Chapter 1
Data-Driven Method for Reduced Order Modeling of Blisks with Large and Small Mistuning

Mihai Cimpuieru, Sean T. Kelly, and Bogdan I. Epureanu

Abstract Turbomachinery blisks are paramount to the safe and efficient operation of gas turbines. They are exposed to large aerodynamic forces while functioning at extreme temperatures and rotating conditions. In addition, blisks are manufactured as one single piece and have low damping, making them susceptible to large vibrations, which are amplified by inherent blisk mistuning, represented by small differences from the nominal design. Furthermore, vibration responses are drastically affected by large mistuning (e.g., cracked or deformed blades) in one or more sectors of the blisk, called rogue sectors. These properties make blisks susceptible to high cycle fatigue, which can result in failure of the structure. Thus, it is essential to predict responses of mistuned blisks with rogue sectors. Tuned blisk responses can be calculated at a low computational cost. However, mistuning and rogue blades break the cyclicity of the system and make response calculations much more challenging. State-of-the-art computational methods address this challenge by using reduced-order models (ROMs) based on tuned modes of cyclic rogue systems. However, computing these ROMs requires formulating one or more sets of full system matrices, which can be computationally expensive. To address this issue, one method developed by the authors utilizes a data-driven approach that enforces known physics derived from equations of motion. Nevertheless, this existing method cannot be applied to blisks with rogue sectors. This manuscript proposes an extension of the state-of-the-art to compute responses of a mistuned blisk with rogue sectors. This approach uses transfer function matrices from both a rogue blisk and pristine blisk to form a linear system of equations to compute blade responses, and is applicable to cases including multiple rogue sectors of various types. This approach is demonstrated for a lumped mass model with 18 blades with one or more rogue sectors. Results show that this method can predict amplitude responses of mistuned blisks with rogue sectors with absolute percent errors below 8%.

Keywords Turbomachinery · Neural Networks · Structural Dynamics · Data Science · Machine Learning

Introduction

Gas turbines are one of the most widely used propulsion systems due to their high power and efficiency. Turbomachinery blisks are often used in the compressor and turbine stages of these systems, interacting with the passing air flow and operating at extreme conditions. Gas turbine efficiency and safety are highly dependent on the air flow-blisk interaction. Hence, it is paramount to understand and continuously develop the state-of-the-art for modeling turbomachinery blisks to ensure efficient and safe operation. These structures are exposed to high aerodynamic forces, are manufactured as one single piece, and have low damping, all of which can result in high amplitude blade vibrations. In addition, as-manufactured blisks are mistuned, i.e., contain small deviations from nominal material properties and/or geometry at the blade level. Furthermore, due to significant wear formation or even damage at one or more blades, blisks can also suffer from so-called large mistuning. Blade vibrations due to either or both of these phenomena can result in high cycle fatigue, causing failure of the structure and potentially disastrous effects on the operational safety of a gas turbine. Hence, it is paramount to predict blisk responses with both small and large mistuning. For a tuned blisk (i.e., no mistuning), the computational effort to compute the blisk response can be highly reduced by leveraging the cyclic-symmetry of the structure using block circulant system matrices and projections onto cyclic coordinates. However, mistuning breaks the cyclic-symmetry of the structure. Thus, one cannot directly leverage the tuned system's cyclic symmetry for reduction, resulting in increased total computational effort.

Mihai Cimpuieru · Sean T. Kelly · Bogdan I. Epureanu
University of Michigan, Ann Arbor, MI 48103
e-mail: mcimpi@umich.edu; seantk@umich.edu; epureanu@umich.edu

Thomas Matarazzo et al. (eds.), *Data Science in Engineering, Vol. 11*, Conference Proceedings of the Society for Experimental Mechanics Series, https://doi.org/10.13052/97887-438-0156-6_1

Thus, formulating reduced-order models (ROMs) to compute mistuned blisk responses has been of great interest in the past decades.

There are several state-of-the-art ROMs to compute mistuned blisk responses. Firstly, small mistuning is commonly defined in literature as small deviations in the cantilever blade alone frequencies with no change in the blade mode shape. Computationally, small mistuning is modeled by stochastically varying the blade's Young's modulus from blade to blade, usually no more than 5% [1]. Multiple ROMs have been developed using physics-based approaches, such as component mode synthesis (CMS) techniques [2]. These methods separately consider the blades and disk sections, utilizing interface constraint modes and normal modes of each section holding the interface between the two sections fixed. Other physics-based ROMs consider a subset of or all the tuned blisk modes when blades are exhibiting cantilever beam motion, while some also include constraint modes associated with the disk-blade interface [3,4]. Secondly, for large mistuning, some methods build upon a CMS-based approach by structuring the mistuned blisk into a tuned blisk component and a mistuned blisk component. However, these substructuring-based approaches require one or more sets of large full-order system matrices. Other approaches have focused on leveraging the cyclic-symmetry of tuned systems [5]. In particular, the pristine-rogue-interface modal expansion (PRIME) method [6,7] uses a basis formed from modes of the tuned system with pristine blades and modes of a tuned system with rogue blades (i.e., all blisk blades have the same large mistuning). Additionally, using an extension of [3], PRIME can predict blisk responses with both small and large mistuning.

