Chapter 12
Mechanical Property Characterization of Small Granular Materials with Laser Doppler Vibrometry and Resonant Ultrasound Spectroscopy

Joshua Bartlett, Caleb Fryer, Dr. Pablo Tarazaga, and Dr. TJ Ulrich

Abstract Resonant Ultrasound Spectroscopy (RUS) is a reliable, non-destructive measurement technique used to investigate the elastic properties and internal structure of various specimens. Widely used in the fields of material science, geophysics, and engineering, it is has become an increasingly popular characterization method due to its high precision, sensitivity, and versatility in measuring both single crystal structures and complex composites. Despite its advancements over the last forty years, RUS is still afflicted by challenging sample preparation, insensitivity to small-scale defects and inhomogeneities, and limitations in soft or highly damped materials. While recent efforts have succeeded in resolving some of these issues using finite element models, these versions of RUS are computationally expensive and slow. This study attempts to develop an alternative, efficient approach for RUS by incorporating interferometric detection, allowing for rapid data collection, reduced material preparation, and increased measurement sensitivity. To accomplish this, a series of modal vibration tests are conducted using a laser doppler vibrometer (LDV) and adjustable scanning mirrors. The test specimen is initially placed in contact with a piezoelectric disk within an acoustically dead platform and excited using a series of swept periodic sinusoidal signals. The recorded measurements are then processed to identify the natural frequencies and vibrational modes of the specimen, where a comparison is formulated against an idealized model. Utilizing initially predicted independent elastic constants; the general eigenvalue problem is iteratively solved using the Levenberg-Marquart minimization scheme to determine the elastic tensor of the test specimen. Understanding the mechanical properties of granular materials allows for quantification of its functionality and longevity, and accurate determination of physical changes over time. Additionally, this study addresses the difficulties of enhancing this technique and explores the potential for its application within structural health monitoring and quality control in high efficiency manufacturing.

Keywords Resonant Ultrasound Spectroscopy · Interferometric Detection · Data Collection · Material Preparation

Introduction & Background

Resonant Ultrasound Spectroscopy (RUS) is a frequency analysis technique capable of determining the mechanical properties of a material sample in a non-destructive manner. By utilizing the normal modes or mechanical resonances of a solid, homogeneous object, it is possible to identify the independent elastic constants C_{ij} that corresponds to its unique geometry, anisotropy, and density.

Before running a RUS test, the sample should follow certain assumptions. It should be a homogenous material with an equally distributed density. The sample should also fall under the umbrella of symmetries as the symmetry of a sample influences the number of independent elastic constants. When conducting the experiment, a sample is placed between an actuator and a receiver. Typically, individual piezoelectric transducers are used for each role. However, a laser vibrometer can be used as a receiver. As the natural frequencies of the samples are actuated across a bandwidth, the receiver can calculate the sample's displacement, velocity, or acceleration from the voltage response produced by the laser effected

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doppler effect. Using this measured response as the dependent variable and the frequency as the independent variable, a Fast Fourier Transform (FFT) can be developed. With the FFT, the modal frequencies can be extracted.

To identify the independent elastic constants of a sample, the factor of error between experimental modal frequencies and theoretical modal frequencies can be calculated through a derivation of the linear elastic Lagrangian energy equation as,

$$L = \int_V (E_k - E_p)\, dV = \int_V \left(\frac{1}{2} \sum_i \rho \omega^2 u_i^2 - \frac{1}{2} \sum_{i,j,k,l} c_{ijkl} \frac{\partial u_i}{\partial x_j} \frac{\partial u_k}{\partial x_l} \right) dV, \tag{1}$$

where the equation takes the integral of the difference between the kinetic energy density, E_k, and the potential energy density, E_p, with respect to the volume of the sample, V. In this case, E_k is determined using the density ρ, angular frequency ω, and i^{th} displacement factor u_i. E_p is determined from the elastic stiffness tensor c_{ijkl} multiplied by partial derivatives of the displacement vectors with respect to their particular Cartesian coordinate, marked by the subscripts i, j, k, and l. After expanding the Lagrangian by using a displacement field, deriving it under stationary conditions, and exploiting symmetries related to the elastic properties, the equation can be simplified into the following eigenvalue matrix equation,

$$\omega^2 \boldsymbol{E}\boldsymbol{a} = \boldsymbol{\Gamma}\boldsymbol{a}, \tag{2}$$

where $\boldsymbol{E}$ is the kinetic term, ω is the frequency, a is the approximate motion, and $\boldsymbol{\Gamma}$ is the elastic energy term. To better understand how this simplification is made, refer to the derivation provided by Visscher et al. [1].

