Chapter 15
Investigating Repeatability of Complex Dynamic Structures with Stochastic Joints

Casey Whitworth, Nicholas Pomianek, Enrique Gutierrez-Wing, Trevor Jerome, and J. Gregory McDaniel

Abstract Understanding and modeling the interfacial dynamics of bolted joints has been the subject of many studies in previous years. However, the effects of increasing the number of bolted joints in a structure on its repeatability and variability has been far less explored. This study investigated these effects utilizing two long beams with a varying number of shorter beams joining them together. The cross-sections of all beams were L-shaped. Two structures with varying geometries and numbers of bolted joints were constructed and tested. Each structure was re-assembled and tested three times. To assess repeatability of more than two transfer functions, a Normalized Standard Deviation Metric (NSDM) is proposed and used to quantify the frequency-dependent similarities of multiple transfer functions. Based on this metric, it was found that a structure with twenty joints is more repeatable, and thus more predictable, than a structure with four joints when they are constructed of the same commercially available parts.

Keywords Joint mechanics · Repeatability · Variability

Introduction

The goal of this work is to investigate the relationship between the number of mechanical joints and the repeatability of measurements for multiple assemblies of the same parts. This goal is motivated by previous work [1] showing that measured transfer functions for multiple assemblies of a structure with a single joint exhibit random variations over reasonably large frequency ranges. These stochastic joints defy deterministic models and are thus unpredictable. However, it is possible that the presence of several stochastic joints might make measured transfer functions more repeatable by effectively averaging over the stochastic joints and therefore converging to a mean value.

For a relatively recent introduction to the field of joint mechanics, see [2]. One goal of research in joint mechanics is given in the beginning of [2] as "... the development of a predictive model for the interfacial dissipation and stiffness of a joint." Overviews of research on jointed structures are given in [3] and [4]. The former includes perspectives on the need for research on joint mechanics and presents several challenges. Of these challenges, the present work pursues "advancements in physics for predictive capabilities" and "advancements in numerical and stochastic techniques." Experimental approaches and issues for mechanical joints are discussed extensively in [5]. Future work in this area is explored in [6], including a discussion of the repeatability question pursued in the present work.

Three specific areas of research are of relevance to the present work, so previous work in these areas will now be reviewed. The first of these is the design of test structures used in experiments with the goal of increasing our understanding of joint mechanics. A variety of test structures have been proposed, fabricated, and experimentally studied to understand various phenomena occurring in the dynamic excitation of mechanical joints. Six test structures are presented in [7] . One of them is the Brake Reuß Beam [1], which is a lap joint with square beam members joined by three bolts on the beam axis. Zaman *et alia* [8] also presented test structures composed of beams and plates bolted together. The beam structure consisted of two beams connected by strips running parallel to the beams which were attached with two bolts. These test structures

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feature a smaller numbers of joints. However, the test structures presented in [9] feature a relatively large number of joints. That work was concerned with understanding the damping due to the joints. It concluded that slip between interfaces at the joint does not generally occur within operational excitations of the joint. For a test structure consisting of a chain of beams connected by joints, it was found that damping in the joints was on the same level as material damping.

The second relevant research area is the repeatability of measurements on mechanical joints. Specifically, this area of research is concerned with changes in measured transfer functions if the test structure is disassembled, reassembled, and retested. The primary work in this area is by Brake and Reuß [1]. They presented a comprehensive study of three test structures with identical shapes but varying construction. The first was a monolithic beam with no joints or bolts. The second was a monolithic beam drilled with three holes and having three bolts secured with washers and nuts. The third test structure was composed of two beams connected by a lap joint with three bolts secured with washers and nuts. The bolt holes, bolts, washers, and nuts of the second and third test structures were identically placed. Measurements of transfer functions showed that the first two test systems experienced a great deal of repeatability over a fairly broad frequency range. When the third test structure was disassembled and reassembled, the transfer functions changed considerably. The transfer functions also changed when the order of bolt tightening was changed. Cooper *et alia* [10] investigated the degree to which the far-field properties of the structure affect the joint dynamics. The third and final relevant research area is the comparison of transfer functions to quantify similarity. Although much work has been done on assessing the similarities of two transfer functions [11, 12], it is less clear how to compare an ensemble of more than two transfer functions.