A primary drawback of physics-based ROMs is that they require the use of high dimensional blisk finite element (FE) models to characterize the system dynamics and build the projection bases used for ROM generation. Furthermore, the accuracy of a physics-based ROM cannot be enhanced using experimental data. Hence, recent work has focused on the development of data-driven methods to predict the response of mistuned blisks [8,9]. In particular, for small mistuning, some methods leverage feedforward neural networks (FFNNs) and a combination of coupling and sector-response networks to predict blade tip responses [9]. Specifically, the method in Ref. [9] uses a sector-level coupling FFNN to predict the responses at sector interfaces while enforcing compatibility conditions, and then uses those responses as input for a separate sector-level FFNN to compute the tip response for each blade. However, this method does not explicitly enforce the system linearity (and hence the governing physics) in the networks nor the solution procedure. As such, a new method to account for this drawback was developed [10,11]. Specifically, this method leverages the cyclic-nature of blisks and enforces system linearity within the loss function of a sector-level, physics-informed neural network (PINN). In particular, this approach introduces two matrix transfer functions: one that relates sector i blade responses to potentially all other blade responses, and one that relates the same sector i blade responses to the forcing applied on sector i. These transfer function matrices are output from the sector-level PINN for each sector and used to form a linear system of equations to predict all blade responses. However, this approach accounts only for small mistuning, without considering large changes in mode shape due to large changes in material properties or geometry. To the authors' knowledge, there is currently no physics-informed data-driven method for predicting blisk responses with both small and large mistuning.

This paper proposes a novel physics-informed data-driven method to compute the response of blisks with both small and large mistuning. Conceptually similar to the PRIME method, the authors utilize two cyclic systems: 1) a pristine blisk with small mistuning, and 2) a rogue blisk with small mistuning. Two sets of sector-level transfer function matrices relating the blade root responses of a sector i to 1) the blade root responses of all other blades and 2) the forcing excitation on sector i are learned. Specifically, one set is learned by a sector-level PINN trained using the cyclic pristine blisk with small mistuning, and the other set is learned by a separate sector-level PINN trained using the cyclic rogue blisk with small mistuning. These two sets of sector-level transfer function matrices are then used to form a linear system of equations that is solved to predict all blade root responses of a blisk with small mistuning and rogue blades. The blisk blade tip responses can then be predicted using the computed blade root responses in a similar manner to that used in Ref. [9]. Prediction results considering a traveling-wave excitation (TWE) with fixed amplitude and a blisk with small mistuning and 1, 2, or 17 rogue blades are presented. Across all presented results, this method predicts blade root responses with absolute percent errors less than 8%.

Methodology

The goal of this approach is to predict blisk responses at the blade roots when one or more blades are rogue (i.e., have large mistuning) and all blades have small mistuning. For this analysis, we will consider that all rogue blades have the same amount of large mistuning. However, this method can be applied to blisks with multiple blades with varying levels of "rogueness".

Firstly, it is critical to define the neural networks used throughout this paper. Two sector-level PINNs (similar to those in [10, 11]]) with 2 fully-connected hidden layers each having 500 nodes are utilized for this work. For a general sector i, each network takes as input the small blade mistuning of sector i, denoted δ_i, and the TWE frequency, ω. The output of each network consists of two matrices, $\mathbf{A}(\omega, \delta_i)$ and $\mathbf{B}(\omega, \delta_i)$, defined as transfer function matrices respectively describing the relationship of the sector i blade root responses to 1) all other blade root responses, and 2) the forcing applied to blade i (see Eq. (1)). In Eq. (1), $\overline{x}_{i,\omega}$ are the blade root responses of sector i for excitation frequency ω, $\overline{f}_{i,\omega}$ is the force applied at the blade tip of sector i, and $\overline{x}_{\neg i,\omega}$ are the blade root responses from all sectors except sector i. The mean squared error (MSE) loss function for a single training sample is defined in Eq. (2), the goal being to learn the optimal $\mathbf{A}(\omega, \delta_i)$ and $\mathbf{B}(\omega, \delta_i)$ that minimize the overall MSE across all training samples. Note that a factor of $1/2n$ is included in Eq. (2). This comes from using n blade root responses per sector, with 2 included in the denominator because, for PINN training, real and imaginary responses are explicitly considered in $\overline{x}_{i,\omega}$ and $\overline{x}_{\neg i,\omega}$ rather than as combined complex-value quantities.

