# Numerical Modeling of Helical External Gear Pump Through a Lumped Parameter Approach

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## Abstract.

External gear pumps are a type of positive displacement machines that combine excellent performances with very competitive costs. These characteristics unfold the necessity of tools to accurately simulate their performance. In this article, a numerical methodology based on a lumped parameter approach has been developed for helical External Gear Pumps (EGPs). The methodology is based on the usage of a tool, that, starting from the pump drawing, thanks to different subroutines developed in different interconnected environments, allows to fully analyze EGPs with Helical. The tool is based on a tool called EgeMATor developed by the research group for spur gears and described in a precedent works [1]. The tool has been further expanded in its functionality and optimized becoming EgeMATor MP+. Indeed, the new release has been expanded not only to include helical gears but also to simulate the bearings' reaction and consequently the gears' eccentricity variations. Results obtained from the simulations have been compared with data obtained thanks to an experimental campaign carried on a series of reference pumps done to further validate the numerical model and the new implemented functionalities.

Then, a new methodology that, for a defined tooth gears profile, is able to automatically obtain an optimized design of the pressure relief grooves of the wear plate has been studied. This methodology, through a flow ripple fluctuation, pressure spikes and crossflow optimization based on geometric assumptions, has the objective to characterize a design that permits to improve the volumetric efficiency as well as to reduce the fluid-borne noise emission of the pump analyzed.

**Keywords**. Helical EGPs, lumped parameter numerical approach, fluid-borne noise, flow ripple reduction, design optimization.

## 1. INTRODUCTION

External gear pumps (EGPs) are a type of positive displacement machines. They are usually chosen thanks to their high resistance and volumetric efficiency combined with simple working principle and low manufacturing cost. Those pumps are widely used in the fluid power field for both industrial and mobile applications. Despite their simple operating principle, obtaining high efficiency levels coupled with low noise emissions is a more arduous task. To assist in reaching these objectives, the scientific community have come with two different approaches. The first utilizes a three-dimensional CFD approaches to analyze the local pressurization and the overall flow behavior. Frosina et al. [2] used 3D-CFD approach to investigate the performance of a high-pressure EGP with spur gears. Heisler et al. [3] and Qi et al. [4] used different approaches to predict the behavior of a helical EGP. This methodology has proved to obtain a comprehensive understanding of those pumps, with excellent prevision of the working characteristics. These results come at the cost of a substantial computational power and time required; thus, to reduce the resources required simplifications are usually employed.

The second methodology implements lumped parameter approaches, where interconnected control volumes with constant properties are chosen to capture pressure variation within the pump. This analysis requires only a small fraction of the computational expenses required by a 3D-CFD approach, permitting to execute rapid simulations. Borghi et al. [5] have developed and validated a lumped parameter model that predicts the general behavior of the EGP. Mucchi et al. [6] have built up a similar model based on the same approach that is also capable to analyze helical gears. Ransegnola et al. [7] compare helical and spur EGPs through a lumped parameter approach in a multi-domain system, demonstrating its flexibility and accuracy.

This research group also has explored the analysis of EGPs by means of a lumped parameter approach, developing a new release of a simulation tool already presented in a previous work [1], called EgeMATor. The tool already permitted to completely analyze and predict both the fluid dynamics and the mechanical properties of EGPs with spur gears. Thus, in this paper, the authors have carried on improvements of such tool with the objective of further optimize it and also to expand the analyses that it could execute. Thus, the EgeMATor tool has been further developed firstly to automatically predict the gear positions, by evaluating the bearings' force reactions, and then to include the analysis on EGPs with helical gears. The new tool release that includes both features has been called EgeMATor MP+.

The novel approach has been also validated comparing the results obtained from the simulations with the experimental data got on a reference helical EGP manufactured by the company Hydreco Hydraulics Italia using a test bench provided by Duplomatic MS, both companies involved in the research.

Finally, the validated approach has been used to devise an optimal pressure relief grooves design based on geometric assumptions.

