MODEL PREDICTIVE CONTROL OF ELECTRO-HYDRAULIC SYSTEMS WITH MULTIPLE DEGREES OF FREEDOM

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ABSTRACT

Modern hydraulic drives have an ever-increasing power density and robustness, however they become more and more complex in their design and control. In many systems, there is the possibility of using an overdetermination of system control inputs to optimize the operating strategy for energy efficiency or tracking error. Many applications are still largely based on empirical and hard coded rules. More advanced methods are using offline optimization algorithms [1] to calculate an optimized trajectory for more than one manipulated variable for a given command trajectory. However, this approach leads to a lack of robustness and flexibility if model equations are not exact enough, real time control is required or the operating point changes. Traditional algorithms lack in standardisation and scalability which is also crucial for success in the industry. To overcome the disadvantages of rule based or offline optimization methods this paper presents the fundamentals and the application of Model Predictive Control (MPC) with respect to electro-hydraulic drives. Furthermore, possibilities are described to make advanced algorithms economically transferable into series production.

Keywords: model predictive control, digitalization, sustainability, optimization

1. INTRODUCTION

1.1. Motivation

The system under investigation comprises two variable displacement hydraulic axial piston pumps attached to one single speed variable motor shaft. The pumps drive respective flows for two chambers of a differential hydraulic cylinder. Leveraging the flexibility of this setup, MPC is used to exploit the available input degrees of freedom (DOF) to match and weight different performance goals and constraints [2]. The algorithm demonstrates the effectiveness of optimization-based control in managing the system for optimal energy strategies and control under state constraints.

2. OPTIMAL CONTROL OF AN ELECTROHYDRAULIC SYSTEM WITH MULTIPLE INPUTS

2.1. Problem Description

Model predictive control is a broad family of methods which solve an optimal control problem iteratively. If one stays in matured space of MPC a well-established set of standardized implementations is at hand. Here, the application is a complex electro-hydraulic system with three-degree-of-freedom (3DOF) in its control inputs. The system consists of a permanent magnet synchronous motor, two displacement variable axial piston pumps on a single speed variable shaft and a differential hydraulic cylinder. The basic idea is to actively use the shaft speed and the two pump displacements to control the cylinder position while optimizing a cost function and complying with the defined boundary conditions.

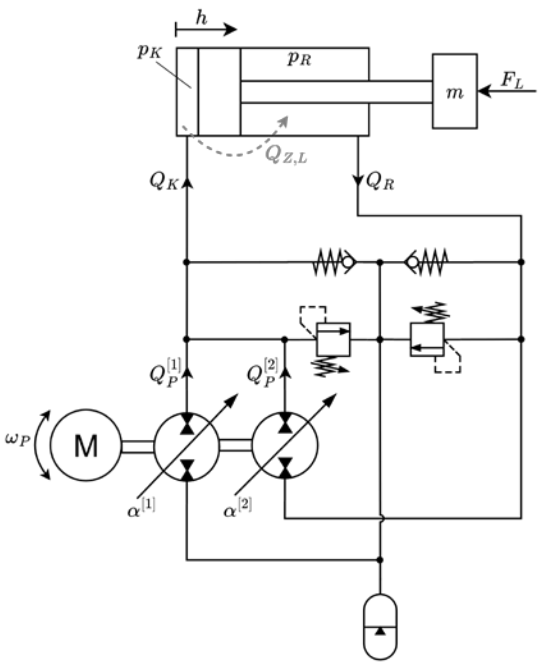

Figure 1: Scheme for the electrohydraulic drive system – summating transformer

Table 1: Parameters of the system

Cylinder parameter	Value	
Area A_K	85.33×10^{-3}	[m²]
Area A_R	42.66×10^{-3}	[m²]
Mass m	3.0×10^{3}	[kg]
Volume pump 1,2	4.0×10^{-5}	[m³]
Piston stroke	0.4	[m]