To effectively calculate the theoretical frequencies, the mass, dimensions, symmetry condition, and estimated independent elastic properties must be used to numerically converge to a solution using a set of basis functions (i.e., Rayleigh-Ritz Method). With these, the calculated frequencies, ω_C, can be compared with the experimental frequencies, ω_E, using the residual error equation.

$$F = \sum_{n=1}^{N} w_n (\omega_E - \omega_C)^2 \tag{3}$$

The weighted variable, w_n, is set to one for measured frequencies and zero for missed frequencies. Frequencies that were not captured experimentally can be identified through models or predicted frequencies and should be included if there are experimental frequencies missing inside the measured bandwidth. With the error function, the Levenberg-Marquardt algorithm is used to minimize the factor of error by iteratively adjusting the elastic constants until the predicted frequencies closely match with the experimental frequencies.

Due to RUS's high accuracy and efficiency, it has become particularly useful for identifying the independent elastic constants for advanced or complex materials that cannot be traditionally evaluated through standard destructive methods. However, it does require an initial guess on the independent elastic constants. Additionally, collecting optimal frequency responses requires the sample actuated under free boundary conditions with the assumption that it is symmetric and linearly elastic.

Experimental Design & Experimental Data Collection

Collecting experimental data through RUS requires an impact or frequency sweep to actuate a sample's resonance frequencies. To capture the sample's response, a transducer is typically used. However, based on the work from Adebisi et al. [2], the apparatus for collecting the frequency response of the sample was the VibroFlex QTec laser from Polytec. When developing the experimental setup, there were three major factors that were considered with respect to this laser: the orientation of the laser with respect to the sample, a means of controlling the laser, and designing a suitable sample mount.

For laser-based RUS to work effectively, not only does the laser have to shine on the sample, but the reflection must also be clearly registered by the infrared sensor. For the laser doppler vibrometer to work effectively, the laser should point at a location on the sample that yields the most resonance frequencies while the signal received is still normal to the receiver of the laser [3]. Inspiration was drawn from the work of Beardslee et al. [4] by implementing the use of external controllable mirrors that can redirect the laser orthogonally. This allows the laser to rest at a horizontal orientation while still pointing the beam onto the sample from a vertical orientation. The schematics of the test setup with the laser configuration can be seen in Figure 1. The same could be obtained with a scanning laser vibrometer and a stationary mirror aimed at the sample.

There are two subset configurations to this experimental setup: The data acquisition system and excitation system. The data acquisition system includes a computer that utilizes the Resonance Inspection Techniques and Analysis (RITA©) software which serves as the signal processing system and the means of controlling the conditions for the RUS experiment [5].