## 2. PUMP ANALYZED AND EXPERIMENTAL TEST SETUP

The analysis proposed in this paper has been performed on a reference external helical gear pump designed by the company Hydreco Hydraulics Italia; the pump has a displacement of 14.5 cm3/rev.

The basic technical data of the pump are listed in Table 1.

The pump has a housing made from die-cast alloy, making it ideal for mobile application. The exploded view of the pump geometry is shown in Figure 2.1.

**Table 1. PUMP TECHNICAL DATA**

| ***Description*** | ***Value*** | ***Unit*** |
|---|---|---|
| Nominal displacement | 14.5 | $cm^3$/rev |
| Max. continuous pressure | 260 | bar |
| Max. intermitted pressure | 290 | bar |
| Max. peak pressure | 310 | bar |
| Min. rotational speed | 500 | rev/min |
| Max. rotational speed | 3500 | rev/min |

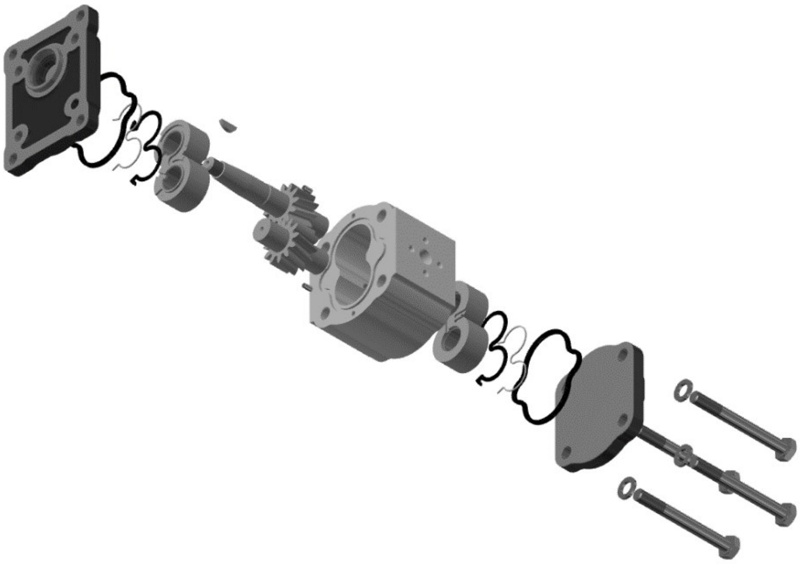

Figure 2.1. Exploded view of the reference pump

The experimental campaign to validate the model has been done on a dedicated test rig of the Italian company Duplomatic MS, involved in the research project. The test bench layout is shown in Figure **2.2**. There are two strain gauge sensors, P1 at the inlet side (Duplomatic model PTH, scale: -1÷10 bar and ±0.25% FS) and P2 at the delivery side (Duplomatic model

PTH scale: 0÷400 bar and ±0.25% FS). The flow-rate meter Q1 is a VSE VS 0.4 (VSE.flow®, Neuenrade, Germany), scale 0.03÷40 L/min, 0.3% measured value accuracy.