2.2. Modelling of the drive system

The continuous-time model of the hydraulic cylinder drive system, with two pumps on a speed variable shaft, can be expressed in state space as in figure 2. The control inputs are $\omega_{P,cmd}$, $\alpha_{cmd}^{[1]}$ and $\alpha_{cmd}^{[2]}$. From top to bottom $\dot{p_K}$ and $\dot{p_R}$ describe the pressure dynamics of the respective chamber. The piston velocity $\dot{h}$ and piston acceleration $\ddot{h}$ describe the mechanical equation of the cylinder. The swivel rate $\dot{\alpha}^{[1,2]}$ and swivel acceleration $\ddot{\alpha}^{[1,2]}$ describes the swivel angle dynamics of both pumps, simplified to a second order system. The rotational speed ω_p and torque generating current I_q finishes the set of equations. Several simplifications were done by neglecting the suction circuit and pressure relief valve in the prediction model. Both assumptions are permissible because the operating strategy will be chosen to avoid the opening of the suction valves by penalising the controller to drop below defined threshold pressure.

$$\dot{x} = \begin{pmatrix} \dot{p}_K \\ \dot{p}_R \\ \dot{h} \\ \ddot{h} \\ \dot{\alpha}^{[1]} \\ \ddot{\alpha}^{[1]} \\ \dot{\alpha}^{[2]} \\ \ddot{\alpha}^{[2]} \\ \dot{\omega}_P \\ \dot{I}_q \end{pmatrix} = \begin{pmatrix} \frac{E'(p_K)}{V_K(h)} \cdot \left(-G_{Z,L}(p_R - p_K) + Q_P^{[1]}(\eta_v) + Q_P^{[2]}(\eta_v) - \frac{\rho(p_K)}{\rho_0^{[f]}} A_K \dot{h} \right) \\ \frac{E'(p_R)}{V_R(h)} \cdot \left(G_{Z,L}(p_R - p_K) - Q_P^{[2]}(\eta_v) + \frac{\rho(p_R)}{\rho_0^{[f]}} A_R \dot{h} \right) \\ \frac{\mathrm{d}}{\mathrm{d}t} h \\ \frac{1}{m} \cdot (p_K A_K - p_R A_R - F_R - F_L) \\ \frac{\mathrm{d}}{\mathrm{d}t} \alpha^{[1]} \\ \frac{1}{\left(T^{[1]}\right)^2} \alpha_{cmd}^{[1]} - \frac{2D^{[1]}}{T^{[1]}} \dot{\alpha}^{[1]} - \frac{1}{\left(T^{[1]}\right)^2} \alpha^{[1]} \\ \frac{\mathrm{d}}{\mathrm{d}t} \alpha^{[2]} \\ \frac{1}{\left(T^{[2]}\right)^2} \alpha_{cmd}^{[2]} - \frac{2D^{[2]}}{T^{[2]}} \dot{\alpha}^{[2]} - \frac{1}{\left(T^{[2]}\right)^2} \alpha^{[2]} \\ \frac{1}{J} \cdot \left(k_M I_q - M_{M,R} - M_L^{[1]}(\eta_m) - M_L^{[2]}(\eta_m) \right) \\ \frac{1}{T_q} \cdot \left(k_n \left(\omega_{P,cmd} - \omega_P\right) + \frac{k_n}{T_n} \int \left(\omega_{P,cmd} - \omega_P\right) \, \mathrm{d}t \right) - \frac{1}{T_q} I_q \end{pmatrix}$$

Figure 2: Detailed dynamical model for the electro-hydraulic system

M_L represents the load torque, including mechanical efficiency model of the axial piston pumps. The function is described with the help of mechanical efficiency maps (3).

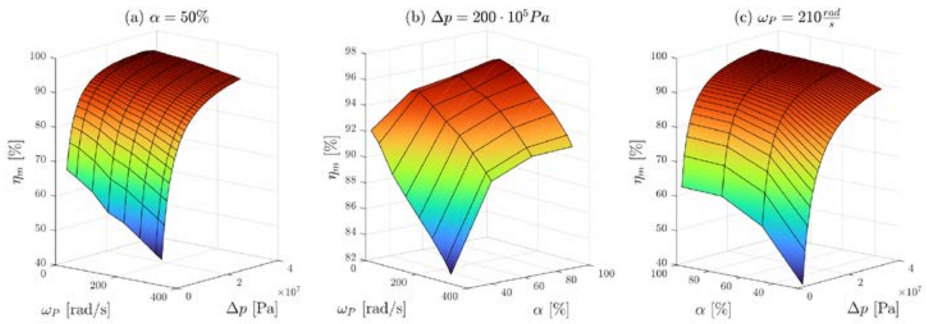

Figure 3: Mechanical efficiency maps regarding the axial piston pumps M_L

Q_p represents the pump flow, including the volumetric efficiency model of the axial piston pumps, which are described by volumetric efficiency maps (4).

