Chapter 8
Investigation of Experimental Modal Substructuring for Constrained Modal Testing of Aerospace Structures

Adam Johnson and Ivan Kurakin

Abstract With the rapid expansion of the commercial aerospace industry, there is an increased need for more efficient and cost-effective approaches for component modal testing. Modal testing is typically used to reduce uncertainty in finite element model (FEM) predictions, proving to be a useful risk reduction tool in many programs whether it's a satellite preparing for a vibration qualification test or characterization of component structural dynamics for use in control loop simulation. Despite this, modal testing is often not a part of the workflow for smaller programs due to budgetary and schedule constraints. Relative to "free-free" tests, constrained modal testing poses significant challenges implementing a boundary condition that reveals accurate constrained modes for the unit under test (UUT). However, empirical substructuring methods exist for correcting an imperfectly constrained boundary condition in post processing of test data. These methods can be used to reduce the requirements on a test fixture, leading to shorter set-up times and higher confidence in the test results. However, many of these methods are very sensitive to measurement error and can involve logistical challenges. The authors of this report previously performed a study that identified experimental modal substructuring (EMS) to be a strong candidate for reducing boundary effects during a constrained modal test subject to practical budgetary and schedule constraints. This method is primarily selected due to its ease of implementation into an existing modal test plan. This paper represents a follow-on study whose focus is to better understand the reliability of using modal substructuring constraints (MSCs) to calculate constrained mode frequencies and identify post-processing techniques that improve confidence in the corrections. Since this study is focused on revealing the efficacy of MSCs under a typical modal test plan, no special accommodations are made to an otherwise typical modal test procedure outside of adding a few additional triaxial accelerometers to the fixture. The impacts of various UUT configurations & boundary conditions are studied. Corrected results are compared to a "truth" reference based on the same test articles grounded to a seismic table. In general, the use of MSCs is found to be highly effective at reducing the boundary effects for a constrained modal test. However, the approach is most beneficial for tests where a larger correction is needed. This study shows that test set-ups requiring finer corrections $<5\%$ may require a test plan that emphasizes enhanced measurement sensitivity and tighter requirements on test fixturing beyond the scope of this study. Finally, a workflow is proposed as to how substructuring uncertainty margins can be used to make decisions during a modal test campaign for FEM correlation.

Keywords Experimental Modal Substructuring (EMS) · Experimental Modal Analysis (EMA) · FEM Correlation · Modal Testing · Spacecraft Testing

Introduction

Constrained modal testing is often desired for improving the accuracy of FEA models such that accurate predictions of a structure's dynamics can be made. In the aerospace industry, some practical examples include:

1. Component level modal testing before integration into spacecraft or launch vehicle
2. Spacecraft modal screening ahead of a high-risk vibration test
3. Modal test of large appendage, such as solar array, for the purposes of guidance navigation and control (GNC) simulations
4. Aircraft component testing prior to integration (wing, blade, rotor, etc.)

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The largest challenge in constrained modal testing is properly controlling or quantifying the boundary condition. Various approaches exist to attempt to isolate the dynamics of the unit under test (UUT) from the fixturing to ensure accurate correlation of key properties in a FEM. However, all these approaches have their own associated uncertainties resulting in varying levels of sensitivity to detect relevant modeling errors in the FEM. The most common approach is to incorporate the fixturing structure into the FEM and include its properties in the model correlation. However, in doing this, the complexity of the model increases and a large part of the correlation often depends on the fixture properties and interface to the UUT, reducing the test's sensitivity to modeling errors within the UUT itself. Another approach is to perform a "free-free" test where the UUT is suspended by comparatively "soft" cables. This approach has a similar draw-back in that most of the modal strain energy is concentrated in joints or parts of the UUT that have little relevance to the constrained modes [2]. Recently, a family of substructuring approaches have been used to remove the effects of a flexible boundary or fixture in post-processing. These approaches fall into two primary categories: frequency domain substructuring and modal substructuring. All these approaches have robust mathematical frameworks allowing them to function well on idealized datasets. In practice, however, many of these approaches are highly sensitive to measurement error or require numerous drive points resulting in increased test scope. The authors of this report previously identified EMS as strong candidate for extensive and practical applications in the industry based on its ease of implementation into an existing test plan. This approach utilizes a synthesized modal parameter model reducing its sensitivity to measurement error and requires nothing more than a few additional accelerometers to measure the dynamic response of the fixture. However, preliminary findings published by the authors [2] show that the applied MSCs do not always reveal the correct constrained modes in practice. The motivation of this study is to quantify the uncertainty in predicted constrained modal frequencies when using MSCs and propose ways to improve confidence in the constrained modes. A MATLAB algorithm is developed to reduce some of the impacts of measurement noise on the results. It is important to note that this study applies typical modal test best practices to UUTs mounted to non-ideal boundary conditions. No special modifications are made to the modal test setup to minimize or optimize the error. This is done to reflect common industry applications of the approach in which budgetary or schedule constraints are present. In some industry applications, the need for boundary correction is not realized until after a test is complete.