Test Structure

The test structure was constructed by attaching two long beams, each $6\,\mathrm{ft}$ long, to each other using a varying number of 1-foot long beams. All beams used in the structure had the same cross-section and came with arrays of holes for fastening. A schematic of the test structure and test equipment is shown in Figure 3. A photo of the four-joint test structure is shown in Figure 1 and a photo of the twenty-joint structure is shown in Figure 2. Commercially available parts used in the design are listed in Table 1. The test structure was designed using commercially available parts that would typically be used by engineers in construction so that the results could be immediately applicable to engineering practice and the structures could be easily replicated by other researchers at a relatively low cost. The beams, nuts, washers, and bolts were all chosen from a standard catalog of parts without any special machining or surface treatments. The L-shaped cross-section was chosen to introduce complexity. The walls of the L-shaped beams were $0.188\,\mathrm{cm}$ thick. Each test structure used washers between the head of the screw and frame and between the nut and frame. Joints were tightened with an instrumented torque wrench set to a torque of $4\,\mathrm{N}-\mathrm{m}$. The accelerometer used was a PCB Model Y352C66 with a sensitivity of $100\,\mathrm{mv/g}$. The instrumented hammer was Dytran Model 5800B2 with a sensitivity of $98\,\mathrm{mV/lbf}$. The accelerometer was attached to the beam using an adhesive mounting base.

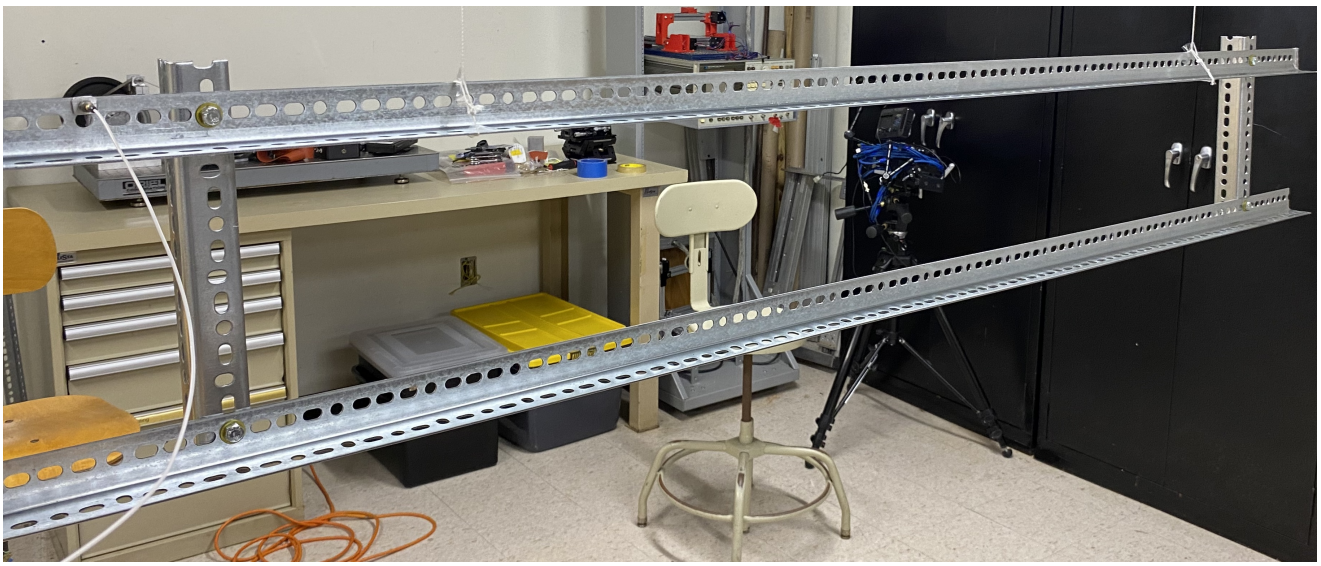

Fig. 1 Photograph of the 4 joint test structure used in this study.