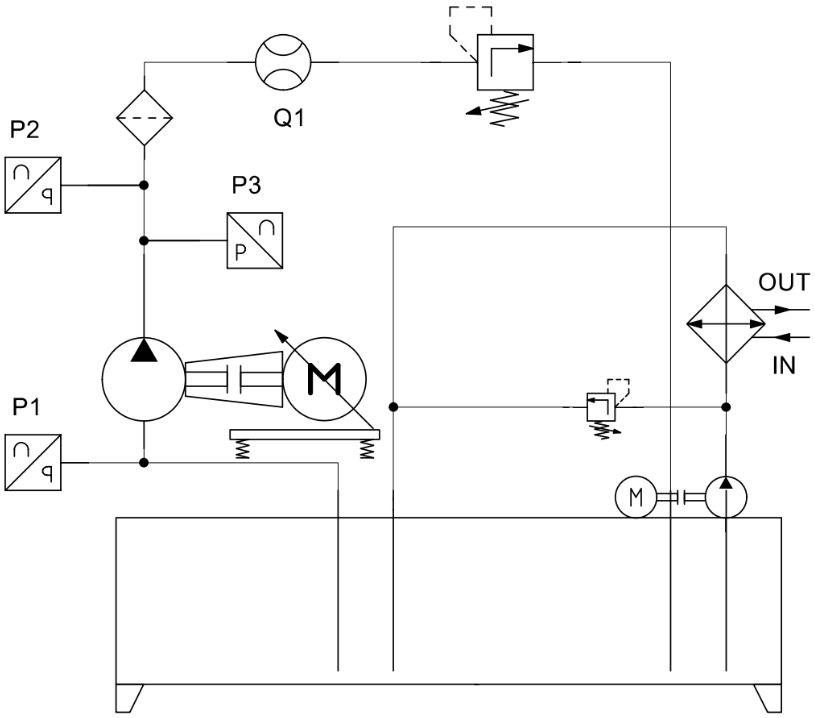

Figure 2.2. Test bench layout

The tests have been carried out with an oil ISO 46, grade HL, maintained at a temperature of 40° C ± 1°C. The pump has been tested varying the delivery pressure at different speeds. The experimental data has been compared to the results obtained from EgeMATor MP+, results will be presented in the section 4.

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# 3. The Tools: EgeMATor and EgeMATor MP+

## *3.1. Overview*

EgeMATor (External Gear Machine Multi Tool Simulator) is a tool completely developed by the authors for numerical simulations of external gear machines. In particular, the tool is composed of different subroutines developed in different environments, interconnected to each other, as visible from its workflow Figure 3.1, to study the EGMs in depth.

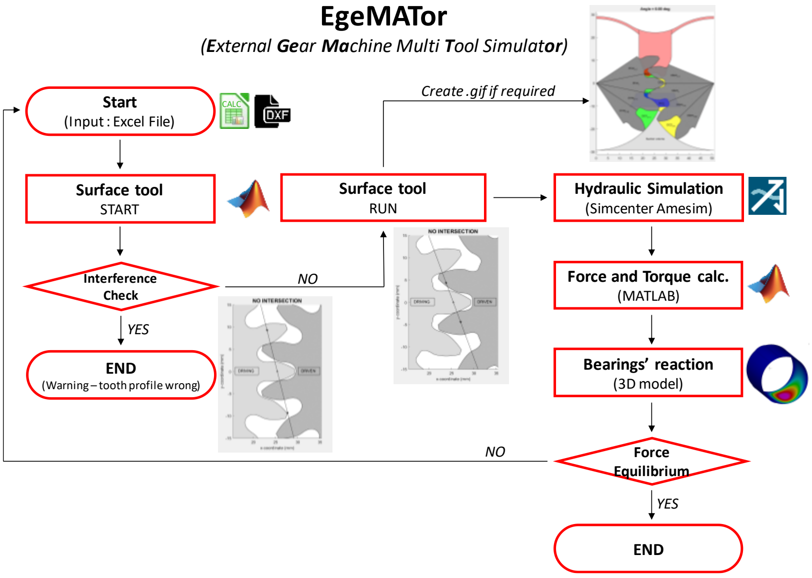

Figure 3.1. The EgeMATor's (External Gear Machine Multi Tool Simulator) workflow

The hydraulic simulation main subroutine, visible in Figure 3.1, runs a numerical simulation of the pump in Simcenter Amesim®, using a lumped parameter method based on the control volume approach. This step is preceded by a code, written by the authors in MATLAB®, called Surface Tool, that provides all the information requested by the hydraulic simulation subroutine. The Surface Tool [1] generates automatically tables with all the inputs requested by the code developed in Simcenter Amesim® like the displacement volumes as the gears rotate and their connection with the contiguous elements.

EgeMATor works in closed loop, therefore, after the hydraulic simulation, it is possible to activate a subroutine that calculates gears' forces and torques. Next, another subroutine runs the bearings' reaction calculation through a 3D-CFD numerical model; finally, results are compared and, if a different gear positioning is necessary, the Surface Tool starts the process again. Further details are available in [1].