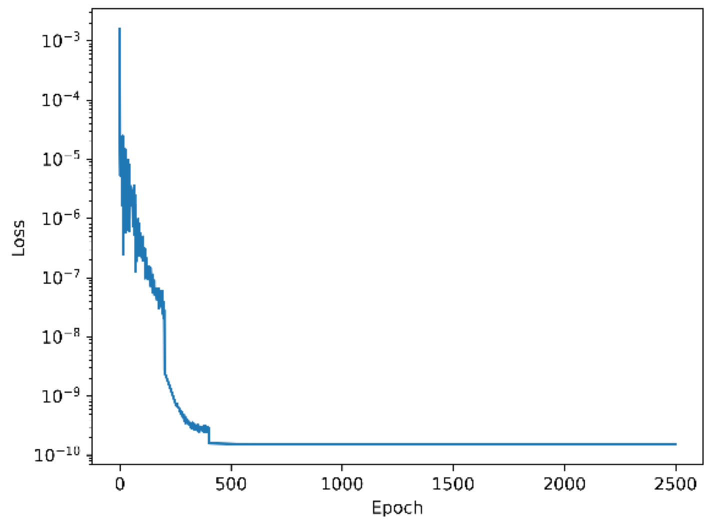

Fig. 1 Loss function during training of pristine blisk PINN.

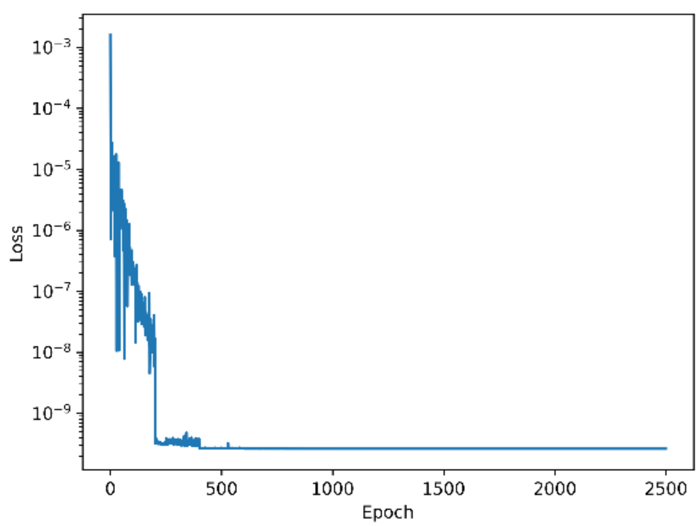

Fig. 2 Loss function during training of rogue blisk PINN.

The sector-level formulation in Eq. (1) leverages the cyclicity of the blisk and is explained in more detail in Refs. [10,11]. However, it is important to mention that the PINNs in [10,11] take as inputs the mistuning values of all or of a subset of the other blades since all responses are measured at the tip of the blades where the responses are significantly affected by mistuning. The formulation the authors propose in this work utilizes responses only from the blade roots. Note that the response of blade i at its root is only dependent on 1) the mistuning of blade i since the force is applied at its tip, and 2) the root responses of all other blades due to coupling through the disk section. Thus, the input to the network proposed here only consists of 1) the excitation frequency, and 2) the mistuning of the sector i. The network is trained for a frequency range 940 Hz – 1060 Hz, with a frequency resolution of 0.3 Hz. Training is performed using a total of 577,440 individual samples.

Secondly, it is important to clarify what systems are considered for the proposed method. As mentioned in the previous section, there will be two systems analyzed, each with a respective PINN: 1) a blisk with pristine blades (i.e., no large mistuning) and small mistuning, and 2) a blisk with rogue blades and small mistuning, each blade having the same level of "rogueness" (i.e., same large mistuning applied at each blade). The respective PINN training curves (i.e., overall MSE loss vs. epoch) are shown in Figs. 1 and 2. For both networks, the learning rate is reduced in 200-epoch increments to reach convergence faster. The training curves in Figs. 1 and 2 show that both networks converge successfully.

Next, a linear system of equations is formed combining the $\mathbf{A}(\omega, \delta_i)$ and $\mathbf{B}(\omega, \delta_i)$ matrices obtained from the two PINNs as presented in Eq. (3). Blade root response predictions for a blisk with both pristine and rogue sectors are obtained by solving the linear system in Eq. (3), where to predict sector i blade root responses and, for example, considering rogue blisk coefficients, we have $\mathbf{A}(\omega, \delta_i)_R = \left[\mathbf{A}(\omega, \delta_i)_R^1 \; \mathbf{A}(\omega, \delta_i)_R^2 \ldots \mathbf{A}(\omega, \delta_i)_R^{i-1} \; \mathbf{A}(\omega, \delta_i)_R^{i+1} \ldots \mathbf{A}(\omega, \delta_i)_R^{17} \; \mathbf{A}(\omega, \delta_i)_R^{18}\right]$ with $\mathbf{A}(\omega, \delta_i)_R^j$ being the sector j blade root response coefficients used to predict sector i's blade root response when blade i is rogue. Similarly, the same definition applies for pristine sectors. Within this system of equations, for rogue blades we use the $\mathbf{A}(\omega, \delta_i)$ and $\mathbf{B}(\omega, \delta_i)$ matrices output from the rogue blisk PINN, while for pristine blades we use the matrices output from the pristine blisk PINN. In Eq. (3), blades i and $i+1$ are selected to be rogue to exemplify how the blade "rogueness" is taken into account in the mathematical formulation using the learned transfer function matrices. These specific equations are boxed in Eq. (3).