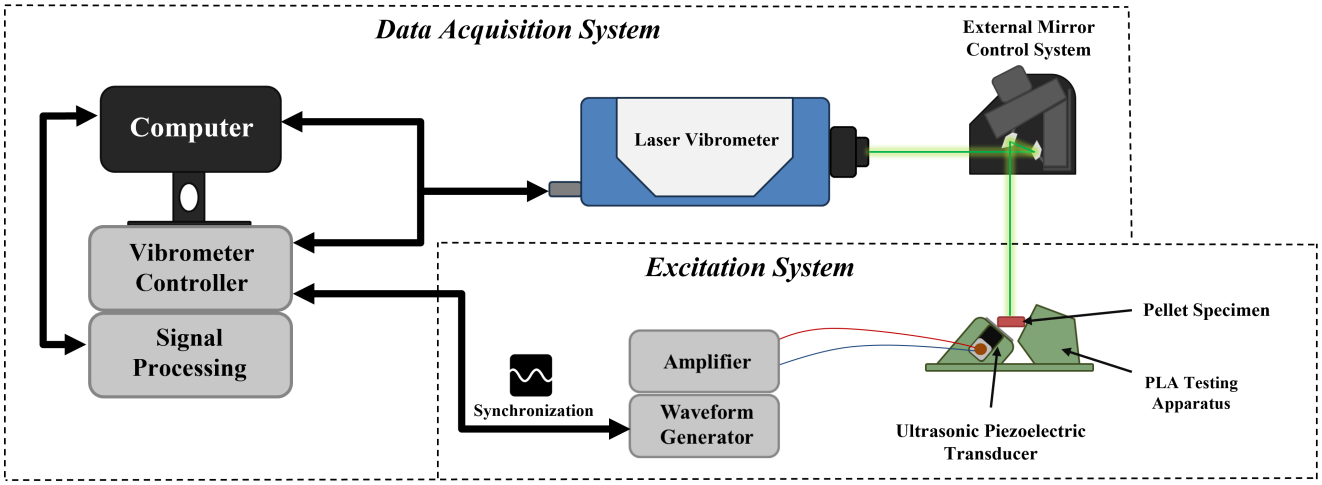

Fig. 1 Diagram of experimental setup for measuring experimental response of a pellet specimen.

From this, a waveform is generated across the desired bandwidth and is amplified by a Tegam high-voltage amplifier, model 2350, with a gain of 50 volts. Granulated aspirin pellets are used as the samples for this experiment and a 1 MHz V103-RM Olympus piezoelectric transducer was used to adequately drive the resonant peaks for the sample's modal responses. This ultrasonic transducer was chosen as the excitation response is flat or vibrationally unresponsive within the measured bandwidth, making it useful for capturing the FRF without interference.

The sample mount was designed specifically for this experiment to hold the sample in an assumed unconstrained boundary condition. There are three contact points with the sample attempting to minimize points of contact on the sample. First is the piezoelectric transducer, the other two are 3D printed fins. As seen in Figure 1, the transducer and fins form a profile like a chuck. The distance between transducer and fin decreases, allowing for samples of various diameters to neatly fit into the sample holder. With this design, samples between 4mm and 13mm diameters fit into the holder while contacting the piezoelectric transducer. For this experiment, samples measured were compressed, finely-granulated aspirin pellets with a diameter of 10.15mm and a thickness of about 2.37mm.

With the help of the external mirror control system the laser can be directed onto any point on the top face of the sample and, with the aid of a python code, it can also be systematically used to quickly scan the surface of a sample. The surface scan can be compiled into a raster image of the operation deflection shape (ODS). This, in turn, can then be used in a modal assurance criteria (MAC) function to confirm the mode shapes of the sample with respect to their resonance frequencies.

Integration of Mode Shape Identification & Automation

The identification of eigenmodes within the analysis of a RUS inversion is not a particularly novel concept. Studies have been conducted to determine the elastic constants of geometrically complex materials with multi-axis scanning [3], incorporate optimal measurement point locations [5], and improve the error minimization function [6, 7]. While these proposed techniques show potential in improving the accuracy and processing time of RUS, they still require manual interpretation by an experimentalist and do not directly attempt to analyze any potential modes that are absent or swapped for a provided test sample. This section presents a new method for systematically identifying and sorting the modes of a material specimen by comparing its operational deflection shapes to numerically approximated resonant modes through the modal assurance criterion.

When conducting a basic experimental measurement using RUS, four standard steps are typically completed to achieve the desired elastic constants: measure the sample behavior to a presented excitation, compute the frequency response function, isolate the natural frequencies, and conduct the RUS inversion algorithm. This process is effective but can only be easily automated for the second and third steps using frequency response function calculators and rational function approximations. Assuming that the experimental design for the system is sufficient to allow for the operational deflection shape to accurately represent the resonant modes (i.e., well separated modes), a new fifth step can be implemented into this process to automate the input parameter for the RUS algorithm. The procedure this identification process would follow is shown in Figure 2.