Background

EMS is a substructuring technique that leverages modal parameters extracted during a modal test to build and modify a modal model. In the case of constrained modal testing, we are often interested in applying a constraint to the developed modal parameter model to remove the influence of the fixture on the UUT's modal response. In this study, this is referred to as a MSC. To do this, a simple requirement must be added to the scope of the modal test: the instrumentation set must include a set of accelerometers on the fixture itself that reveal the motion of the fixture that is to be constrained. The modal parameters that are extracted from the test include natural frequencies (ω_n) , modal damping fractions (ζ), and unity-mass-normalized modes shapes (ϕ) of the UUT and the fixture. These parameters can be derived in various ways but are most commonly derived by performing EMA transfer function curve fitting techniques. Ultimately, a modal model must be derived that consist of "N" normal modes that approximate the measured system drive point transfer functions using the form shown in Eq. (1). An example curve fit is shown in Fig. 1.

$$\frac{\boldsymbol{Y}_{\boldsymbol{k}(\boldsymbol{\omega})}}{\boldsymbol{f}_{\mathbf{j}}(\boldsymbol{\omega})}=\sum_{\boldsymbol{n}=1}^{\boldsymbol{N}}\frac{\boldsymbol{\phi}_{\boldsymbol{j},\boldsymbol{n}}\boldsymbol{\phi}_{\boldsymbol{k},\boldsymbol{n}}}{\boldsymbol{\omega}_{\boldsymbol{n}}^{2}-\boldsymbol{\omega}^{2}+2\boldsymbol{\zeta}\boldsymbol{\omega}_{\boldsymbol{n}}\boldsymbol{i}\boldsymbol{\omega}} \tag{1}$$

Once the modal parameters are extracted, a transformation matrix must be found and applied to the modal model that satisfies a compatibility condition as explained in [1]. As outlined by Allen and Mayes [3, 5], this is done by finding a matrix, $\boldsymbol{B}$, that spans the null space of a set of physical equations transformed to modal coordinates. In practice, this can be done using various approaches, however in this study, $\boldsymbol{B}$ is found in the null space of a set of constraint shapes transformed to modal coordinates as shown in Eq. (2). The constraint shapes, $\mathbf{U_c}$, are a set of orthogonal motions that describe the unique motions of the fixture calculated from a singular value decomposition of the extracted mode shapes at the fixture degrees of freedom [3]. The transformation matrix $\boldsymbol{B}$ can then be applied to the modal parameters as shown in Eq. (3) and Eq. (4). Note that the matrices $\boldsymbol{M}$ and $\boldsymbol{K}$ in these equations represent the modal mass and modal stiffness matrices such that $\boldsymbol{M}$ is unity and $\boldsymbol{K}$ is a diagonal matrix with entries of $\boldsymbol{\omega_n^2}$. To find the constrained modal parameters, an eigenvalue decomposition is performed on the transformed $\boldsymbol{K}$ and $\boldsymbol{M}$ matrices as in Eq. (5). The constrained modal frequencies are found directly from this eigenvalue decomposition, and the constrained mode shapes are found as shown in Eq. (6).