$$\overline{x}_{i,\omega} = \begin{bmatrix} \mathbf{A}(\omega, \delta_i) & \mathbf{B}(\omega, \delta_i) \end{bmatrix} \begin{bmatrix} \overline{x}_{\neg i,\omega} \\ \overline{f}_{i,\omega} \end{bmatrix} \tag{1}$$

$$L = \frac{1}{2n} \left\| \overline{x}_{i,\omega} - \begin{bmatrix} \mathbf{A}(\omega, \delta_i) & \mathbf{B}(\omega, \delta_i) \end{bmatrix} \begin{bmatrix} \overline{x}_{\neg i,\omega} \\ \overline{f}_{i,\omega} \end{bmatrix} \right\|_2^2 \tag{2}$$

$$\begin{bmatrix} 0 & \mathbf{A}(\omega,\delta_1)_P^2 & \mathbf{A}(\omega,\delta_1)_P^3 & \ldots & \mathbf{A}(\omega,\delta_1)_P^{16} & \mathbf{A}(\omega,\delta_1)_P^{17} & \mathbf{A}(\omega,\delta_1)_P^{18} \\ \mathbf{A}(\omega,\delta_2)_P^1 & 0 & \mathbf{A}(\omega,\delta_2)_P^3 & \ldots & \mathbf{A}(\omega,\delta_2)_P^{16} & \mathbf{A}(\omega,\delta_2)_P^{17} & \mathbf{A}(\omega,\delta_2)_P^{18} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \mathbf{A}(\omega,\delta_i)_R^1 & \mathbf{A}(\omega,\delta_i)_R^2 & \mathbf{A}(\omega,\delta_i)_R^3 & \ldots & \mathbf{A}(\omega,\delta_i)_R^{16} & \mathbf{A}(\omega,\delta_i)_R^{17} & \mathbf{A}(\omega,\delta_i)_R^{18} \\ \mathbf{A}(\omega,\delta_{i+1})_R^1 & \mathbf{A}(\omega,\delta_{i+1})_R^2 & \mathbf{A}(\omega,\delta_{i+1})_R^3 & \ldots & \mathbf{A}(\omega,\delta_{i+1})_R^{16} & \mathbf{A}(\omega,\delta_{i+1})_R^{17} & \mathbf{A}(\omega,\delta_{i+1})_R^{18} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \mathbf{A}(\omega,\delta_{17})_P^1 & \mathbf{A}(\omega,\delta_{17})_P^2 & \mathbf{A}(\omega,\delta_{17})_P^3 & \ldots & \mathbf{A}(\omega,\delta_{17})_P^{16} & 0 & \mathbf{A}(\omega,\delta_{17})_P^{18} \\ \mathbf{A}(\omega,\delta_{18})_P^1 & \mathbf{A}(\omega,\delta_{18})_P^2 & \mathbf{A}(\omega,\delta_{18})_P^3 & \cdots & \mathbf{A}(\omega,\delta_{18})_P^{16} & \mathbf{A}(\omega,\delta_{18})_P^{17} & 0 \end{bmatrix} \begin{bmatrix} \bar{x}_{1,\omega} \\ \bar{x}_{2,\omega} \\ \vdots \\ \bar{x}_{i,\omega} \\ \bar{x}_{i+1,\omega} \\ \vdots \\ \bar{x}_{17,\omega} \\ \bar{x}_{18,\omega} \end{bmatrix} + \begin{bmatrix} \mathbf{B}(\omega,\delta_1)_P \overline{f}_{1,\omega} \\ \mathbf{B}(\omega,\delta_2)_P \overline{f}_{2,\omega} \\ \vdots \\ \mathbf{B}(\omega,\delta_i)_R \overline{f}_{i,\omega} \\ \mathbf{B}(\omega,\delta_{i+1})_R \overline{f}_{i+1,\omega} \\ \vdots \\ \mathbf{B}(\omega,\delta_{17})_P \overline{f}_{17,\omega} \\ \mathbf{B}(\omega,\delta_{18})_P \overline{f}_{18,\omega} \end{bmatrix} = \begin{bmatrix} \bar{x}_{1,\omega} \\ \bar{x}_{2,\omega} \\ \vdots \\ \bar{x}_{i,\omega} \\ \bar{x}_{i+1,\omega} \\ \vdots \\ \bar{x}_{17,\omega} \\ \bar{x}_{18,\omega} \end{bmatrix} \tag{3}$$

Analysis

To validate this approach, an 18-sector blisk lumped mass model is considered, with a sector-level illustration of this model shown in Fig. 3. Small mistuning is introduced at the blade level by proportionally varying both blade-specific spring constants (i.e., $k_{b_0}^i$ and $k_{b_1}^i$) by the same value. Here, large mistuning (for a rogue blade) is introduced by increasing both of these spring constants by 15%, though other values could be considered to generate large mistuning. A TWE with a vibration amplitude of 1 N is applied to mass $m_{b_2}^i$.