### *3.2. The new bearings' model approach*

The bearings' reaction subroutine included in the first release of EgeMATor was based on a 3D-CFD numerical approach using the commercial code Simerics MP+®. This subroutine required considerably more computational time than all the other subroutines included in EgeMATor. To remove this limitation and to gain more flexibility, a new subroutine has been studied and implemented in the tool.

This subroutine has been written in MATLAB®; it needs as input the geometrical data of the bearings and the load applied on them, giving as output the force reactions, the eccentricity value, and the angle of minimum film thickness.

The approach is based on the application of the Reynolds equation (3.1) under an iso-viscous approximation [8-9]:

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(h^3\frac{\partial p}{\partial y}\right)=6U\eta\frac{dh}{dx} \tag{3.1}$$

where $h$ is the hydrodynamic film thickness, $p$ is the film pressure, $U$ is the bearing entraining velocity and $\eta$ is the dynamic viscosity.

The code implements a discretization of the solution domain and resolves, with a finite difference method, the Reynolds equation in its non-dimensional form:

$$\frac{\partial}{\partial x^*}\left(h^{*3}\frac{\partial p^*}{\partial x^*}\right)+\left(\frac{R}{L}\right)\frac{\partial}{\partial y^*}\left(h^{*3}\frac{\partial p^*}{\partial y^*}\right)=6U\eta\frac{\partial h^*}{\partial x^*} \tag{3.2}$$

where $R$ is the bearing radius and $L$ is the bearing axial length.

To further improve the accuracy of the numerical solutions, a numerical parameter developed by Vogelpohl [10] has been implemented. This parameter, $M_v$, is defined as follows:

$$M_v=p^*h^{*1.5} \tag{3.4}$$

Substitution into the non-dimensional form of Reynolds equation (3.1) yields the so called “Vogelpohl equation”:

$$\frac{\partial^2 M_v}{\partial x^{*2}}+\left(\frac{R}{L}\right)^2\frac{\partial^2 M_v}{\partial y^{*2}}=FM_v+G \tag{3.5}$$

where parameters $F$ and $G$ are evaluated as follows:

$$F=\frac{0.75\left[\left(\frac{\partial h^*}{\partial x^*}\right)^2+\left(\frac{R}{L}\right)^2\left(\frac{\partial h^*}{\partial y^*}\right)^2\right]}{h^{*2}}+\frac{1.5\left[\frac{\partial^2 h^*}{\partial x^{*2}}+\left(\frac{R}{L}\right)^2\frac{\partial^2 h^*}{\partial y^{*2}}\right]}{h^*} \tag{3.6}$$

$$G=\frac{\left(\frac{\partial h^*}{\partial x^*}\right)}{h^{*1.5}} \tag{3.7}$$

Therefore, giving as input the geometrical data of the bearing, the pump speed rotation and load, the subroutine evaluates the pressure field along the bearings, as it can be seen in Figure 3.2.

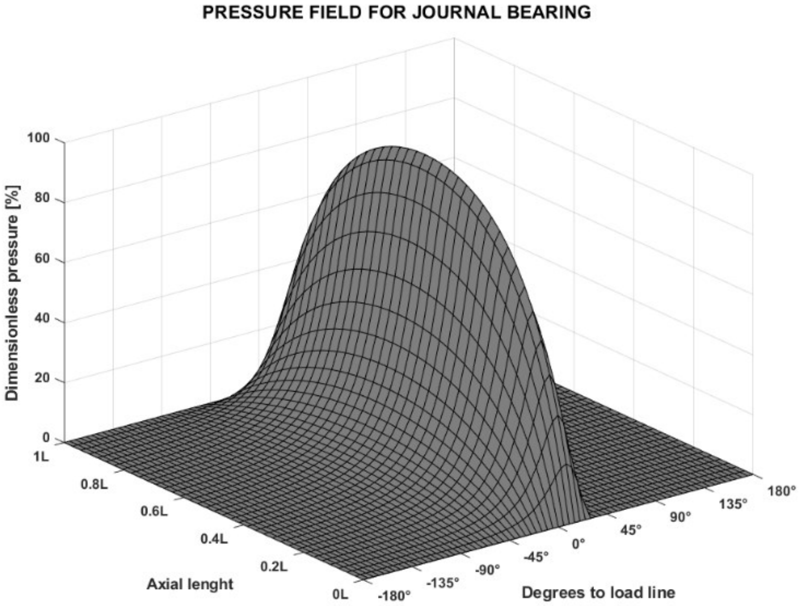

Figure 3.2. The non-dimensional pressure field evaluated by the new MATLAB® subroutine