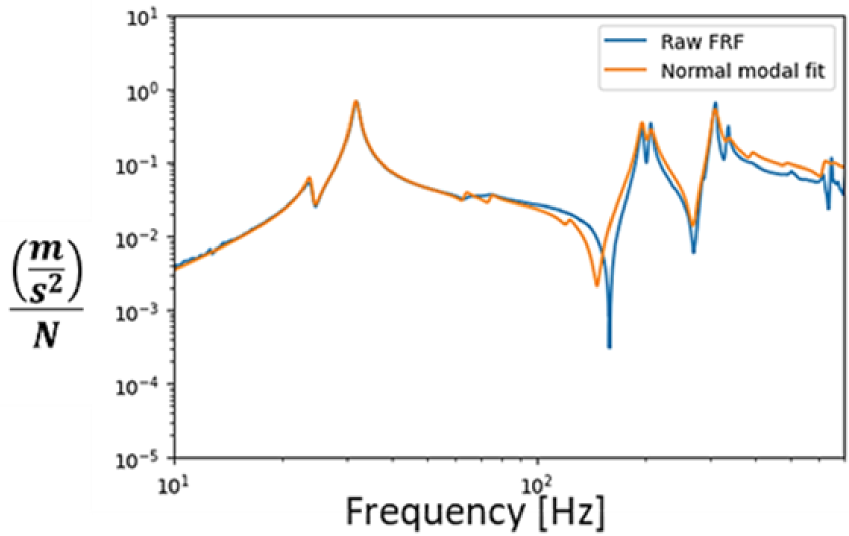


Fig. 1 Example Fit of Drive Point Transfer Function Using Normal Modes

$$\boldsymbol{B} = \emptyset(\boldsymbol{U}_c^T \boldsymbol{\phi}) \tag{2}$$

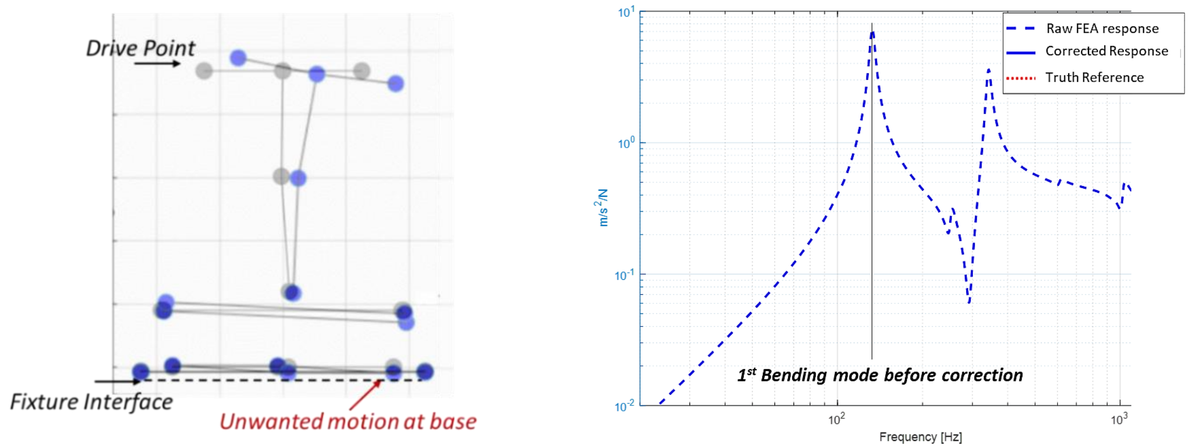
$$\boldsymbol{M_c} = \boldsymbol{B^T M B} \tag{3}$$

$$\boldsymbol{K_c} = \boldsymbol{B^T K B} \tag{4}$$

$$\boldsymbol{\omega_c^2}, \widehat{\boldsymbol{\phi}}_{\boldsymbol{c}} = \boldsymbol{eig}(\boldsymbol{K_c}, \boldsymbol{M_c}) \tag{5}$$

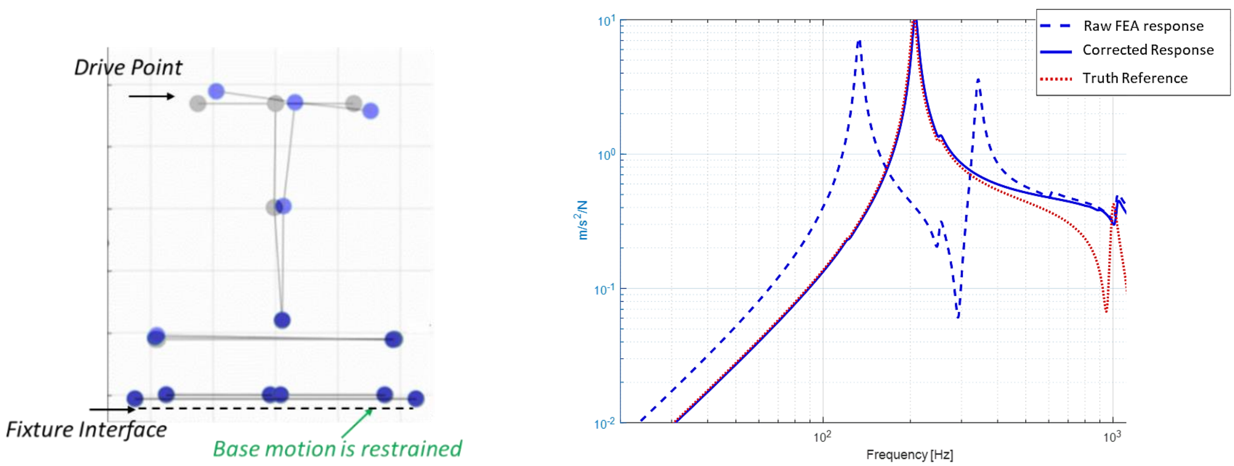
$$\boldsymbol{\phi_c} = \boldsymbol{\phi B} \widehat{\boldsymbol{\phi}}_{\boldsymbol{c}} \tag{6}$$

