Chapter 1
Theoretical Foundations and Practical Applications of Damage Detection Using Autocovariance Functions

Jyrki Kullaa

Abstract Autocovariance functions (ACFs) can be utilized as damage-sensitive features in structural health monitoring. They have several advantages: they include the necessary information about the dynamic characteristics of the structure for damage detection, they are easy to estimate, they contain many data points, their accuracy can be controlled, and they are spatiotemporally correlated. Damage detection is performed in the time domain. ACFs have the same form as a free decay of the system for a stationary random process. A major issue of ACFs is that the free decay at a natural frequency includes cross-terms also from other modes. Therefore, the data lie in a high-dimensional space. For spatial correlation, the required number of sensors therefore increases considerably with an increasing number of active modes. Fortunately, the number of sensors can be reduced if temporal correlation is also utilized. Using spatiotemporal correlation, it is possible to detect changes in both natural frequencies and mode shapes. The objective is to pursue data redundancy, so that damage can be detected in the noise space. A simple and useful formula is presented, relating the number of active modes, the number of sensors, and the model order for damage detection. The conditions are also given for the design parameters limiting detection to frequency changes only or mode shape changes only. It is also shown that the variability of the excitation characteristics between measurements has no effect on damage detection. Numerical experiments were performed to validate the theoretical considerations.

Keywords Autocovariance function · Spatiotemporal correlation · Damage detection · Data redundancy · Model order

Introduction

Structural health monitoring (SHM) aims at receiving an early warning about structural damage or failure. Damage detection is based on vibration data measured from the structure during normal operation. Therefore, the excitation cannot be controlled or measured, and it may also vary between measurements. If the changes of the dynamic characteristics of the structure are statistically significant, it is an indication of damage. Training data are first acquired under different environmental or operational conditions to distinguish changes due to damage from changes due to environmental or operational variability.

Damage detection in the time domain is an attractive alternative to parameter-based methods, because time domain data include more data points and system identification is not needed. Covariance functions have been used in damage detection [3–12]. In this study, autocovariance functions (ACFs) are used as damage-sensitive features. They have been shown to work with numerical and experimental data [11].

In this paper, the applicability of ACFs for damage detection is theoretically investigated. More specifically, a relationship between the number of active modes, the number of sensors, and the model order in the data analysis is explored. Additionally, the effect of the variability of the excitation characteristics is studied (operational variability).

The objective of the proposed SHM system design is to make the measurement data redundant. Then, some variables can be computed using the remaining variables. For a redundant system, the data space can be divided into two subspaces, the signal space and the noise space. The signal space contains useful information of the vibration and the environmental or operational effects. The noise space consists mostly of measurement error. This was also discussed in [12]. Once damage appears, the new data are assumed to appear also in the noise space of the training data, which is easier to detect, because the variance is assumably much smaller in the noise space than in the signal space. It should be noted, however, that the

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existence of the noise space does not guarantee that the data from the damaged structure enters the noise space. Therefore, the degree of redundancy should be increased if possible so that the dimension of the noise space is greater than one.

The paper is organized as follows. The autocovariance functions are introduced in the next section, in which important formulas are given for the SHM system design. The proposed damage detection and localization algorithm in the time domain follows. A numerical experiment is performed to validate the theoretical consideration. Finally, concluding remarks are given.

Autocovariance Functions for Damage Detection

If the excitation is stationary random, the autocovariance functions of the structural responses (e.g. acceleration, velocity, displacement, strain) have the same form as a free decay of the system, which is also the foundation of operational modal analysis (OMA) [13].

The autocovariance function of a zero-mean random process $\{y(t)\}$ is

$$R_{yy}(\tau) = E\left[y(t)y(t+\tau)\right] \tag{1}$$

where $E[\cdot]$ is the expectation operator and τ is the time lag. Using modal superposition of linear structures, the displacement response $\mathbf{y}(t)$ at time t can be written as

$$\mathbf{y}(t) = \sum_{r=1}^{n} \boldsymbol{\phi}_r q_r(t) \tag{2}$$

where n is the number of active modes, $\boldsymbol{\phi}_r$ is the mode shape vector of mode r, and $q_r(t)$ is the modal coordinate of mode r. If we denote ϕ_{ir} as the ith component of vector $\boldsymbol{\phi}_r$, the autocovariance function of the displacement degree-of-freedom (DOF) $\{y_i(t)\}$ is

$$R_{ii}(\tau) = E\left[\sum_r \phi_{ir} q_r(t) \sum_s \phi_{is} q_s(t+\tau)\right] = \sum_r \sum_s \phi_{ir}\phi_{is} E\left[q_r(t) q_s(t+\tau)\right] \tag{3}$$