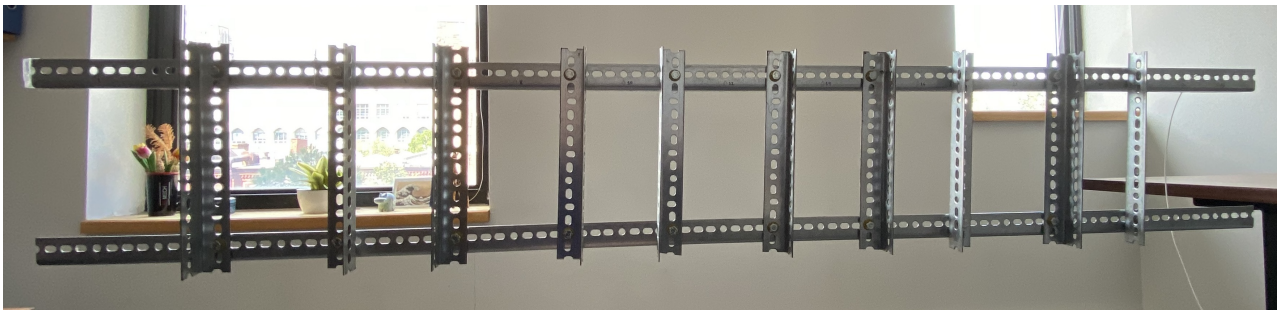

Fig. 2 Photograph of the 20 joint test structure used in this study.

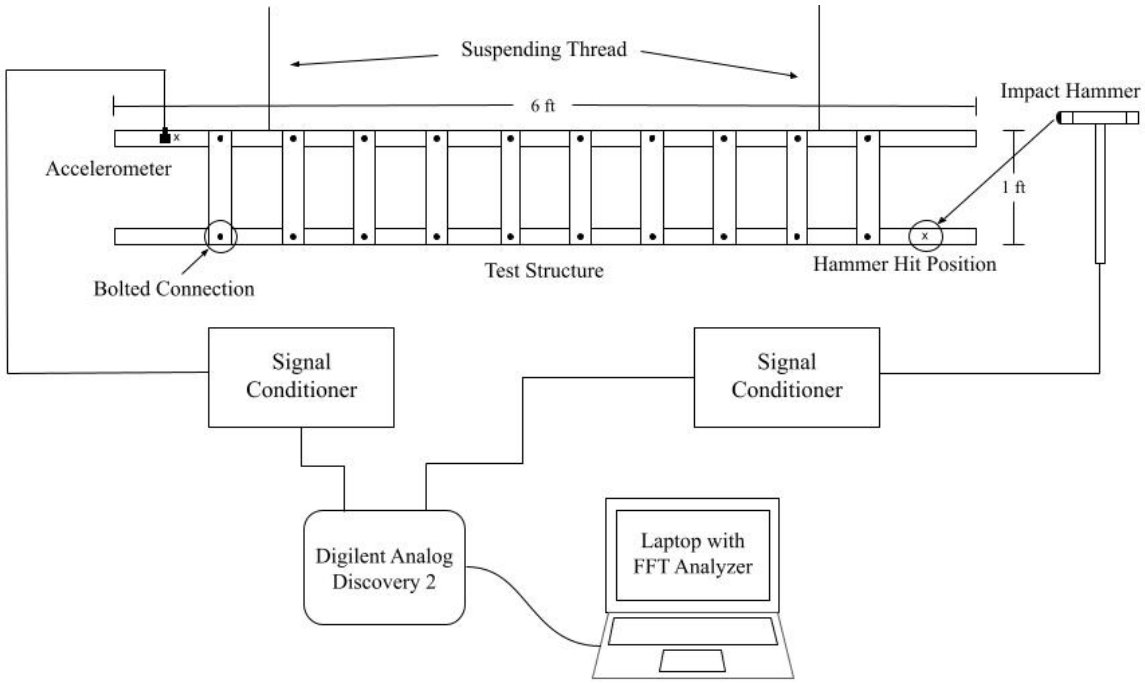

Fig. 3 Schematic of the experiments. The two impact locations marked by the symbol **x** very near the accelerometer.

Table 1 Table of components used to construct test structure

Part	Material	Dimensions
Bolt-Together Framing, L-Shaped Rail	Zinc-Plated Steel	$1-1/2$ inches $\times$ $1-1/2$ inches $\times$ 6 ft
Bolt-Together Framing, L-Shaped Rail	Zinc-Plated Steel	$1-1/2$ inches $\times$ $1-1/2$ inches $\times$ 1 ft
Hex Head Screw	Zinc-Plated Steel	$5/16$ inches-18 $\times$ $3/4$ inches
Hex Nut Grade 2	Zinc-Plated Steel	$5/16$ inches-18
Grade 9 Steel Washer	Zinc Yellow-Chromate Plated Steel	$5/16$ inches Screw Size $\times$ 0.703 inches OD

Test Procedure

Vibration tests were conducted as follows:

1. One of the two structures shown in Figures 1 and 2 was chosen.
2. One of the two impact locations shown in Figure 3 was chosen.