In the ideal case, this approach achieves zero motion at the constraint points and a well-formed constrained mode shape. However, in practice, various types of measurement error accumulate to make the method imperfect. An example of this can be seen by constraining the first bending mode of an idealized FEA model of one of the test items examined in this study. In this example, an FEA model of a UUT 3 (later described and shown in Fig. 5a) is joined to a constrained flexible plate such that the interface is allowed to move. A Guyan Static Reduction followed by an Eigenvalue Decomposition is performed to achieve a mass normalized modal model at a set of response locations for mode shape visualization. The flexible interface response (before correction) can be seen in Fig. 2. Note the presence of unwanted motion at the base interface (Fig. 2a). Figure 3 shows the results of applying a MSC to twelve degrees of freedom (DOFs) at the fixture interface (three directions at four locations). The response of the first bending mode is then fully corrected and matches what we expect in the ideally constrained structure overlaid in red (18-bolt ring fully constrained). Adding additional constraint points would improve agreement at higher frequencies, but this example is focusing only on 1st bending. Figure 4 demonstrates that adding measurement noise to the mode shapes and truncating the number of retained modes corrupts the process resulting in a constrained mode frequency that is considerably higher than the true frequency as well as an irregular appearing mode shape. In a real test scenario, similar behavior is observed when applying MSCs. Because of this error accumulation, care must be taken in selecting the minimum number of measurement degrees of freedom that need to be constrained for the modes of interest. Further, it can be advantageous to add constraint shapes one at a time to determine if any of the shapes are creating spurious solutions. However, even when exercising these cautions, a considerable amount of uncertainty can remain in the true constrained mode frequency. Because of this, one of the primary objectives of this study is to quantify uncertainty as it manifests in practical scenarios when applying MSCs to constrained tests.



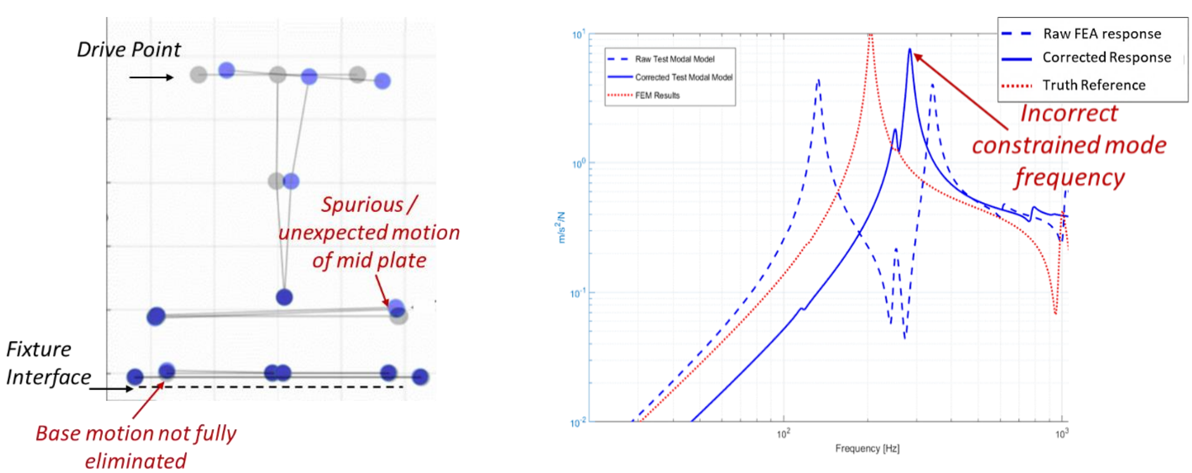
(a) Uncorrected Mode Shape (b) Drive Point X Transfer Function (DPTF)

Fig. 2 FEA Lateral Bending Response with Flexible Fixture Interface Motion (Before Correction)



(a) Corrected Mode Shape (b) Corrected DPTF

Fig. 3 Idealized Correction of 1st Bending Mode of FEA Model



(a) (b)

Fig. 4 1st Bending Mode Correction of FEA Model with Gaussian Noise and Truncated Modal Model