For a stationary random process with white noise excitation, the ACF can be derived [13, 14, 5]:

$$R_{ii}(\tau) = \sum_{s=1}^{n} \left[A_{is}\cos\left(\omega_{ds}\tau\right) + B_{is}\sin\left(\omega_{ds}\tau\right)\right] e^{-\zeta_s\omega_s} \tag{4}$$

where

$$A_{is} = \sum_{r=1}^{n} F_{rs}\phi_{ir}\phi_{is}\boldsymbol{\phi}_r^T \mathbf{B}\mathbf{Q}\mathbf{B}^T \boldsymbol{\phi}_s \tag{5}$$

and

$$B_{is} = \sum_{r=1}^{n} G_{rs}\phi_{ir}\phi_{is}\boldsymbol{\phi}_r^T \mathbf{B}\mathbf{Q}\mathbf{B}^T \boldsymbol{\phi}_s \tag{6}$$

where

$$F_{rs} = \frac{1}{2m_r\omega_{dr}m_s\omega_{ds}}\left[\frac{-\zeta_r\omega_r-\zeta_s\omega_s}{\left(\zeta_r\omega_r+\zeta_s\omega_s\right)^2+\left(\omega_{dr}+\omega_{ds}\right)^2} + \frac{\zeta_r\omega_r+\zeta_s\omega_s}{\left(\zeta_r\omega_r+\zeta_s\omega_s\right)^2+\left(\omega_{dr}-\omega_{ds}\right)^2}\right] \tag{7}$$

and

$$G_{rs} = \frac{1}{2m_r\omega_{dr}m_s\omega_{ds}}\left[\frac{\omega_{dr}+\omega_{ds}}{\left(\zeta_r\omega_r+\zeta_s\omega_s\right)^2+\left(\omega_{dr}+\omega_{ds}\right)^2} + \frac{\omega_{dr}-\omega_{ds}}{\left(\zeta_r\omega_r+\zeta_s\omega_s\right)^2+\left(\omega_{dr}-\omega_{ds}\right)^2}\right] \tag{8}$$

where m_r, ω_r, ω_{dr}, ζ_r are, respectively, the modal mass, the undamped natural circular frequency, the damped natural circular frequency, and damping ratio of the mode r. Matrix $\mathbf{B}$ is the load distribution matrix, and $\mathbf{Q}$ is a diagonal matrix where the diagonal entries are the spectral densities of the independent loads. $\mathbf{B}$ is assumed to be constant during a single measurement. An example of an ACF with 300 lags is shown in Fig. 1, when the number of modes is 7.

From Equation (4), it can be seen that the ACF has the same form as a free decay of the system. However, the decay at natural frequency ω_{ds} includes also cross-terms from other modes, as seen in Equations (5) and (6). This can make damage detection challenging, because the modal amplitudes ratios between sensors (A_{is}/A_{js} and B_{is}/B_{js}) may be different in each measurement.