3. Five impacts were applied at that location, each impact being near the corner of the L-shaped cross section.
4. A new assembly was obtained by loosening and re-tightening all bolted joints to $4\,\mathrm{N-m}$.
5. Steps 3 and 4 were repeated three times for each structure and each hit location.

Two impact locations were chosen because the resulting accelerances were expected to be affected by different propagation paths of vibration. For the location near the accelerometer, the accelerance was expected to be a strong function of the local dynamics. For the impact location far away from the accelerometer, the accelerance was expected to be a strong function of the very long propagation path through most of the structure. It is understood that the two structures will have different dynamics such as natural frequencies and mode shapes. However, both structures had many natural frequencies that were excited over the frequency range of interest, therefore general conclusions about repeatability are valid as they did not require identical modal properties.

Post Processing

The acceleration and force were measured over a period of $10\,\mathrm{s}$ with a sampling frequency of $12\,\mathrm{kHz}$, which was sufficiently long that the acceleration signal had decayed to the noise floor by the end of the measurement. For each impact, a time window of duration $12\,\mathrm{ms}$ was applied to the measured force with the window centered around the impact. An FFT of the time signals was then applied to yield the frequency-domain acceleration $\tilde{a}(f)$ and force $\tilde{F}(f)$. In what follows, the subscript m shall refer to an assembly (1–3) and the subscript n shall refer to an impact (1–5). Accelerance for assembly m and impact n was computed by

$$A_{mn}(f) = \tilde{a}_{mn}(f)/\tilde{F}_{mn}(f), \tag{1}$$

The average accelerance over all five impacts for assembly m was computed by

$$\alpha_m(f) = \frac{1}{5}\sum_{n=1}^{5} A_{mn}(f). \tag{2}$$

An average over all three assemblies was computed by

$$\mu(f) = \frac{1}{3}\sum_{m=1}^{3} \alpha_m(f). \tag{3}$$

Deviation of the real and imaginary parts from their means was computed by

$$\sigma_r(f) = \sqrt{\frac{1}{3}\sum_{m=1}^{3} \left(\mathrm{Re}\left\{\alpha_m(f)\right\} - \mathrm{Re}\left\{\mu(f)\right\}\right)^2}, \tag{4}$$

$$\sigma_i(f) = \sqrt{\frac{1}{3}\sum_{m=1}^{3} \left(\mathrm{Im}\left\{\alpha_m(f)\right\} - \mathrm{Im}\left\{\mu(f)\right\}\right)^2}. \tag{5}$$

The magnitude of the complex-valued standard deviation was computed as a frequency-dependent assessment of repeatability,

$$\sigma(f) = \sqrt{\sigma_r^2(f) + \sigma_i^2(f)}. \tag{6}$$

Equations (4) and (5) are recognized as the standard deviation formula used in statistics, however those equations were used here for a very different purpose. While it is understood that the standard deviation in statistics requires an appropriately large number of realizations to be meaningful, we only used the formula as a metric of repeatability that increases with less repeatability. It is therefore a suitable metric even when the number of transfer functions is small, such as the three accelerances (for varying assemblies) used here. A mean value μ_0 of the accelerance magnitude was computed over all frequencies and assemblies and used to normalize the standard deviation. The normalized standard deviation (NSDM) is

$$\alpha(f) = \sigma(f)/\mu_0. \tag{7}$$

Results

A central question was whether a sufficient amount of non-repeatability was present in the data to allow for comparisons between the two structures. This question is addressed in Figures 4 – 6, Figure 4 shows five impacts while Figure 5 shows three assemblies. Visual comparison indicates that the repeatability between impacts was greater than the repeatability between assemblies. This observation is confirmed in Figure 6, where two differences in accelerance are plotted. The first accelerance