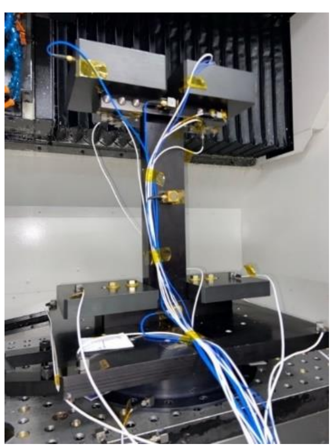
Test Setup

The authors of this study developed four UUTs with various boundary conditions resulting in eight distinct modal tests summarized in Table 1. For each test, the constrained mode frequency is estimated based on the algorithm discussed in the previous section. The calculated frequencies are compared to a "truth" reference to assess the resulting accuracy. The "truth" reference is based on the observed structural responses when the UUTs are clamped to a seismic table. For this study, no corrections are performed on the data received from the UUTs mounted to the seismic table, but it is expected that its efficacy as a "truth" reference below 200Hz is appropriate. For this reason, this study focuses on extracting constrained first bending modes in each axis of the UUTs. This provides fifteen samples of "primary modes" for comparison with the "truth" reference since one of the test items only has pronounced bending modes in a single axis. Some "Secondary Modes" (second bending and axial modes) were also processed during this study, but it was expected that the estimates of their constrained frequencies are significantly more erroneous.

Table 1 Executed Test Matrix

Test Number	DUT	Boundary Condition
1	Small Sat 1	1" Vibe Plate on Floor
2	Small Sat 2	1" Vibe Plate on Floor
3	Small Sat 3	1" Vibe Plate on Floor
4	Small Sat 1	Heavy Vibe Fixture on Springs
5	Small Sat 2	Heavy Vibe Fixture on Springs
6	Small Sat 3	Heavy Vibe Fixture on Springs
7	Small Sat 1	Heavy Vibe Fixture on Floor
8	Composite Blade	Steel Fixture with Elastomeric Element

UUTs 1-3 are various mass configurations of a small satellite dynamic simulator with adjustable ballast as shown in Fig. 5a. This test item is intended to identify logistical and technical challenges of performing constrained tests of smaller satellites. For this reason, a realistic separation ring simulator is incorporated into the design. The inclusion of a separation ring, is easily instrumented for EMS. However, for many frequency domain substructuring approaches, drive points would need to be measured in the narrow clearance of the separation ring which is found to be impractical. Additionally, the first three resonant modes and the CG of the satellite simulator are tuned to match a typical small satellite of that size. UUT 4 is a light composite blade clamped to a steel fixture, shown in Fig. 5b. This test item is chosen to add variance to the dataset and demonstrate an additional application for constrained modal testing such as identifying appropriate FEA properties for custom materials on aerospace components.



(a) Small Sat Dynamic Simulator with Adjustable Ballast: 3 Mass Configurations for 3 Unique Test Items

(b) Light Composite Blade Clamped to Steel Fixture on Elastomeric Elements

Fig. 5 UUTs used for this study

UUTs 1-3 are tested on two different flexible boundary conditions. The first, is bolted to a 1” plate that is resting on an uneven cement floor. This creates a quasi-linear flexible response of the plate boundary since the ground does not fully support the bottom of the plate. The second boundary condition for these test items is a 3” vibe fixture which rests on compression springs. This is selected to provide a repeatable linear boundary condition that is more controlled than the first. This boundary condition, with its springs, poses a challenge for the correction algorithm because it creates a nearly free-free response of the test item, but with rigid body modes within the measurable frequency range. A third boundary condition is tested on UUT1 where the 3” vibe fixture is placed directly on the uneven cement floor. UUT 4 is tested on a steel fixture on elastomeric elements.

Analytical Procedure

An analytical procedure is developed to reduce the influence of measurement noise on the resulting constrained frequencies. As discussed, when measurement noise is present, the extracted modal parameters and constraint shapes have inherent small errors. In these real scenarios, leveraging all the constraint shapes almost always results in an over-constrained response because of the inevitably imperfect orthogonalization of the mode shapes. In general, if the constraint shapes are removed one by one, at some point a “best estimate” of the constrained mode is found. Further, it was found in [2] that seemingly equivalent sets of constraint points result in slightly different solutions. The authors previously proposed in [2] that a proxy for accuracy of each result can be calculated by comparing the resulting shapes to an analytical or expected shape using modal assurance criterion (MAC) or cross orthogonality. If numerous perturbations of the constraint set are run in a sensitivity study, a repeatable constrained mode that matches the expected shape can indicate a “best estimate” solution. This procedure assumes that the influences of perturbations to the constraint during the sensitivity study are substantially larger than the true discrepancies between the expected and actual structural mode shape.