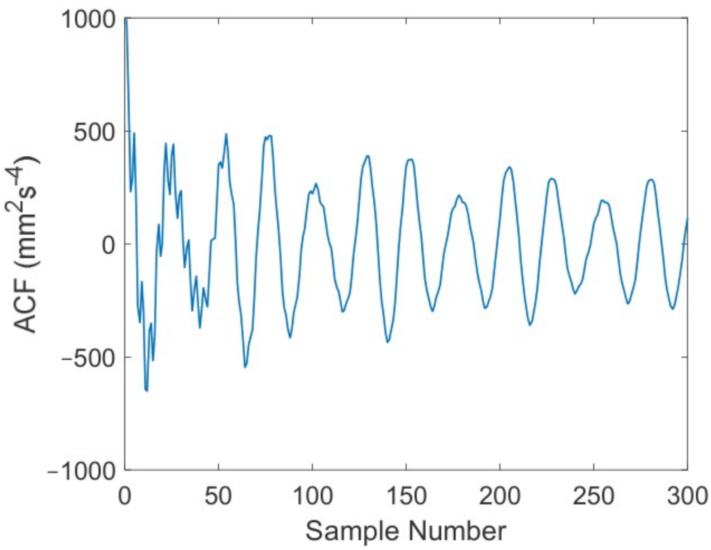

Fig. 1 Autocovariance function with 7 active modes

In the design of an SHM system, three parameters are important for damage detection: the number of active modes *n*, the number of sensors *p*, and the model order *m* used in the data analysis. The number of active modes cannot be controlled, but it affects the selection of the other two parameters. If the number of active modes is too large, filtering could be applied to remove the contribution of some modes. Once *n* is known, the number of sensors can be determined. This decision influences the hardware and maintenance costs and should be kept to a minimum. Optimal sensor placement is beyond the scope of this study but should also be considered. If changes in both the mode shapes and frequencies must be detected, this number must be at least equal to *n*. Once the first two parameters are fixed, the model order can be determined. It only affects the duration of the data analysis and the memory usage without additional hardware costs.

The selection of design parameters is based on redundancy of the ACF data. The redundancy can be spatial, temporal, or spatiotemporal. The results are only presented here without lengthy derivations.

Spatial correlation ($m = 0$) detects only changes in the mode shapes. The required number of sensors *p* is

$$p > \frac{1}{2} n (n + 1) \tag{9}$$

For many real structures, this condition may yield too large a sensor network and high costs.

Temporal correlation ($1 \leq p \leq n$) detects only changes in the natural frequencies or modal damping. The model order used in the analysis must be

$$m > 2n \tag{10}$$

Spatiotemporal correlation ($p \geq n$) can detect both changes in the frequencies, damping, and mode shapes. The model order *m* must be

$$m > \frac{2n^2}{p} - 1 \tag{11}$$

These formulas, especially Eq. (11) can be applied to the design of an SHM system. Note that environmental influences were not considered in the formulas. Their contribution can be taken into account by increasing the number of sensors *p* and the model order *m*.

Damage Detection Algorithm

The damage detection algorithm is briefly introduced. More details can be found in [15]. For each data set and for each sensor separately, the ACFs are estimated. The user has to determine the number of lags in the ACFs. The training data are comprised of the ACFs from the undamaged structure under different environmental or operational conditions. The data are copied and shifted *m* times for spatiotemporal correlation. The model order *m* is determined by the user with the help of

Eqs. (10) or (11). A covariance matrix of the time-shifted training data is estimated. The covariance matrix is subjected to a whitening transformation for data normalization. When new data (ACFs) arrive, the same whitening transformation is applied to these data. If the dynamic characteristics of the structure have changed, the new data are assumed to appear in the noise space and thus outside the hypersphere (for Gaussian data). This can be detected by applying principal component analysis to all data. The first principal component is now dictated by the data points outside the hypersphere. The first principal component scores are only retained, reducing the data dimensionality to one. Extreme value statistics (EVS) distributions [16] are identified for the scores of the training data by dividing the data into equal sized subgroups. The extreme values of each subgroup are plotted on a control chart with control limits determined from the probability of a false alarm [17]. If the data points of the test data are outside the control limits, an alarm is raised.

Numerical Experiment

The structure being monitored was a bridge deck with a concrete slab and steel stiffeners (Fig. 2). A detailed description of the model can be found in [18]. The first 7 modes were active, having natural frequencies of 3.95 Hz, 5.35 Hz, 13.7 Hz, 15.4 Hz, 17.9 Hz, 24.1 Hz, and 24.8 Hz. The corresponding damping ratios were 0.01, 0.01, 0.02, 0.02, 0.03, 0.03, and 0.03.

The sampling interval was $\Delta t = 0.01$ s and the measurement period was almost 44 min including $2^{18} = 262144$ samples. A long measurement period is necessary to provide accurate ACF estimates [19].