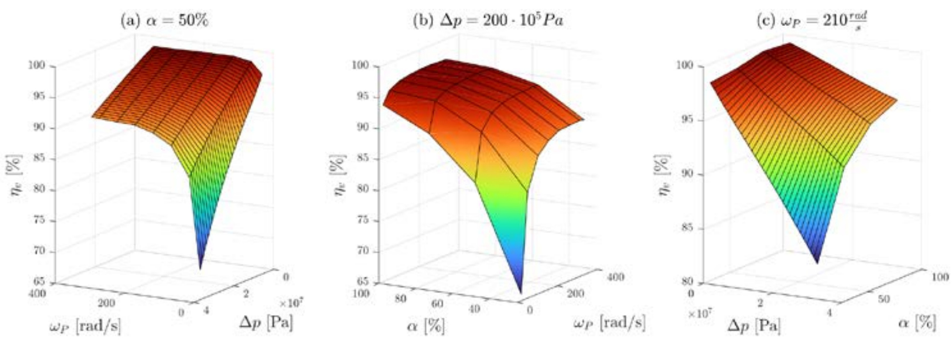

Figure 4: Volumetric efficiency maps regarding the axial piston pumps Q_P

The electric power losses are comprised by simplified formula for copper (1) and iron losses (2).

$$P_{Mot,Cu,eff} = \frac{3}{2} \cdot \mathrm{R_S} \cdot \left(\mathrm{I_{q,eff}}\right)^2 \tag{1}$$

Where R_S is the electrical resistance of the copper windings and $I_{q,eff}$ is the effective torque generating current.

$$P_{Mot,Fe} = \mathrm{k_{Hys}} \cdot \omega_M \cdot sign(\omega_M) + \mathrm{k_w} \cdot (\omega_M)^2 \tag{2}$$

ω_M is the rotational motor speed and $\mathrm{k_{Hys}}, \mathrm{k_w}$ are fitted motor factors. All these loss-models build the foundation for an operating strategy which allows the efficient operation of the system [1]. The aggregate loss model (3) consists of the volumetric and mechanical losses for both pumps, the iron and copper losses of the motor, the inverter losses and mechanical cylinder losses. For further details, we recommend studying [2].

$$P_{loss} = P_{P1,Vol} + P_{P2,Vol} + P_{P1,Mech} + P_{P2,Me} + P_{Mot,Fe} + P_{Mot,Cu} + P_{Mot,Me} + P_{Cv,f} + P_{Cyl,Me} \tag{3}$$

3. CONTROL DESIGN

3.1. MPC and adaptive linear MPC

Model Predictive Control (MPC) is an advanced control strategy that computes control inputs by solving an optimization problem at each time step. It involves the use of a model of the system to predict its future behaviour over a finite horizon. The MPC algorithm computes the control inputs in a way that minimizes a cost function, typically by finding a compromise between the deviation from a desired command trajectory and the control effort. Once the optimization problem is solved, only the first control input of the optimized sequence is applied to the system. Then the whole process is repeated at the next time step. This 'receding horizon' strategy allows MPC to handle multivariable systems with constraints on inputs, system states, and outputs.