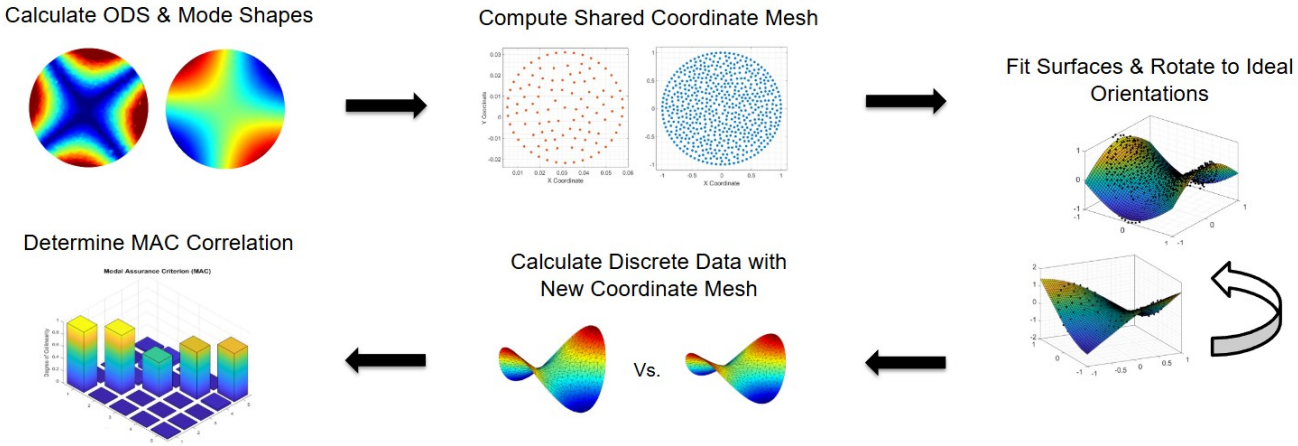

Fig. 2 Representative flowchart of automatic MAC identification process for one natural frequency.

As seen in Figure 2, a fast-scanning raster image of the material specimens' surface was taken for each identified natural frequency. The operational deflection shapes were then computed within the frequency domain and rotated to match the same orientation of an analytically approximated mode shape using the Rayleigh-Ritz numerical method. A new shared discrete mesh was then applied to each surface to produce a common set of discrete positional coordinates. The colinearly of the out-of-plane displacement for each mode was then calculated using the modal assurance criterion function in Equation (4).

$$MAC\left(\{\varphi_E\},\{\varphi_A\}\right) = \frac{\left|\{\varphi_E\}^T\{\varphi_A\}\right|^2}{(\{\varphi_E\}^T\{\varphi_E\})(\{\varphi_A\}^T\{\varphi_A\})}, \tag{4}$$

where this function is a normalized inner product of the surface vectors of the experimental operational deflection shapes, φ_E, and the analytical mode shapes, φ_A, at each common nodal mesh point.

Results & Discussion

The measured material specimen locations produced the average vibrational response seen in Figure 3. Six natural frequencies were identified at 43.7, 69.2, 85.4, 94.6, 121.7, and 129.5 kHz. Note that the fourth natural frequency was determined to be a measured in-plane response and was not included due to concerns about reliability from our experimental design.

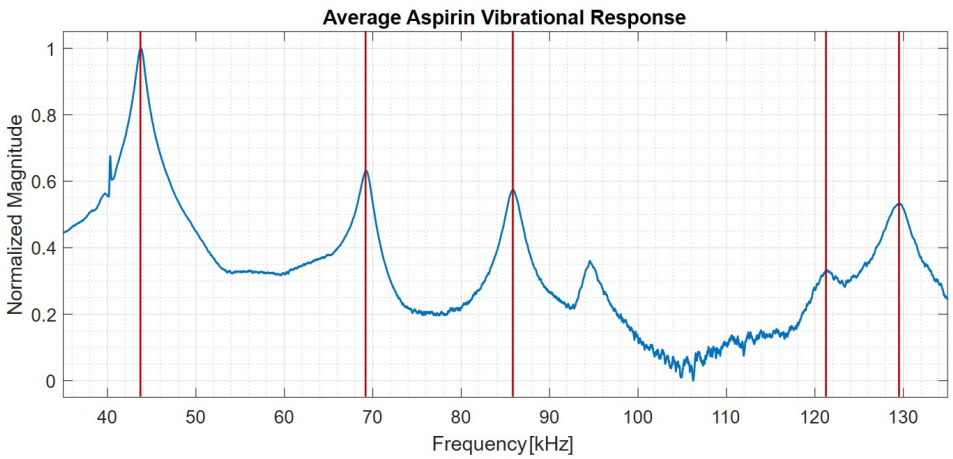

Fig. 3 Frequency response function and first five out-of-plane natural frequencies of aspirin pellets.