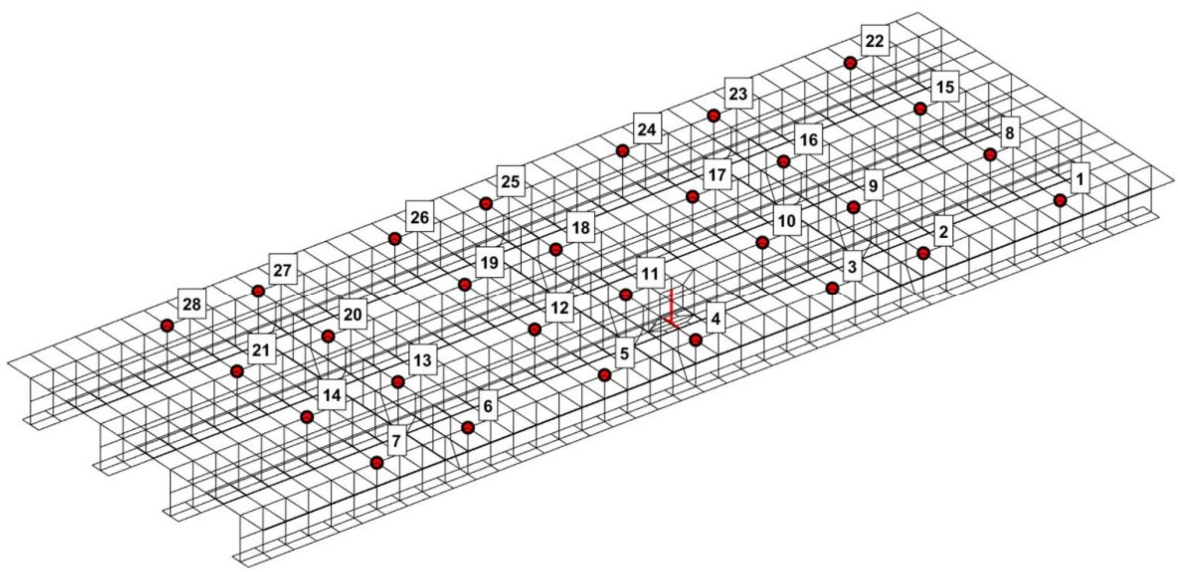

Fig. 2 Finite element model of the bridge deck with labeled sensor positions. Damage location is close to sensor 11

Because environmental influences were not considered in this study, the excitation was the primary cause of varying the ACFs (operational variability). Some variability came from the measurement noise, which was also different in each measurement. The number of independent excitations varied randomly between 1 and 3. Furthermore, the excitation points were randomly selected from all nodes at the joints between the steel webs and flanges. The load histories were generated in the frequency domain between 0 and 25 Hz. Each measurement had different load functions with random amplitudes and phases at each frequency pin. In addition, it was randomly determined if a loading had zero contribution at a randomly located 5 Hz interval. This simulated measurements in which some modes were not excited.

The FE analysis was performed in the frequency domain [20]. Measurement noise was simulated by adding Gaussian noise to each sensor. The average signal-to-noise ratio (SNR) was 30 dB. The ACFs were estimated from the long response time histories at lags τ between 0 and 3 s, resulting in a length of 300 data points in each measurement (Fig. 4a).

In the data analysis for damage detection, the ACFs were concatenated to form a long data matrix (Fig. 4b). To remove discontinuities between measurements, m data points had to be removed from each data set.

The first 100 data sets were from the undamaged structure and the next 36 data sets from the damaged structure with an increasing crack length (Fig. 3). The number of different crack lengths was 6, and each damage level was monitored with 6 simulations. The first 70 data sets were used as training data.

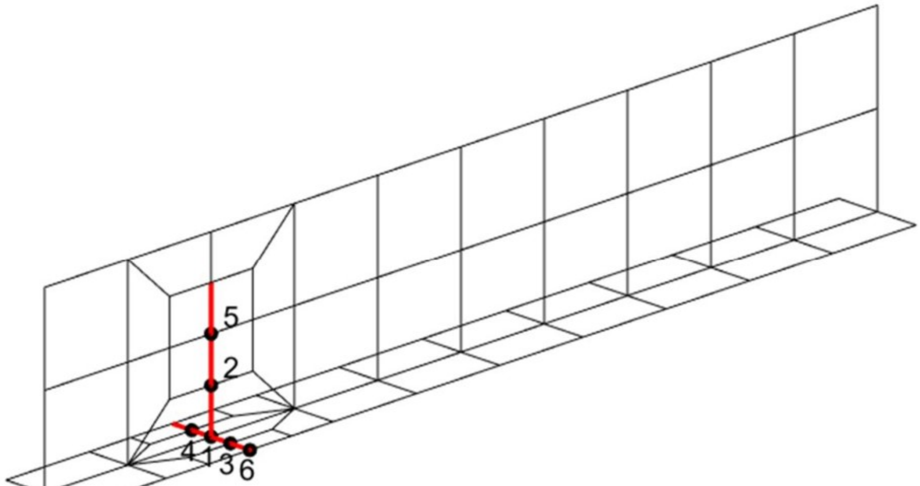

Fig. 3 A detail of the steel girder with a sequence of crack propagation [18]