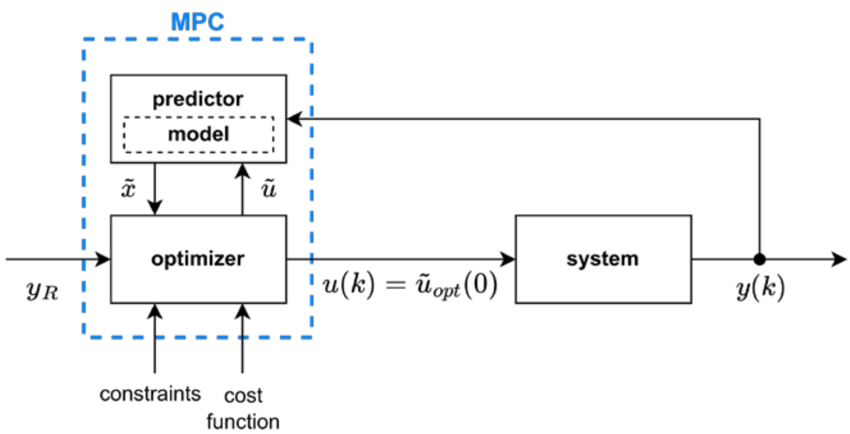

Figure 5: Traditional MPC scheme

Adaptive Linear MPC (ALMPC) on the other hand, is an extension of the traditional MPC approach, that is tailored for systems with time-varying linear dynamics. It involves an online update of the linear model used in the MPC optimization as new data becomes available. This enables the controller to adjust to changes and nonlinearities in the system dynamics.

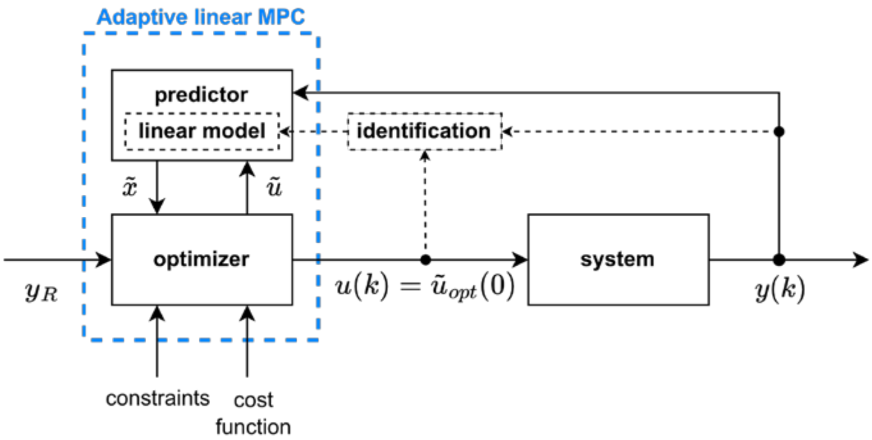

Figure 6: Adaptive linear MPC scheme

A general disadvantage of MPC is the high computational demand. It scales significantly with number of states N, number of constraints m and number of inputs n. For example, for the interior point method the complexity is $\mathcal{O}(\mathrm{N}^3 \cdot (n+m)^3)$ [4].

Additionally, the prediction horizon n_P, the control horizon n_c and the controller sample time T_s are also critical parameters in MPC frameworks. n_P limits the time frame for forecasting the system's trajectory, while n_c defines the duration over which control actions are optimized. A higher n_P leads to a more forward-looking but also increases the computational burden exponentially. Similarly, extending n_c generally results in more optimized control sequences but at the cost of higher computational demand. A smaller T_s leads to a higher computational burden but also to more accurate control behavior.

Therefore, the target is to use the least number of states, constraints, and inputs as well as the biggest control sample-time, the shortest prediction and control horizon as possible, while preserving the advantages of the optimal control strategy. To reach both, accuracy and computational viability for industrial control systems, a detailed operating strategy (reference) and a reduced operating strategy are implemented and validated in simulation.

3.2. Full order model operating strategy

Optimization problem formulation

The detailed variant is benchmarked against a state-of-the-art P-controlled system [5] (with acceleration feedback). The controller is only utilising the speed input of the pump and the swivel angle is set to maximum for both pumps.