To evaluate this method's ability to predict blisk responses with both small and large mistuning, multiple rogue blisks with different small mistuning patterns were tested. Initially, a blisk with one small mistuning pattern (i.e., 18 small mistuning values, each value applied to one of the blisk blades) and varying number of rogue blades was considered. Small mistuning values are randomly sampled from a normal distribution with 0 mean and 1% standard deviation. Considering the same small mistuning pattern, the following results are obtained for blisks with 1 rogue blade, 2 rogue blades, and 17 rogue blades. It

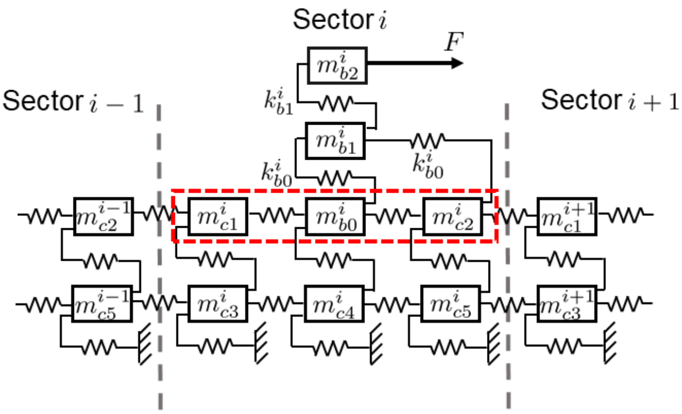

Fig. 3 Blisk lumped mass model – sector level viewpoint.

is important to emphasize that this approach was also tested by the authors for multiple other small mistuning patterns with the number of rogue blades ranging from 1 to 18. However, these tests are not shown in here for brevity.

For all three cases presented (i.e., same small mistuning pattern with 1 rogue blade, 2 rogue blades, and 17 rogue blades), Figs. 4, 5, and 6 show blisk response envelope predictions (i.e., the maximum response across all blades at each excitation frequency) with absolute percent errors all less than 8%. In particular, with only 1 or 2 rogue blades, the envelope response errors are below 6%. Future investigation is needed to better understand the higher errors associated with adding more rogue blades. Figures 7, 8, and 9 show response predictions for each blade root degree of freedom, where the dotted lines are the predicted responses using the proposed approach and the solid lines are the actual (i.e., reference) responses. These results show that this method accurately predicts the actual blade root responses, not only for the maximum values of the responses shown in Figs. 4, 5, and 6, but for all blade responses in the considered frequency range. The blade real and imaginary

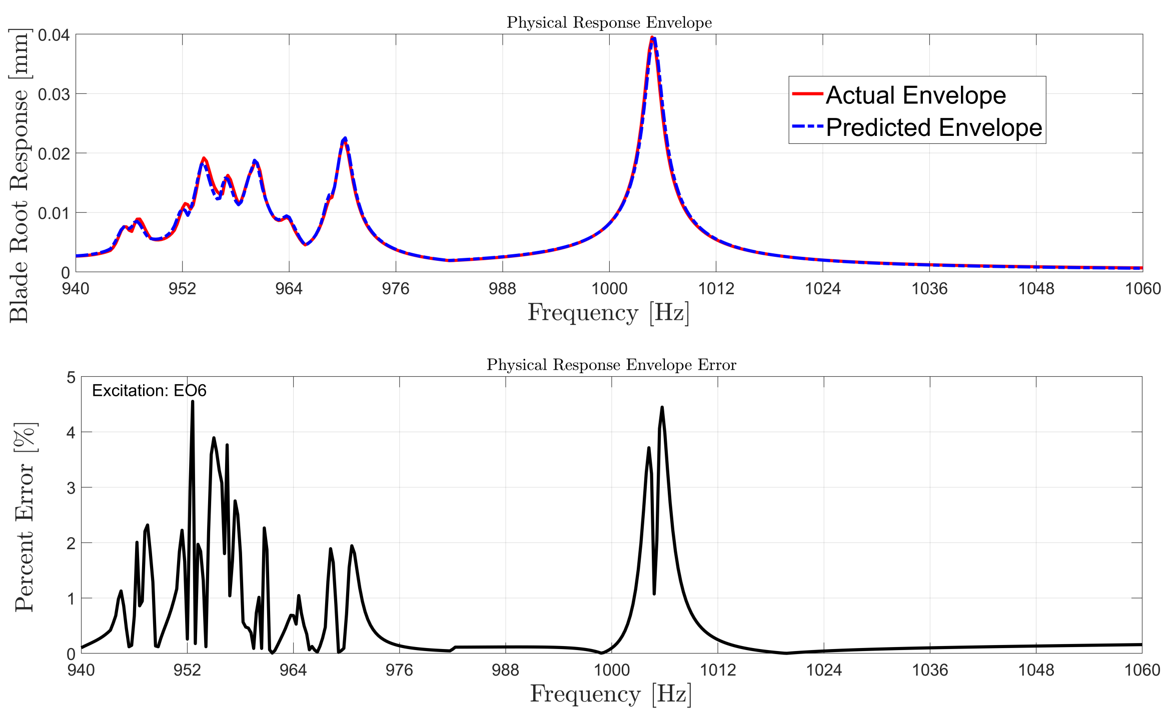

Fig. 4 (Top) Blisk response envelope prediction and (bottom) absolute percent error with small mistuning and 1 rogue blade.