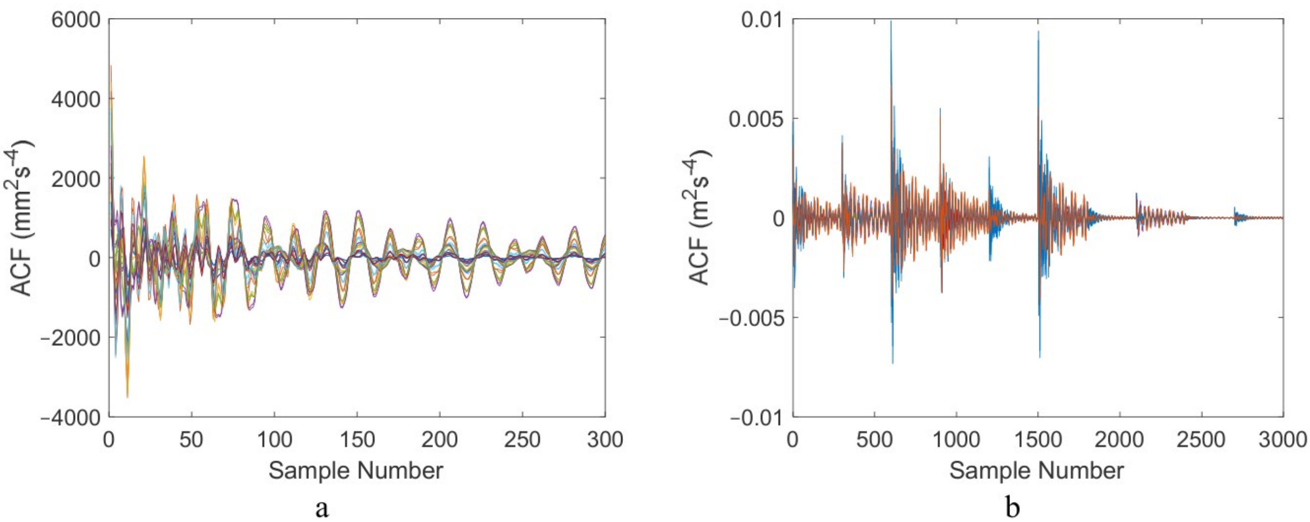

Fig. 4 a) Autocovariance functions of 28 accelerometers from measurement 1. b) Autocovariance functions of 9 accelerometers [2:3:28] from measurements 1 to 10

The whitening matrix was identified using the training data, but principal component analysis was applied to all data. The first principal component scores were only retained and divided into subgroups each consisting of all data from a single measurement. The minimum and maximum of each subgroup were used for damage detection by identifying extreme value distributions for the training data and determining the probability of a false alarm equal to 0.001 to compute the control limits. Finally, a control chart for the extreme values was plotted.

Damage localization was based on the projection of the first principal component on the sensor coordinates and selecting the largest length, which was supposed to reveal the sensor closest to damage.

SHM systems for damage detection for different sensor arrays were designed. The number of active modes was assumed known ($n = 7$).

In the first case, spatial correlation ($m = 0$) was only considered. According to Eq. (9), the number of sensors had to be greater than 28. If all 28 accelerometers were used, the control chart in Fig. 5a resulted. Damage detection was successful even if the number of sensors was not sufficient. This is probably because distant modes had only a small contribution to the ACFs. Damage was localized to sensor 12 (Fig. 5b). Sensor 11 also had a high damage index. Because the crack was located between sensors 11 and 12, damage localization was considered successful. Note that with spatial correlation, changes in the mode shapes could only be detected.