The computational time required to resolve the numerical solution with a residual inferior to $10^{-5}$ is lower than a minute, streamlining considerably the iterative procedure to verify the force equilibrium condition present on the EgeMATor's workflow and, thus, identifying the real position of the gears inside the housing.

### *3.3. EgeMATor MP+*

As said, the EgeMATor was capable to study only EGPs with spur gears; this limitation had to be overcome. The new release of the tool, named EgeMATor MP+ (External Gear Machine Multi Tool Simulator for Multiple Gears' Profiles), permits to simulate also pumps with a gears' profile that shifts along the axial dimension, like the helical gears profile.

The new tool has a different workflow described in Figure 3.3. The new capability can be seen in figure after the interference check, the tool now, in fact, verifies also the gears' profile typology: in presence of spur gears, the workflow proceeds as in the older version; while, if gears are helical, the new approach is activated running the new subroutines of the Surface Tool developed in EgeMATor MP+.

This new approach is based on considering gears as composed of a series of slices, with a limited axial extension, and each slice is shifted of a certain degree compared to the others. The relation between the slice axial extension $dz$) and its angular shift ($d\varphi$) against a reference slice is the following:

$$dz = \frac{b}{\beta} \cdot d\varphi \tag{3.8}$$

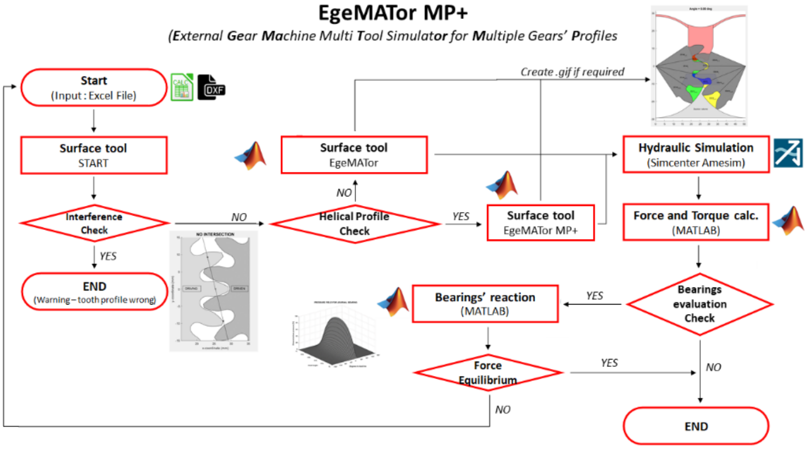

Figure 3.3. The EgeMATor MP+ (External Gear Machine Multi Tool Simulator for Multiple Gears' Profiles) workflow

where $b$ is the axial length of the gears and $\beta$ is the total helical angle. To obtain the total values of the geometric characteristics of the pump analyzed, like for example the volume of the displacement chamber, an integration is performed:

$$V = \int_{z_1}^{z_2} A dz = \frac{b}{\beta} \int_{\varphi_1}^{\varphi_2} A(\varphi) d\varphi \tag{3.9}$$

where the $z_1$ and $z_2$ are respectively the top and bottom surfaces of the gear; $\varphi_1$ and $\varphi_2$ are their respective profile angle on the area curve, and the function $A(\varphi)$ is the area curve, that could represent a displacement area or a variable area.

When the new subroutine included in the *Surface Tool* has terminated the evaluation of the geometrical properties of helical gears' profile, the workflow processes with the same iterations done in the previous version of the tool, giving to the hydraulic simulation the requested inputs.