Using these values the new modal assurance identification process was conducted, producing Figure 4. The five operational deflection shapes were determined to have a degree of collinearity of 98.2, 87.8, 75.3, 2.4, and 90.1 percent. This correlation was found to be logical as the first, second, third, and fifth natural resonances were well-defined. The fourth operational deflection shape had minimal correlation to its expected mode due to its high degree of coupling. This comparison can be visually confirmed when reviewing the compared operational deflection shapes and analytical modes found within the appendix. Additionally, it was found that no mode swapping was present and that only the second experimental operational deflection shape was non-degenerate.

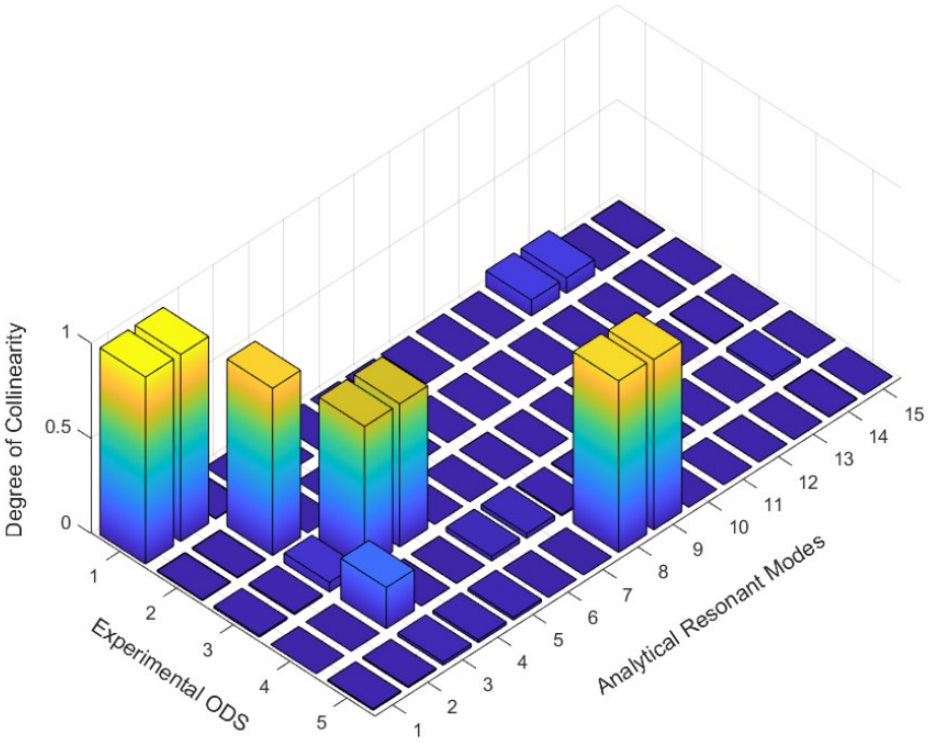

Fig. 4 Bar chart comparison of experimental ODS & numerical modes of aspirin pellets.

Considering these observations, only four measured natural frequencies are reliable for usage within the RUS algorithm. Since the general rule of thumb for RUS states that a total of ten unique modes are required to properly define the elastic tensor solution surface for an isotropic material [8], the elastic properties for this data set cannot reliably be determined. Nevertheless, the proposed identification techinique using modal assurance criterion was successful and shows promise for automating the RUS inversion process.

Challenges and Future Work

Several challenges arose during this research, with one key difficulty being the precise alignment of the laser with the external mirror control unit. Due to the weight of the external mirrors, drooping occurred throughout the experiment and led to the laser doppler vibrometer becoming misaligned. This affected the accuracy of the measurements and introduced error through the introduction of a laser incidence angle. Additionally, material samples composed of a larger granular size proved to be more difficult in scanning during the raster imaging process because of increased surface irregularities and texturing, scattering the reflection of the laser and disrupting the signal quality.

Future work will focus on the further optimization and automation of this process by reducing the total measurement and computational time. Enhancing the precision of the laser alignment with the external mirror control unit and ensuring the measurement efficacy for material samples will be prioritized. There is also potential for the integration of continuous scanning laser doppler vibrometer (CLSDV) or machine learning algorithms to rapidly extract ODS and generate the proper

indexing of modes for automated comparison to the model. This will allow for more accurate, generalized surface scanning that is not dependent on specified measurement locations and reduced mathematical complexity when determining the degree of collinearity between the experiment and numerical model. Finally, implementing an optical lens to adjust the laser's incidence angle would allow for the increased recognition of in-plane operational deflection shapes. These advancements would enhance the overall reliability and versatility of the experimental system for future applications.