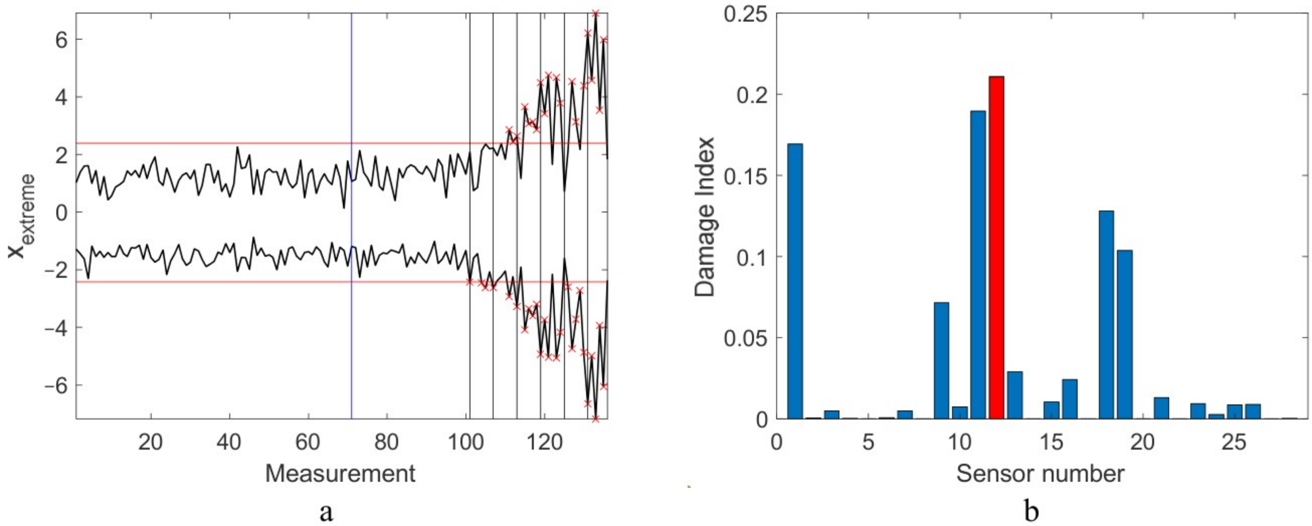

Fig. 5 a) EVS control chart for 28 accelerometers and spatial correlation ($m = 0$). Data on the left side of the leftmost vertical line were used as the training data, and the other vertical lines indicate the six damage levels with increasing crack lengths. Logarithmic scaling was applied to the extreme values. b) Damage localization

The second system had 14 accelerometers (sensors 2:2:28). If spatial correlation was used, damage detection failed (Fig. 6). Using Eq. (11), the condition for the model order was $m > 6$. For the model order equal to $m = 10$, the control chart in Fig. 7a was plotted. Damage was successfully detected with occasional false positives. Damage was incorrectly localized to sensor 16 (Fig. 7b).

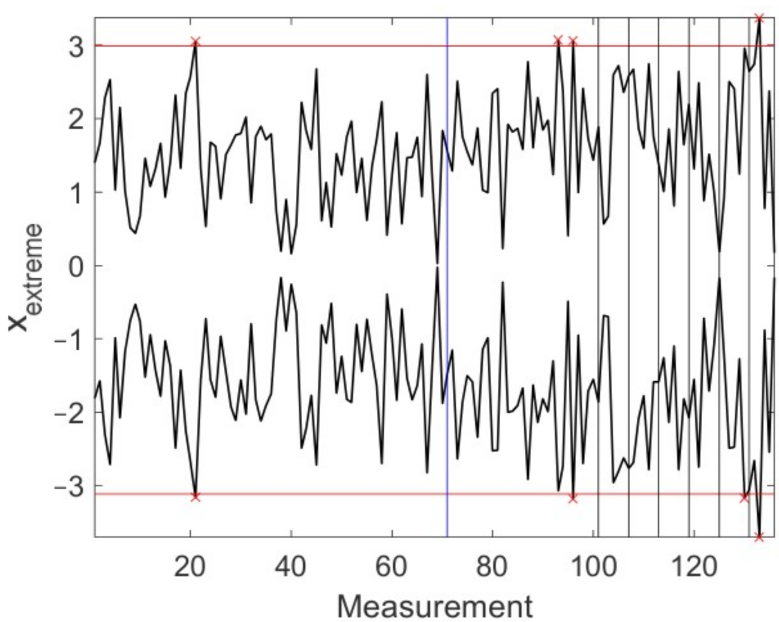

Fig. 6 EVS control chart for 14 accelerometers [2:2:28] and spatial correlation ($m = 0$)

For the third system, hardware costs were further reduced by installing only 9 accelerometers (sensors 2:3:28). Now, the model order in the data analysis had to be increased ($m > 9.9$). If the model order equal to $m = 16$ was used, the result was the control chart in Fig. 8a showing successful detection, but incorrect damage localization to sensor 8 (Fig. 8b). On the other hand, if too small a model order was used ($m = 4$), damage could not be detected (Fig. 9a).