For the strategy itself there are two important aspects, the cost function (4), and the state and input constraints (5) of the problem. The cost function consists of the following parts: control deviation

$e_{Control}$, rate of control inputs du and the total loss energy use $E_{Sys,loss}$.

$$[\mathrm{u,x,s}]^{\mathrm{T}} \ = \ \mathbf{argmin} \ \mathrm{f}_{cost} \ (\mathrm{e}_{Control} \ , \mathrm{du}, \mathrm{E}_{\mathrm{Sys,loss}}) \tag{4}$$

The constraints are set by the system boundaries. It is distinct between hard and soft constraints, whereas hard constraints (5) are mandatory to be hold. In this case minimal and maximal pump speed and minimal and maximal swivel angle are considered.

$$\begin{aligned}
\omega_P - \omega_P^{[max]} &\leq 0 \\
-\omega_P - \omega_P^{[max]} &\leq 0 \\
\pm\alpha_{cmd}^{[1,2]} - 1 &\leq 0 \\
\pm\omega_{P,cmd} - \ \omega_P^{[max]} &\leq 0 \\
-|\omega_P \ | - \omega_P^{[min]} &\leq 0
\end{aligned} \tag{5}$$

Soft constraints (6) are on the pressure A and B port. Soft constraints can be violated if no feasible solution can be found. For this reason, a slack variable is introduced to weight such violations negatively in the cost function. However, the slack variables have been omitted for reasons of clarity.

$$\begin{aligned}
p_K - p^{[max]} &\leq 0 \\
p_R - p^{[max]} &\leq 0 \\
-p_K + p^{[min]} &\leq 0 \\
-p_R + p^{[min]} &\leq 0
\end{aligned} \tag{6}$$

The total loss energy of the system (7) is calculated by the integral over the total loss power equation (3).

$$E_{Sys,loss} = \int P_{loss} \, dt \tag{7}$$

Results

The design and performance evaluation of the MPC algorithm is based on a reference cycle (7). The use of a reference cycle helps to evaluate the advantage of the proposed control scheme for a specific or generic task.

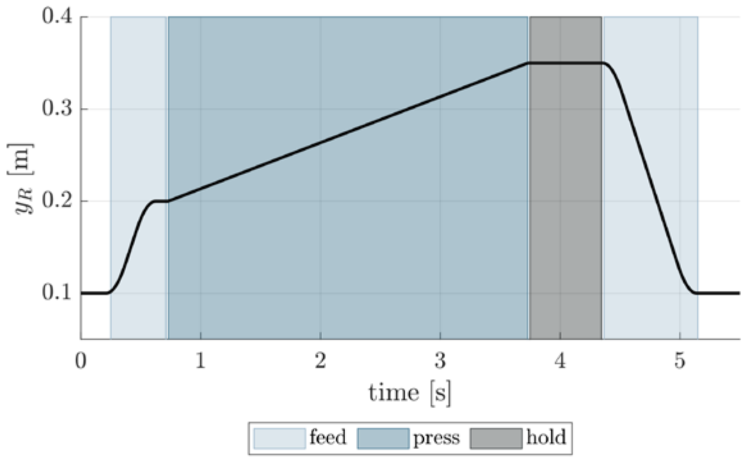

Figure 7: Reference cycle

Figure (8) shows the comparison of three different approaches: A state-of-the art p-controller (with acceleration state feedback), the ALMPC method without energy efficiency optimization and ALMPC method with energy efficiency optimization.