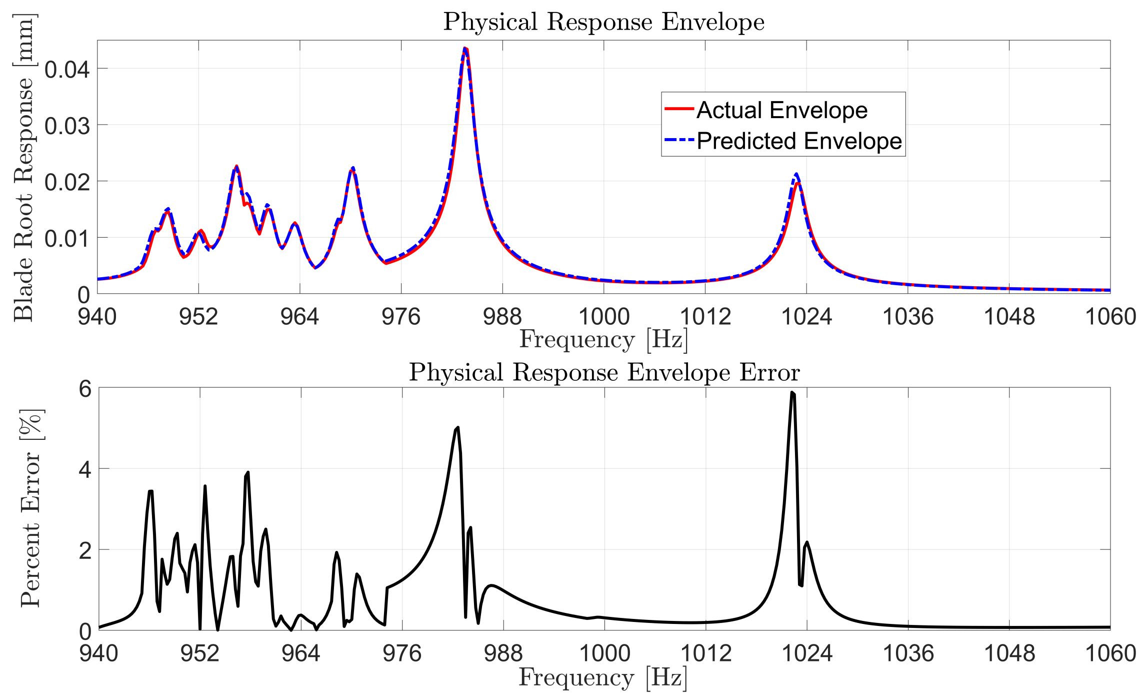

Fig. 5 (Top) Blisk response envelope prediction and (bottom) absolute percent error with small mistuning and 2 rogue blades.

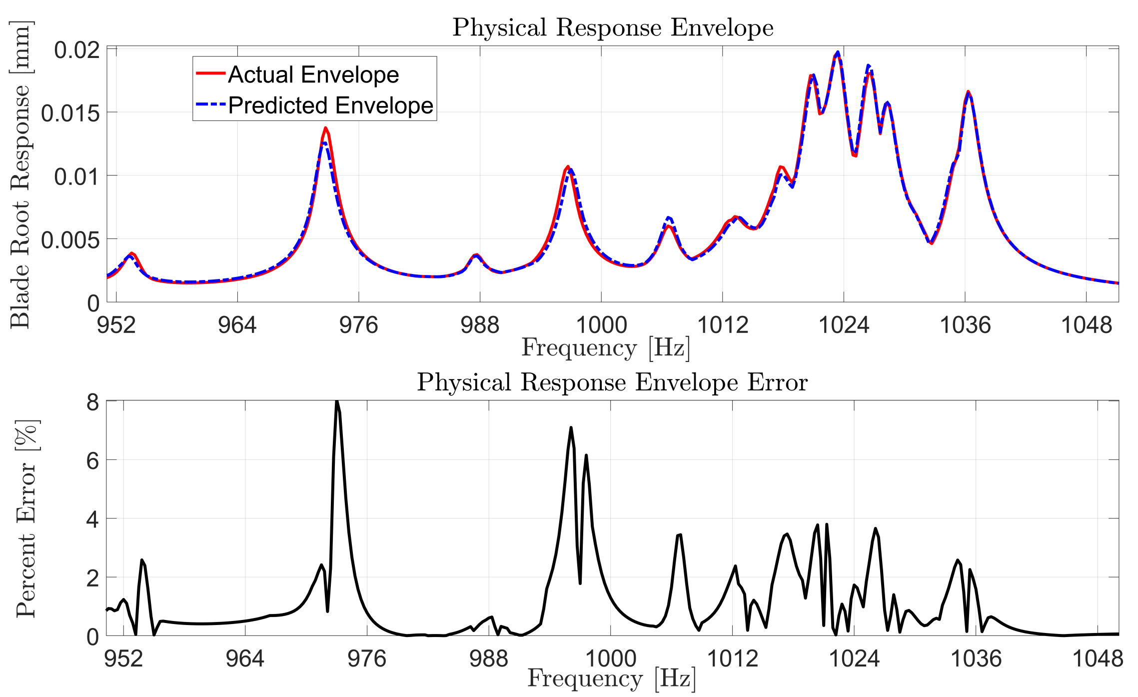

Fig. 6 (Top) Blisk response envelope prediction and (bottom) absolute percent error with small mistuning and 17 rogue blades.