The fourth system had only three accelerometers (sensors 2, 11, and 20), implying that temporal correlation was only possible. According to Eq. 10, the model order had to be greater than 14. Using $m = 20$ resulted in the control chart in

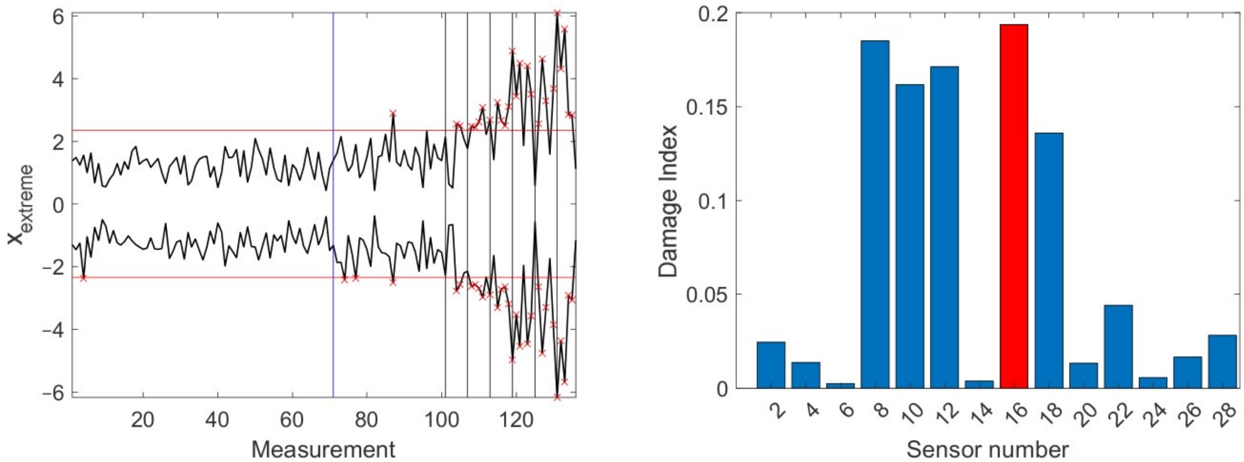

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Fig. 7 a) EVS control chart for 14 accelerometers [2:2:28] and spatiotemporal correlation ($m = 10$). b) Damage localization

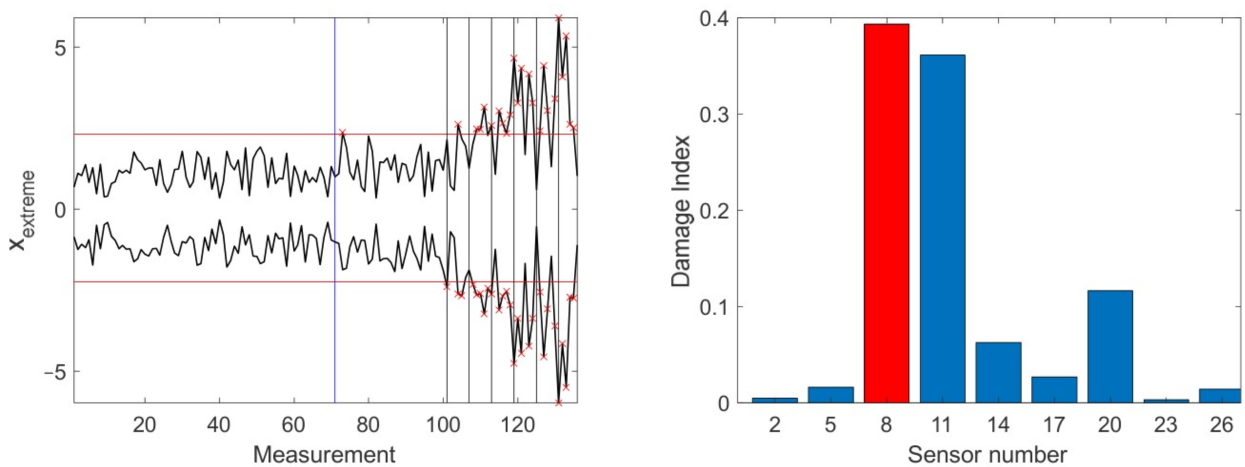

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Fig. 8 a) EVS control chart for 9 accelerometers [2:3:28] and spatiotemporal correlation ($m = 16$). b) Damage localization