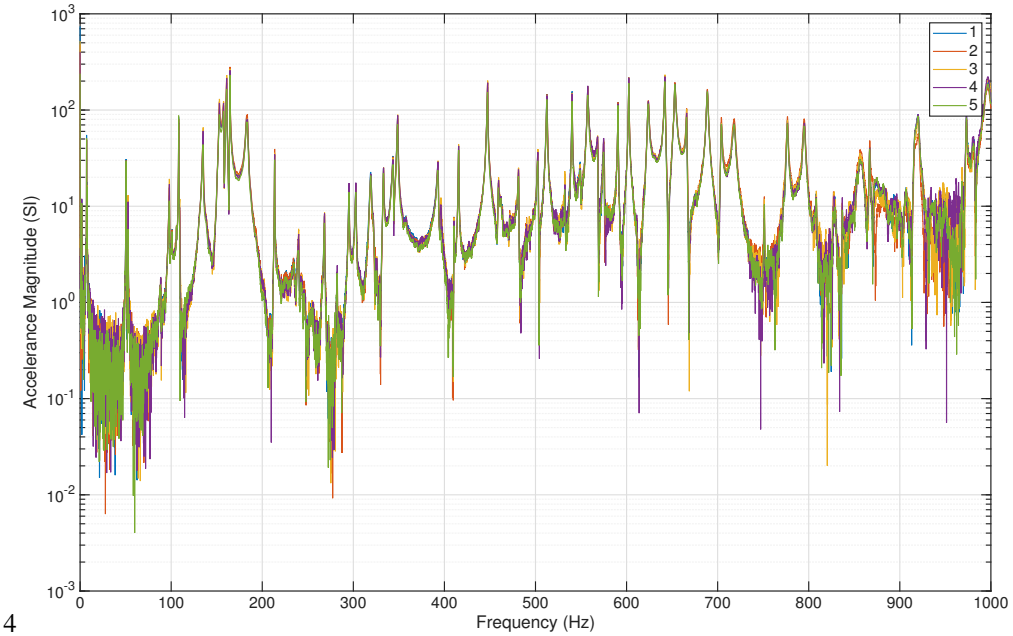

Fig. 4 Plots of accelerance for five impacts. The numbers 1-5 denote different impacts. The impacts were at the same spot near the accelerometer for the four-joint structure.

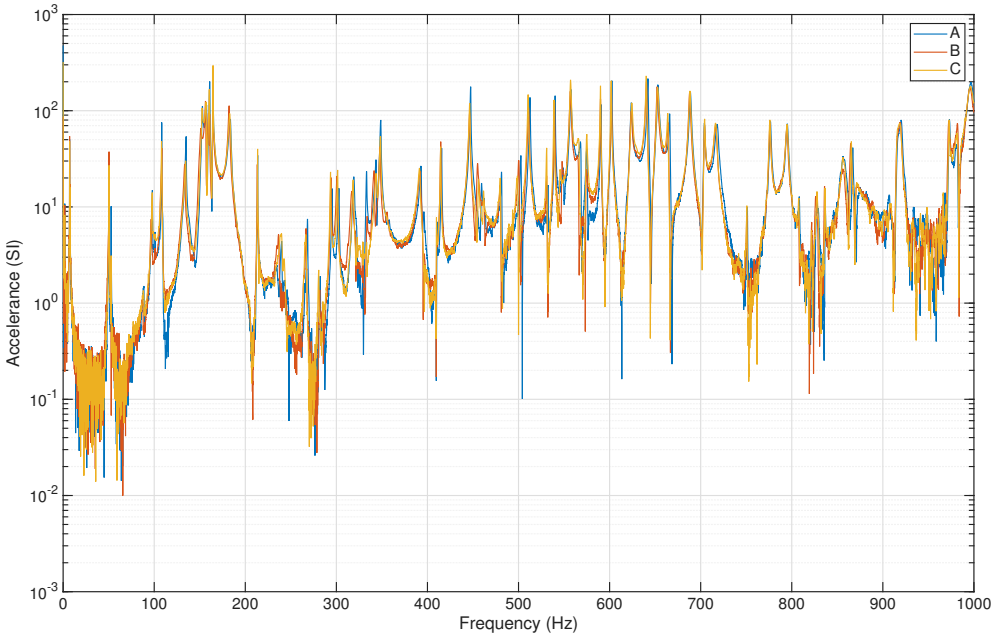

Fig. 5 Plots of accelerance for three different assemblies, where each assembly accelerance has been averaged over five impacts.The letters A, B, and C denote different assemblies. The impacts were at the same spot near the accelerometer for the four-joint structure.