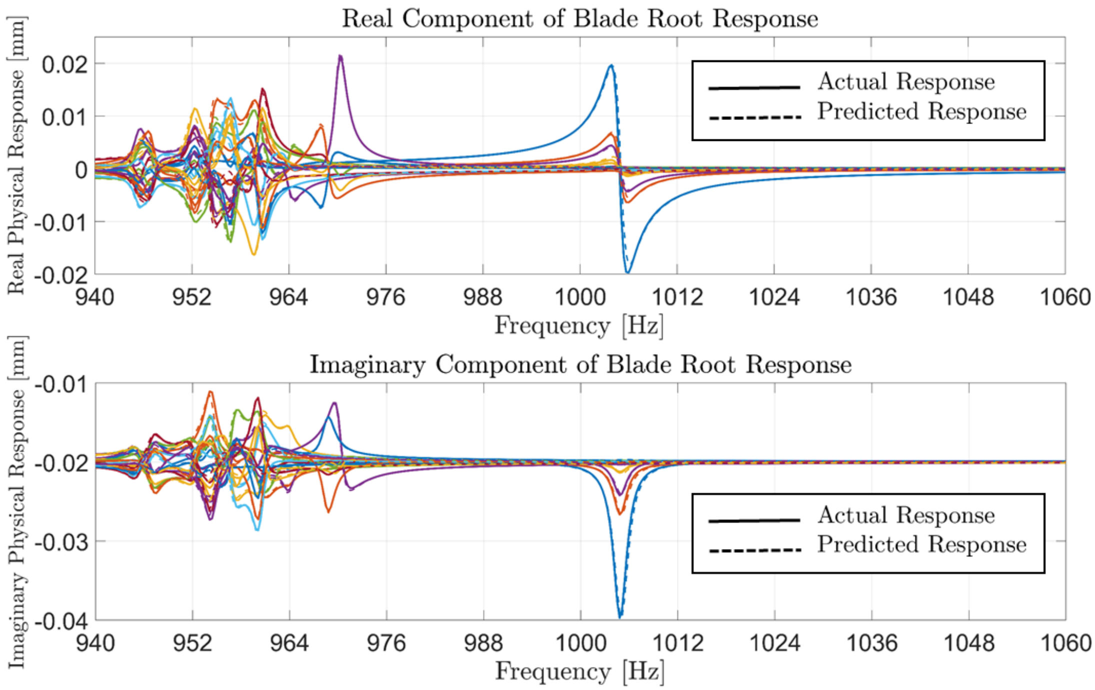

Fig. 7 (Top) Real and (bottom) imaginary blisk response prediction for all blades considering small mistuning and 1 rogue blade.