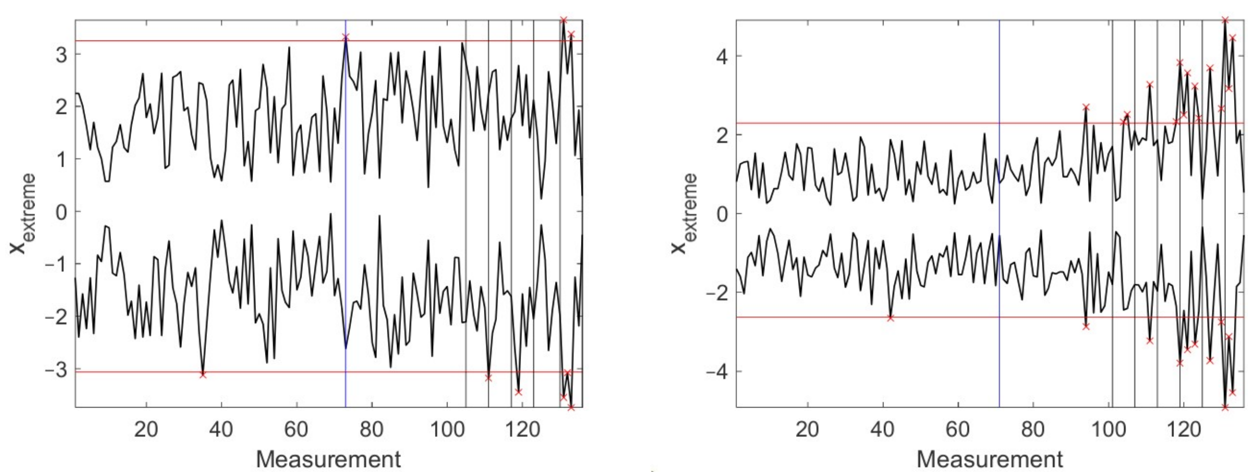

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Fig. 9 a) EVS control chart for 9 accelerometers [2:3:28] and spatiotemporal correlation ($m = 4$). b) EVS control chart for 3 accelerometers [2, 11, 20] and temporal correlation ($m = 20$)

Fig. 9b with reasonable damage detection capability. Detection of natural frequency changes was only possible. Damage was correctly localized to sensor 11 having the largest damage index (not shown).

For systems 2–4, it was observed that the theoretical minimum for the model order m was not optimal, but a slightly larger number yielded better results. The reason was probably an increased dimensionality of the noise space, increasing the probability of the test data to appear in that space. In addition, systems 2–3 with spatiotemporal correlation were able to detect changes in both frequencies, damping, and mode shapes.

Damage localization was correct only when using spatial or temporal correlation. With spatiotemporal correlation, localization failed. On the other hand, the sensor closest to damage also had a high damage index in all cases.

Conclusion

Autocovariance functions are promising damage-sensitive features in the time domain. Their usage was theoretically and numerically explored. Practical formulas for the design parameters were given for the design of an SHM system. If the number of active modes is known, the minimum number of sensors can be computed. Once the number of sensors is determined, the required model order used in the data analysis can be computed.

The main conclusions are:

1. To detect both frequency and mode shape changes, the number of sensors must be at least equal to the number of active modes. The required model order can then be computed. This spatiotemporal correlation model is suggested for most cases.
2. If the number of sensors is less than the number of modes, only changes in frequencies and damping can be detected but not changes in mode shapes.
3. For spatial correlation, changes of mode shapes can only be detected. A large number of sensors is necessary, which is impractical in many cases.
4. In spatiotemporal correlation analyses, a slightly greater model order than the required minimum was found to yield optimal results.
5. The excitation variability has no effect on damage detection, provided the process is stationary random and the design parameters fulfil the given conditions.
6. The proposed method yielded incorrect damage localization in some cases, but a large damage index resulted also for the sensor closest to damage.

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