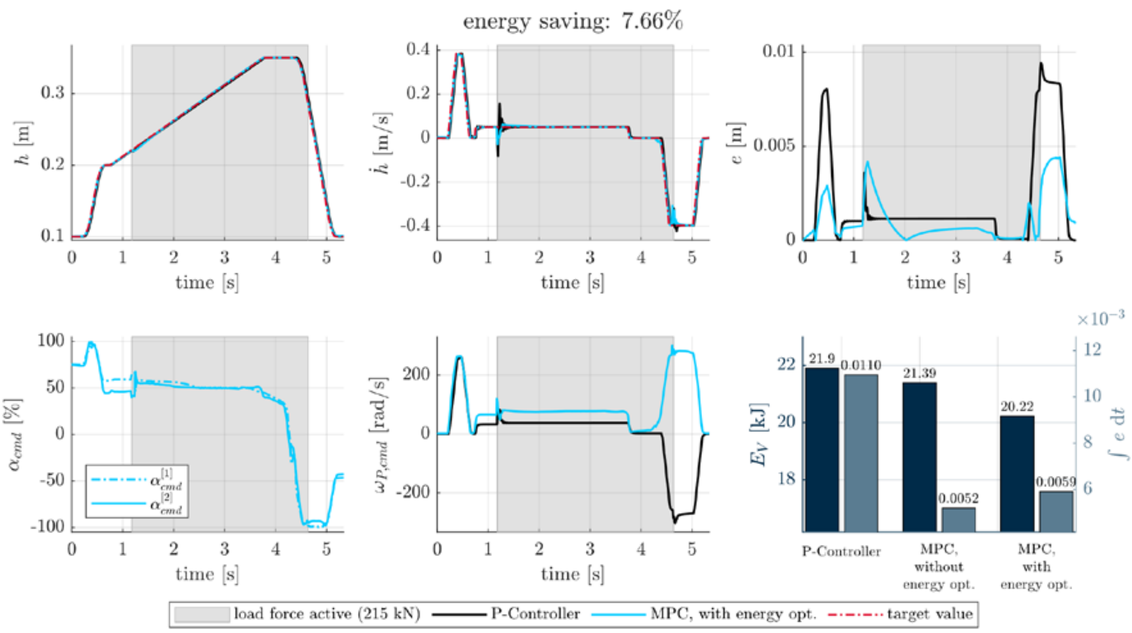

Figure 8: Result for reference cycle n_P = 40, n_C = 12 and T_S = 10ms

After a hyper parameter tuning (parameter set n_P = 40, n_C = 12 and T_S = 10 ms) the results for the benchmark cycle show that both conflicting performance indicators could be improved at the same time. For MPC without energy optimization the control accuracy is slightly better than MPC with energy optimization. Both approaches yield a significant improvement over the P-controller benchmark for both energy usage and tracking performance.

3.3. Reduced Order Model Operating Strategy

Optimization problem formulation

To bring the MPC approach closer to the real-time applicability, computational burden must be reduced. To speed up the embedded optimization problem a reduced order prediction model, fewer constraints and a new reduced cost function is formulated. These simplifications are done while the goal is to preserve most of the results of the high order model.

$$\dot{x} = \begin{pmatrix} \dot{p}_K \\ \dot{p}_R \\ \dot{h} \\ \ddot{h} \end{pmatrix} = \begin{pmatrix} \frac{E'(p_K)}{V_K(h)} \cdot \left(-G_{Z,L}(p_R - p_K) + Q_P^{[1]}(\eta_v) + Q_P^{[2]}(\eta_v) - A_K\dot{h}\right) \\ \frac{E'(p_R)}{V_R(h)} \cdot \left(G_{Z,L}(p_R - p_K) - Q_P^{[2]}(\eta_v) + A_R\dot{h}\right) \\ \frac{\mathrm{d}}{\mathrm{d}t}h \\ \frac{1}{m} \cdot (p_K A_K - p_R A_R - F_R - F_L) \end{pmatrix}$$

Figure 9: Reduced dynamical model for the electro-hydraulic system

First, the required effort to tune the weights of the cost function is reduced considerable by moving the control deviation $e_{Control}$ out of the function and transform it into a soft constraint. Also, the weight tuning is simplified by having a reduced size weight matrices to trade off the opposing optimization objectives [3].

$$[\mathrm{u}, \mathrm{x}, \mathrm{s}]^{\mathrm{T}} = \mathbf{argmin}\, \mathrm{f}_{cost}\, (\mathrm{du}, \mathrm{E}_{\mathrm{Sys}}) \tag{8}$$

As the swash plate dynamics is removed from the model, no constraints are set on the minimum and maximum swivel angle and swivel rate. The control deviation $e_{Control}$ is now formulated as a soft constraint, which penalizes the exceeding of a limit e_{tol}. As in the first optimization problem the slack variables have been omitted for reasons of clarity.

$$\begin{aligned} e_{control} - e_{tol} &\leq 0 \\ -e_{control} - e_{tol} &\leq 0 \\ p_K - p^{[max]} &\leq 0 \\ p_R - p^{[max]} &\leq 0 \\ -p_K + p^{[min]} &\leq 0 \\ -p_R + p^{[min]} &\leq 0 \end{aligned} \tag{9}$$

While searching for controller hyperparameters, of the reduced order problem, we observed better results for lower prediction and control horizons than for long prediction and control horizons. This can be explained by the fact that ALMPC uses linearization. Therefore, the model uncertainty of the prediction model increases with a rising distance to the operating point such that modelling errors and linearizing effects accumulate along the prediction horizon. However, this fact comes in very handy when we try to have the controller run in an embedded environment. The shorter horizons result in less computing effort.

