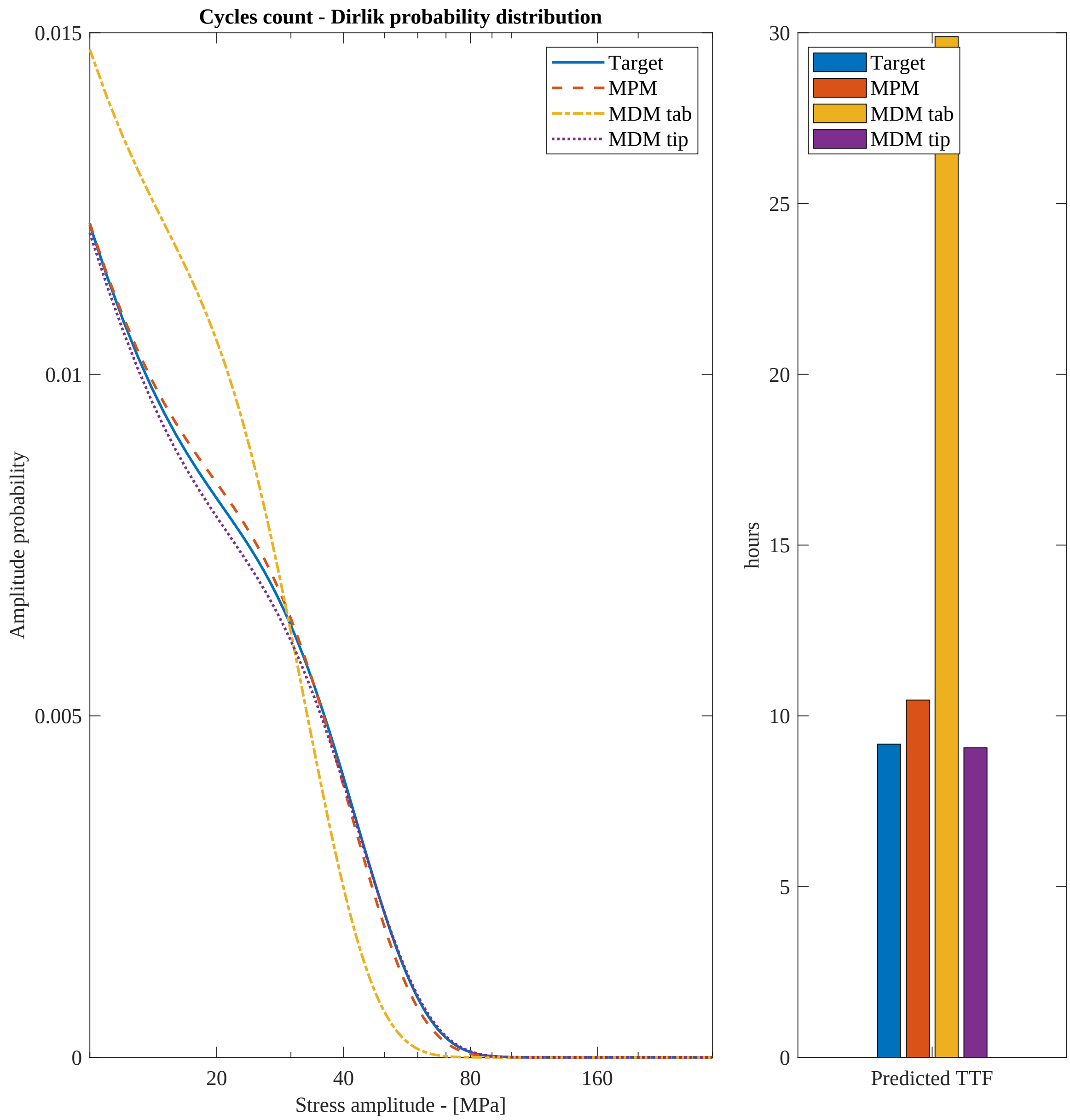

Fig. 6 Amplitude probability distribution (Dirlik) of the Von Mises stress PSDand expected time to failure (TTF) of each test

measurement. Moreover, the *MDM* was not able to simultaneously minimize the drives and maintain the *target* response on the specimen. On the contrary, the *MPM* has completely preserved the damaging effect of the target vibration environment, while demanding the lowest ammount of energy to conduct the test.

Conclusions

This manuscript proposes a novel methodology to define the target for a MIMO random control test, the *Minimum PSDs Method*. The method provides the minimum excitation PSDs to be applied to the DUT that allow to replicate the response of the DUT caused by a different set of excitations while minimizing the voltage required by the shakers to conduct the test. The effectiveness of the *MPM* has been verified through a dedicated experiment. Moreover, the *MPM* has been compared to the *MDM*, a state of the art solution for drive power reduction in MIMO random tests.

The *MPM* determined the highest reduction of the drives, while preserving the response and the fatigue damage induced in the specimen by the *target*. On the contrary, when controlling the *tab* accelerometer, the *MDM* required higher drives the *MPM* and determined a different response of the specimen which lead to a significant under-testing of the DUT. Moreover, when controlling the response of the DUT to match the *target* responses, the *MDM* was able to replicate the *target* responses. However, the drive minimization was not successful because of the physical constrains of the DUT, which lead to an increase in the required voltage. It can be concluded that the *Minimum PSDs Method* allows to replicate a set of responses on the DUT while reducing the total power required to conduct the test. The methodology may allow to re-design multi-axis random control tests, reducing the drive requirements while preserving the damaging effects of a targeted vibration environment.

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