A MATLAB user interface is developed to implement this approach in an automated fashion. The primary motivation for the MATLAB tool is to prevent the user from selecting over-constrained solutions like that shown in Fig. 4 or solutions with insufficient constraint. The tool leverages a sensitivity-stabilization style approach by calculating all possible combinations of constraint points and constraint shapes and highlighting the calculated poles the match the expected analytical shape within a specified MAC tolerance. This approach toward exploring the solution space reveals repeatable solutions to the user to establishing a higher level of confidence in the selected solution. An example of this approach applied to real test data is shown in Fig. 6 applied to a test number four. In this case, the algorithm reveals a well-formed first bending shape is observable at the indicated frequency. The dotted blue line indicates the original test measurement of the modal response which is significantly softer than the “truth” reference shown in red. When the synthesized constrained modal model (blue solid line) is compared to the “truth” reference, it is found that the general shape of the frequency response function matches reasonably well, and the primary bending mode response frequency has an error of 4.5% compared against the “truth” reference.

Results

The procedure described in the previous section is repeated for all tests and all primary modes. Additionally, an attempt is made to extract the higher frequency “secondary” modes. The example shown in Fig. 6 is chosen as a representative example for two reasons: first, it reveals a level of error close to the average of all cases; second, it shows the power of MSCs to make extremely large corrections to a set of modal measurements. Test four places the UUT in a nearly free-free condition, with the first rigid body mode near 3Hz. Given this large dissimilarity from a constrained state, the algorithm is still able to resolve an approximate constrained response.

Figure 7 summarizes the accuracy of MSCs at resolving all primary mode frequencies. The grey data on the plot shows pre-correction results and the blue data shows the results after applying MSCs and estimating the constrained frequencies according to the process outlined above. On average, the mode frequencies are corrected to within 5% of the respective “truth” references and most cases fell within 8% (P80). Figure 8 examines the results from an uncertainty reduction perspective. It compares the needed improvement for each measured mode frequency to the amount of improvement that was actualized when the MSCs are applied. A key takeaway from this visualization is that applying an MSC to modal test results improves the estimate of the constrained frequency in almost all scenarios – even for secondary modes. Ideally, the data in Fig. 8 would follow a perfect 1:1 line that intersects zero (i.e. needed correction is perfectly achieved). However, the fit to the primary mode correction data shows an expected deviation from that ideal. This study finds that using MSCs to make corrections to modes that are already within 8% provides reduced benefit, but still improves results in most cases.

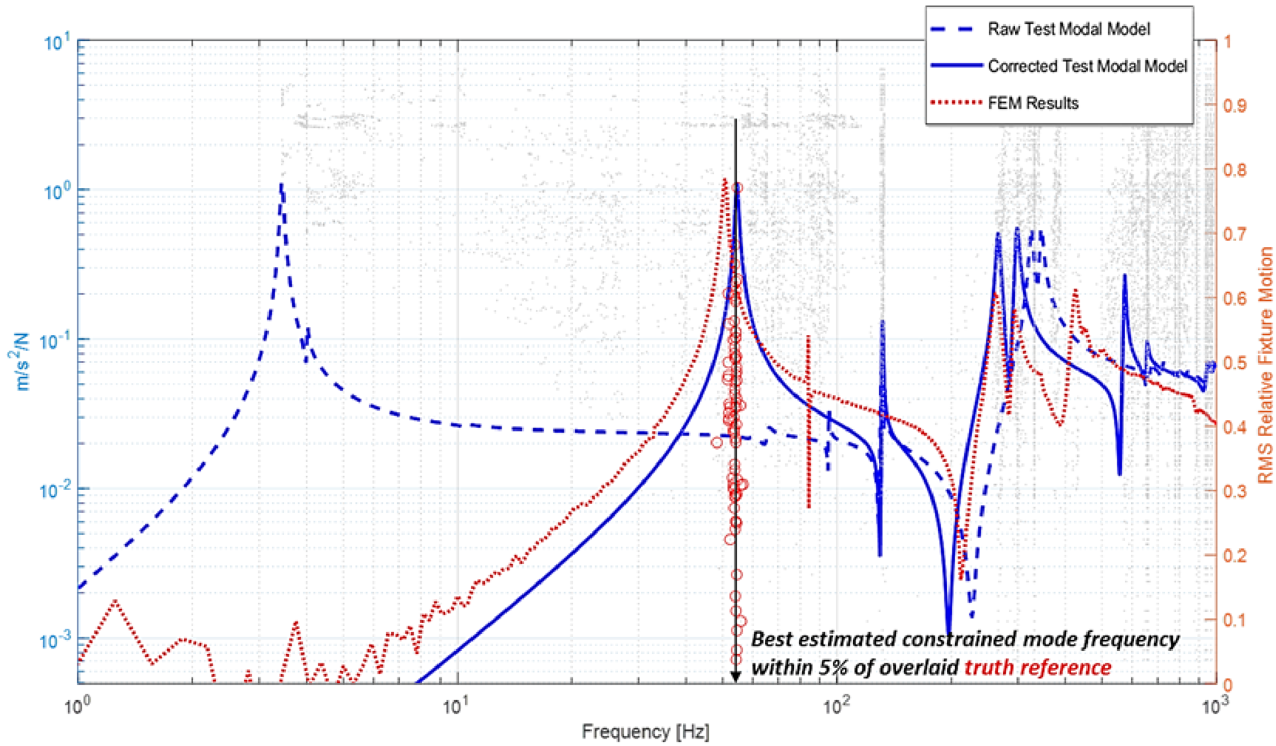


Fig. 6 MATLAB User Interface Results for UUT1 on Test 4, X Axis Drive Point Transfer Function