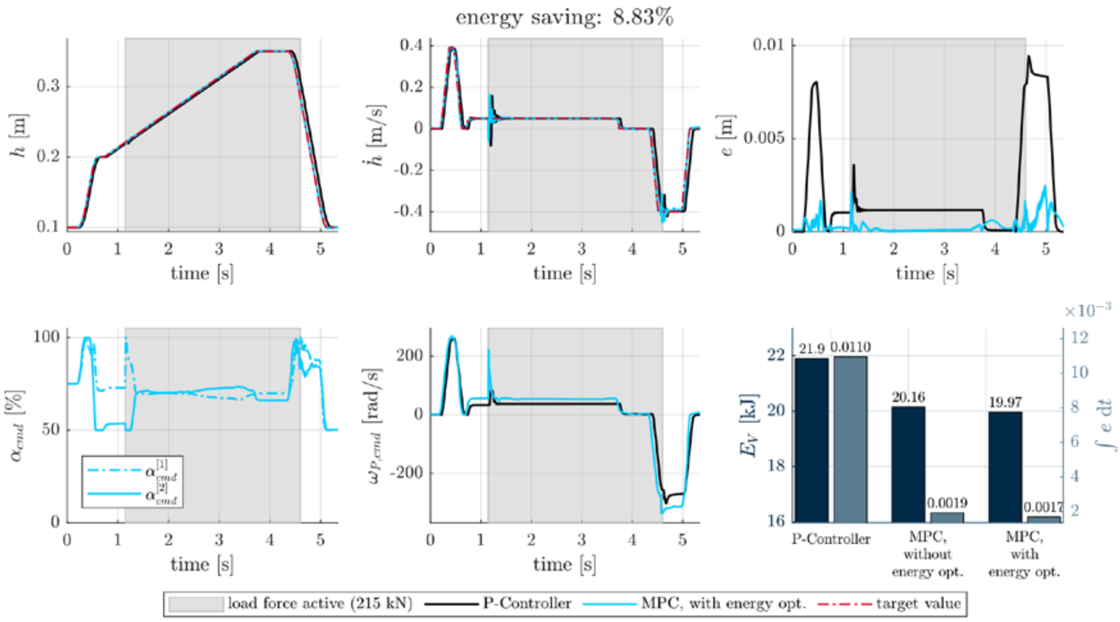

Figure 10: Result for reference cycle n_P = 12, n_C = 6 and T_S = 5ms

Noteworthy is that by reducing the problem complexity the amount of compute dropped significantly and allowed for a wider and more granular hyperparameter search. This ultimately led to hyperparameters (parameter set n_P = 12, n_C = 6 and T_S = 5 ms) which gave similar results on the benchmark cycle versus the full order model. Additionally, the reduced amount of compute brought the software into the realm of real time operation on an industrial computer.

4. CONCLUSION AND OUTLOOK

In summary, our work has demonstrated the scalability and potential of Model Predictive Control (MPC) for control systems with multiple degrees of freedom in its inputs. The optimization-based approach makes it a scalable and versatile tool for achieving different performance goals under constraints and safety margins.

Looking forward, the future of MPC in hydraulic systems is promising. Combining control algorithms with AI and machine learning will further enhance capabilities, and the demand for energy-efficient solutions underscores the importance of MPC in optimizing hydraulic operations. The competencies of hydraulic control engineers need to evolve towards dynamical modelling and optimization-based control to harness the power of discussed approaches. Collaborative efforts between academia and industry will facilitate the transition of these advancements into practical applications, benefiting numerous sectors that rely on hydraulic systems. In conclusion, MPC offers substantial benefits, and its future applications promise improved control performance and energy efficiency in various hydraulic industries, driving innovation in engineering and automation.

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