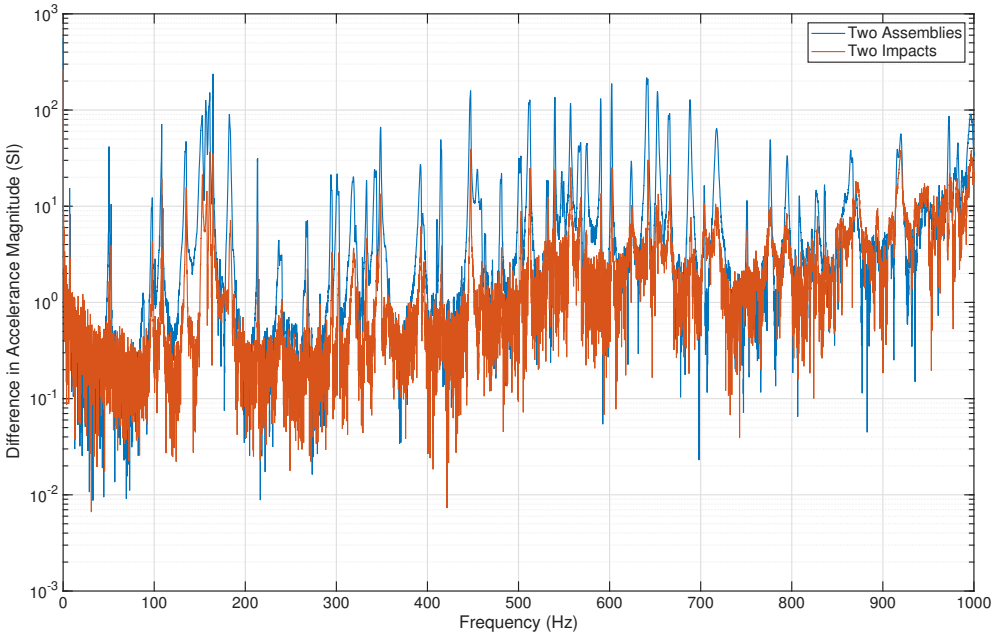

Fig. 6 Plots of the differences between accelerances. The data labeled "Two Assemblies" is the difference between two accelerances measured for two assemblies. The data labeled "Two Impacts" is the difference between two accelerances measured for two impacts. The impacts were at the same spot near the accelerometer for the four-joint structure.

difference is between two assemblies, each averaged over five impacts and the second accelerance difference is between two impacts of the same assembly. It is clear from this figure that that there was significantly more repeatability between impacts than there is between assemblies, thus justifying the use of this data to compare repeatability for the four-joint and 20-joint structures.

Figures 7 and 8 show the normalized standard deviation for the two-joint and four-joint structures. Figure 7 is for the impact near the accelerometer and Figure 8 is for the impact far away from the accelerometer. While there is no clear difference

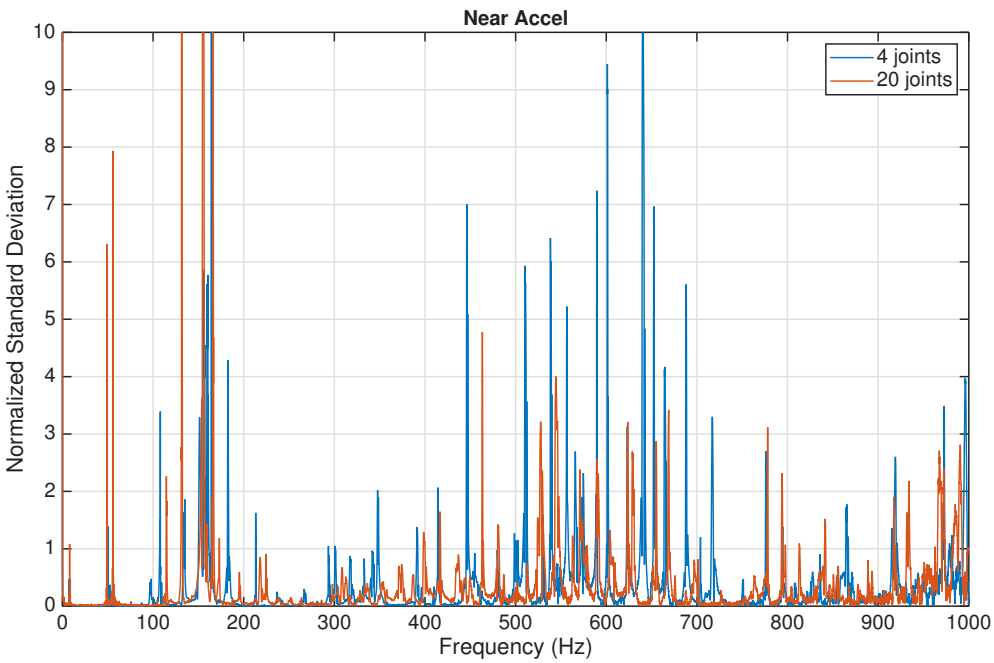

Fig. 7 Normalized standard deviation metric, $\alpha(f)$, versus frequency for an impact near the accelerometer.