Conclusions

This research successfully identified the operational deflection shapes of compressed, granular aspirin samples and accurately correlated them with a numerical model, providing a robust validation of the experimental data used with RUS when determining a material's elastic properties. By addressing the issues of mode swapping and missing modes, this study eliminates significant challenges in analyzing the vibrational behavior of a system and the uncertainty of a material specimen's response. Furthermore, the use of modal assurance criterion values creates a more efficient and streamlined workflow, allowing for the automation of the RUS inversion process and reduction of potential errors in manual input. These advancements contribute to a more precise and confident modal analysis as well as improve the reproducibility of a RUS experiment. While this proposed method was demonstrated to be viable for isotropic samples of cylindrical geometry, further development is necessary to successfully obtain enough vibrational information to conduct a RUS inversion for samples of large damping.

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Declaration of Competing Interests The authors have no conflicts to disclose regarding to any known financial interests or personal relationships that could influence this paper.

Data Availability The supporting data to this study is available upon request.

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Appendix

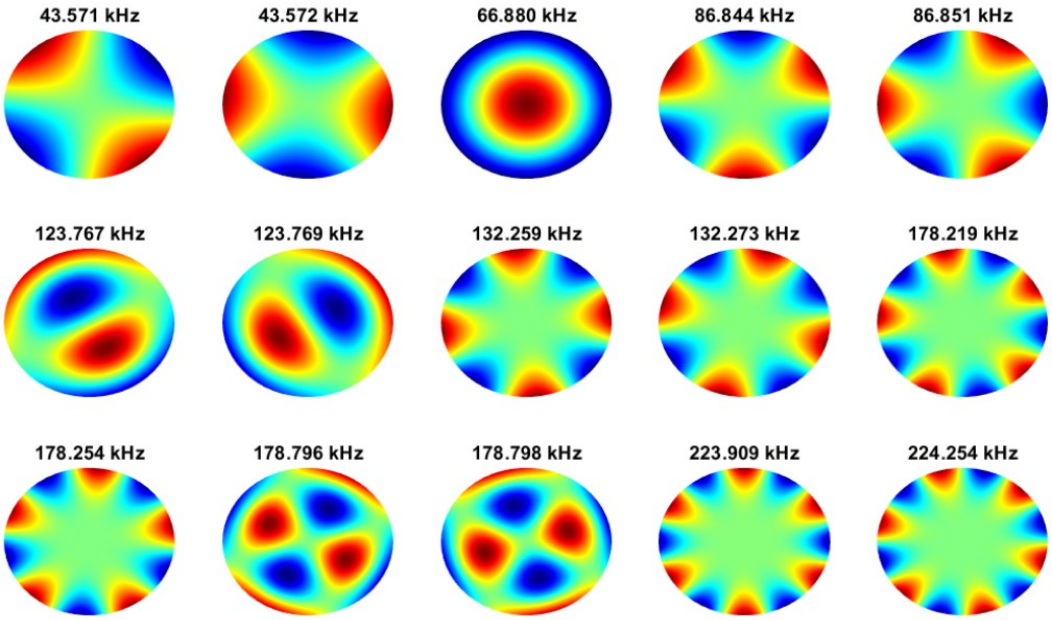

Fig. A First ten out-of-plane analytical modes of aspirin pellets.

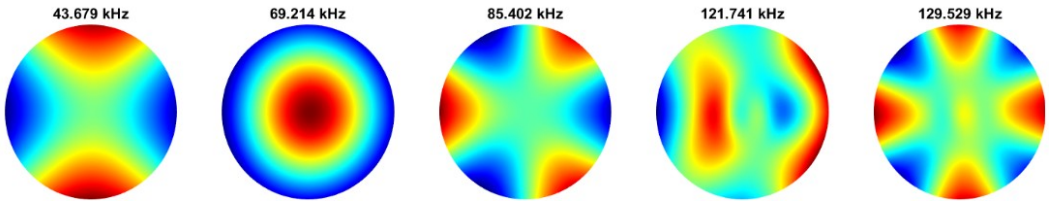

Fig. B Five experimentally measured operational deflection shapes.