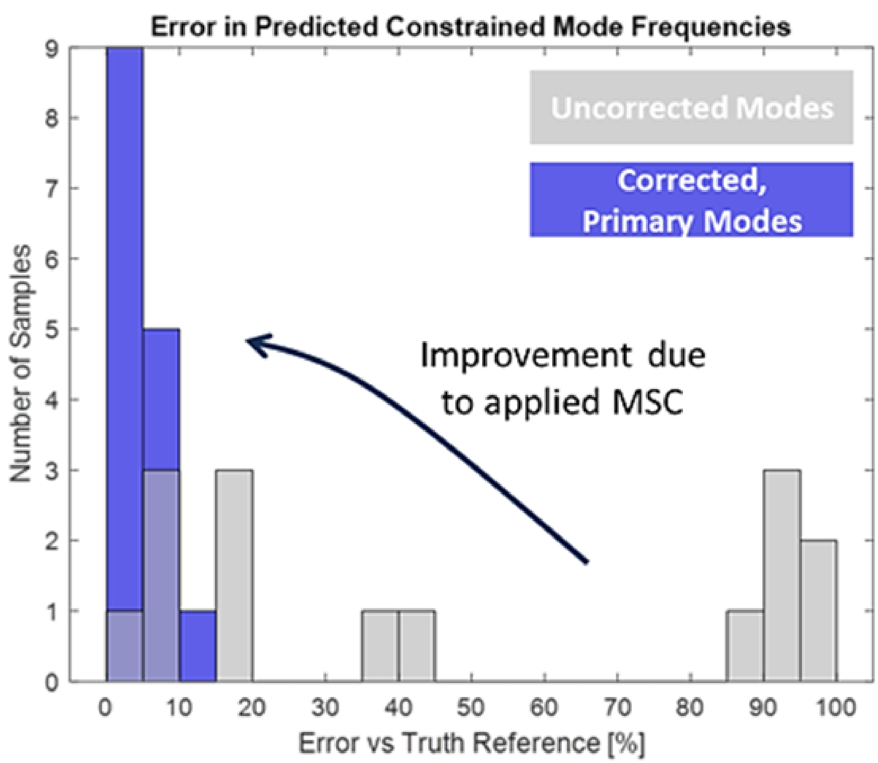


Fig. 7 Effect of Modal Substructure Constraints on Primary Modes Compared to Truth Reference

Further, making finer adjustments <5% may increase risk of "over-correcting" and incorporating more error into results (observed in only one out of thirty instances). It is important to note that in all cases MSCs improve the overall shape of the frequency response functions revealing the nature of the interaction with the test fixture (e.g. which response peaks are driven by fixture vs the UUTs natural response).