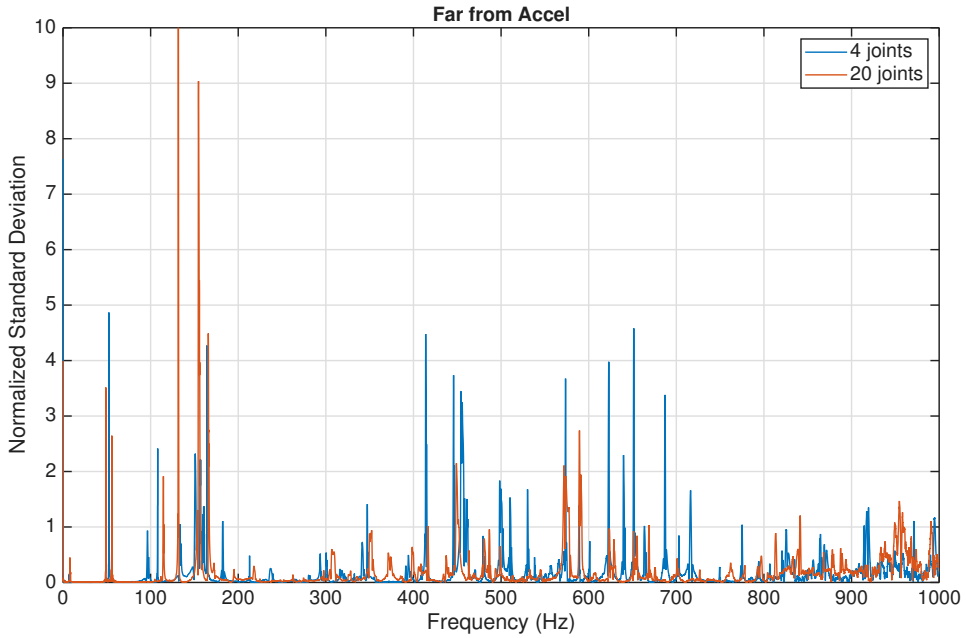

Fig. 8 Normalized standard deviation metric, $\alpha(f)$ versus frequency for an impact far away from the accelerometer.

over the frequency range $0\,\mathrm{Hz}$ to $300\,\mathrm{Hz}$, the four-joint structure generally has higher normalized standard deviation than the twenty-joint structure over the range $300\,\mathrm{Hz}$ to $1000\,\mathrm{Hz}$. A greater number of joints in the structure has made its dynamics more repeatable and thus predictable. No significant difference is observed between the two impact locations shown in the two figures.

Conclusions

It was experimentally demonstrated that the repeatability of a twenty-joint structure is higher than that of a four-joint structure. More work is necessary to determine if this result may be generalized to conclude that increasing the number of joints results in an increase in repeatability. If so, one could approximately assess the repeatability of a structure *a priori* and have a reasonable hope that a deterministic model will accurately predict its dynamics.

Comparison of our results to those of [13] suggests an interesting avenue of future research. The Brake Reuß Beam that was studied in [13] consisted of two beams attached by a lap joint. The metal beams had a $2.5\,\mathrm{cm}$ square cross-section and were attached to each other over a $12\,\mathrm{cm}$ region with M8 bolts. By comparison, the relatively thin L-shaped beams used in the present study had walls that were $0.188\,\mathrm{cm}$ thick, making them 0.075 times the thickness of the Brake Reuß beam. In comparison to the L-angle beams used in the present work, the beams in the the Brake Reuß paper were orders of magnitude stiffer. The dynamics of the structure in the present work were significantly more repeatable than the dynamics of the Brake Reuß beam. This may be due to differences in the relative stiffnesses of beams and bolts as well as the area of overlap. For example, stiff bolts connecting flexible structures may be more repeatable than flexible bolts connecting stiff structures because the former case produces a more consistent joint from one assembly to the next. Future work may investigate this question by studying a collection of structures in which these properties are varied in a systematic way.

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