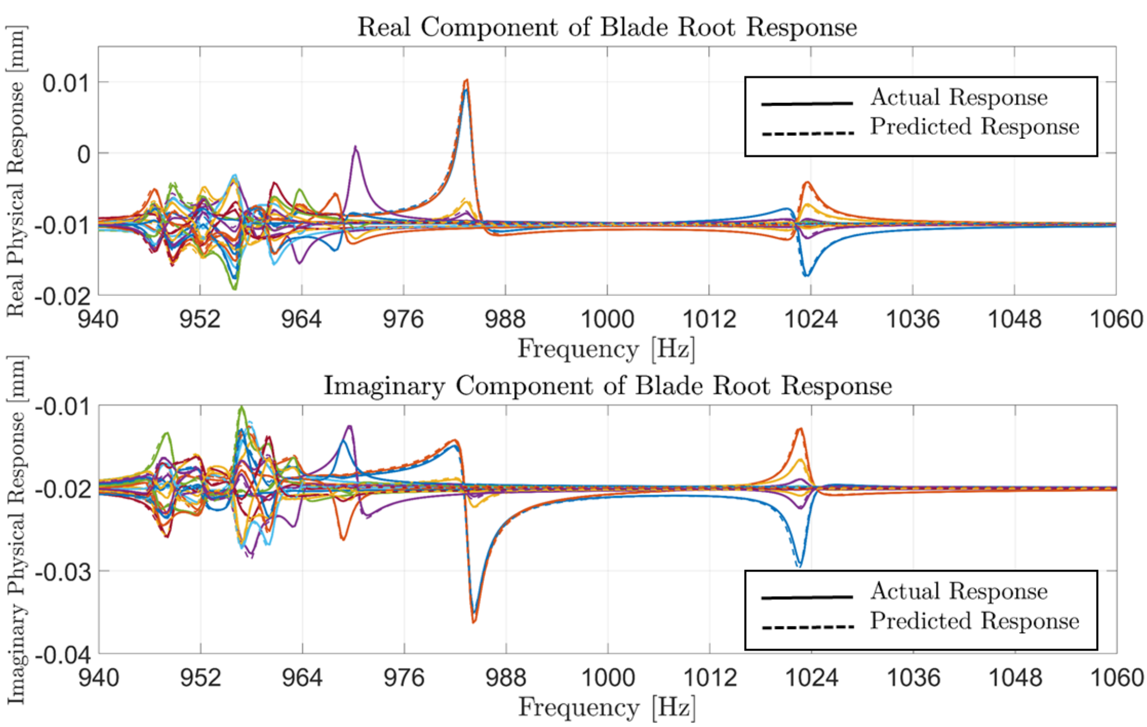

Fig. 8 (Top) Real and (bottom) imaginary blisk response prediction for all blades considering small mistuning and 2 rogue blades.

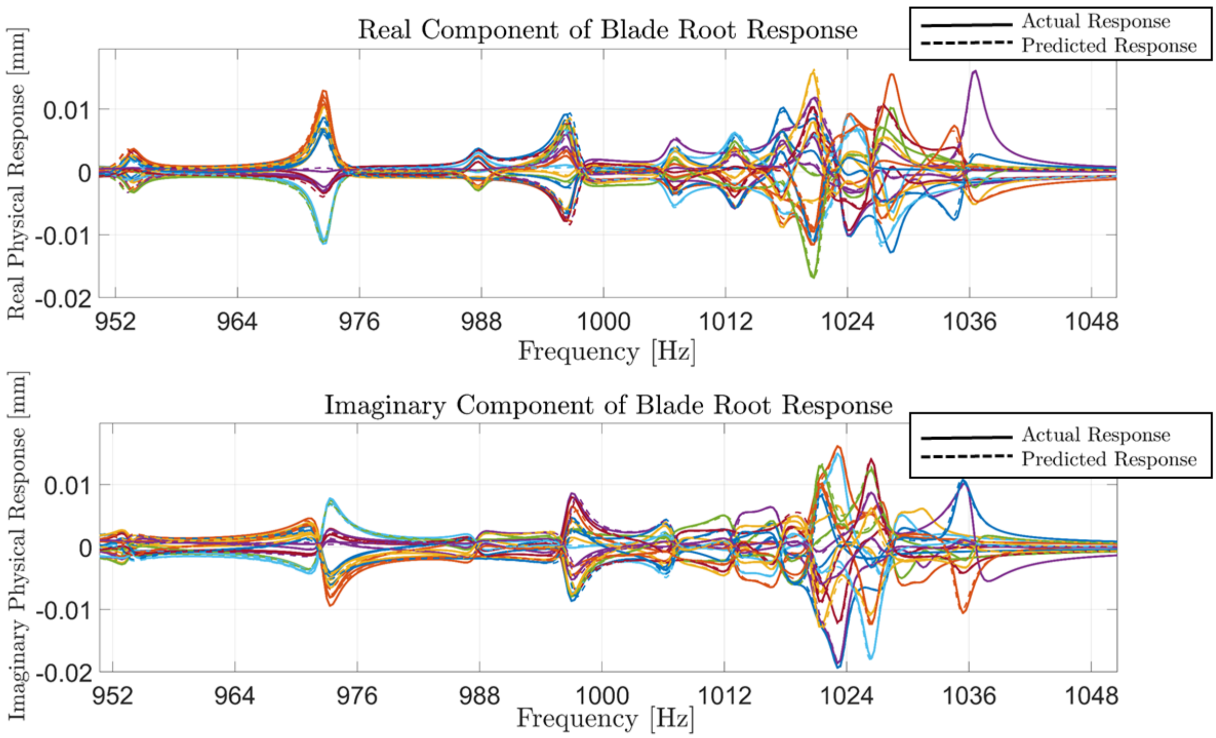

Fig. 9 (Top) Real and (bottom) imaginary blisk response prediction for all blades considering small mistuning and 17 rogue blades.

responses are reported separately because, as stated previously, the response vector $\overline{x}_{i,\omega}$ is split into real and imaginary components for training both PINNs.

Conclusion

To summarize, this paper proposes a physics-informed data-driven reduced-order modeling approach to predict blisk responses with small and large mistuning. This method builds upon a previous technique developed by the authors that leverages the blisk's cyclic properties and enforces system linearity through a sector-level PINN. The proposed method is the first physics informed data driven method able to capture blisk responses with both small and large mistuning. The method combines the transfer function matrices learned for a pristine blisk with small mistuning and a cyclic rogue blisk with small mistuning to predict the blade root responses of a blisk with one or more rogue sectors. The blade root responses can then be used to predict the blade tip response using a previously developed data-driven technique. The presented results show accurate predictions, with absolute percent errors below 8%. Future work includes identifying potential accuracy enhancements through tuning of the network training parameters (e.g., frequency resolution, frequency range, and amplitude normalization).

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