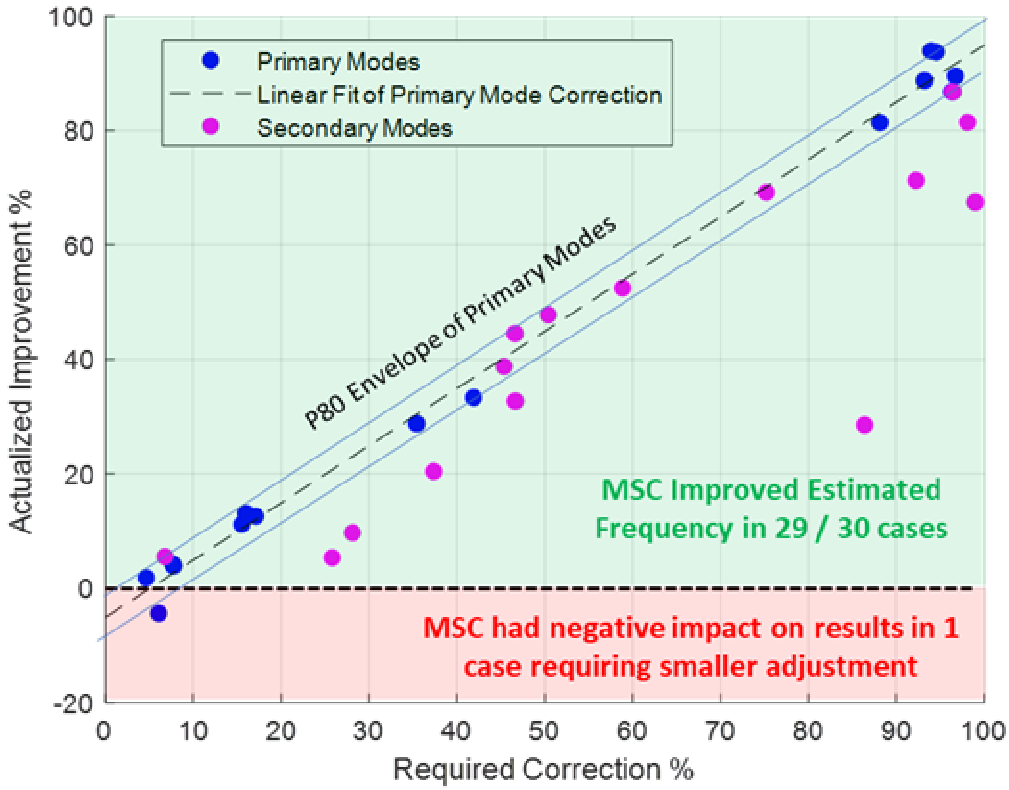


Fig. 8 Actualized Improvement vs Required Correction (%Relative to Truth Frequencies) when MSC's were Applied to Test Results

Conclusion

This study investigates the efficacy of MSCs for removing flexible boundary effects during constrained modal tests of realistic aerospace structures. Four test items are investigated under various boundary conditions. This study finds that MSCs are a powerful tool for reducing risk and uncertainty during a constrained modal test with little additional expense or technical scope. They offer a practical solution when cost, schedule, or logistics limit the efficacy of a constrained boundary, making modal testing accessible to many programs that previously did not utilize it. In this study, MSCs prove capable of making significant corrections to experimentally measured modal parameters, substantially reducing the influence of an unwanted flexible fixture or boundary. However, the application of MSCs is most beneficial when large corrections are needed to primary modes >8% (P80). Further, the observed trends indicate that smaller, fine-tuning adjustments <5% may not add substantial value without special attention to fixture properties or measurement accuracy that fall beyond the scope of this study. A practical interpretation of these error margins is offered in Fig. 9. In this process, a modal test campaign must select a threshold "X%". If the discrepancy between a corrected test mode and FEM mode is greater than that threshold, an adjustment to the FEM is likely to benefit the value of the FEM predictions after a model update is made. This study preliminarily determines that an appropriate value for that threshold lies between 5% and 8%. Using a value lower than 5% will often result in unneeded corrections; using a value higher than 8% will often miss needed corrections. Further research should be continued to investigate ways of reducing these error margins as well as comparing the efficacy of modal substructuring constraints to other leading approaches in a similar light.

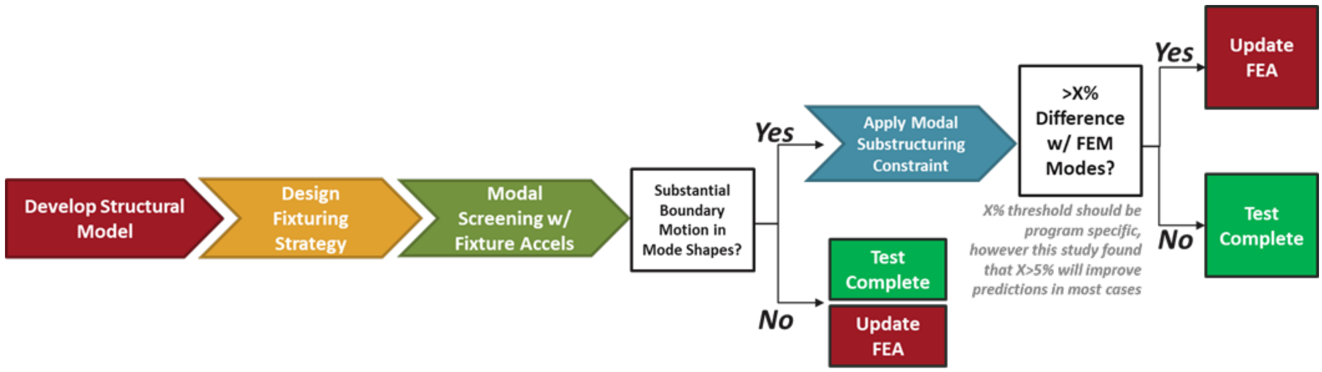


Fig. 9 Decision tree describing practical implementation of modal substructure constraints in a modal test for purposes of FEM updating

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