At the present time, the evaluations of the force and torque in the case of the helical gears' profile is still incomplete because the script does not yet evaluate the component along the axial dimension of the pump, that is present on this typology of gear profiles. Therefore, following the workflow in Figure 3.3, at the stage "bearing evaluation check", in case of spur gears, EgeMATor MP+ works like EgeMATor evaluating the bearing reaction; in case of helical gears, the evaluation of the bearings' reaction is excluded, leading to termination.

Consequently, in the validation phase presented in the next section of this paper, the iterative process to find the force equilibrium has not been executed and the gears are positioned in contact with the housing along the vertical direction.

# 4. MODEL VALIDATION

To validate the new tool EgeMATor MP+, an experimental campaign on the previously mentioned reference helical EGP have been conducted on the test bench already described. The pump has been tested under four different pump speeds: 1000, 1500, 1800 and 2200 rev/min. The values of the delivery flow rate have been acquired for all the delivery pressure conditions in the range of 20÷240 bar. Simulations have been run for a fixed temperature of the hydraulic oil of 40°C.

The low computational time required to complete a simulation permits to run a high number of working conditions; therefore, for each pump speed six discrete value of delivery pressure have been simulated.

In detail, the simulations have been run for delivery pump pressure values of: 20, 50 100, 150, 200 and 240 bar.

In Figure 4.1 the comparison between the numerical model and the experimental results is presented; the flow-rate has been normalized to reference value, $Q_{ref}$.

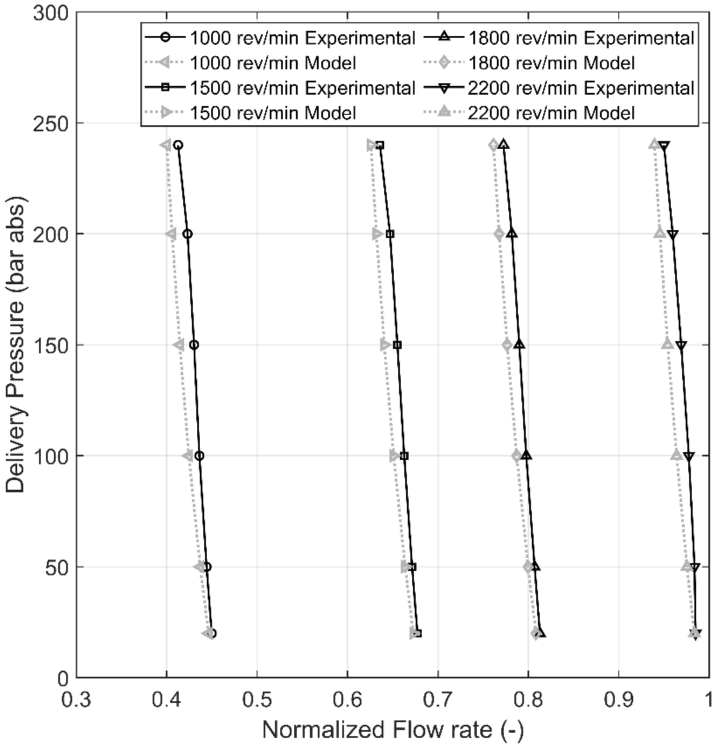

Figure 4.1. Pressure vs. normalized delivery flow rate comparison at different pump speed: 1000 rev/min, 1500 rev/min, 1800 rev/min and 2200 rev/min

It can be noted that EgeMATor MP+ correctly predicts the experimental with percentage errors between model and experimental data always below 3%, staying thus within the uncertainty range of the experimental measurements.

## 5. OPTIMAL GROOVE DESIGN RESEARCH METHODOLOGY

The validated numerical model for studying helical EGPs has been designed to include further capabilities such as optimization procedures. This section presents a preview of an under-development procedure that, for a given gears profile, will be able to optimize the pressure relief grooves inside the pump's wear plates. It is based on geometric assumptions to minimize the crossflow between delivery and suction while limiting pressure spikes. To better understand the methodology of this new procedure, a brief description of the considered internal connections is needed. Figure 5.1 shows a schematization of all the internal connection that have been implemented in the mode.

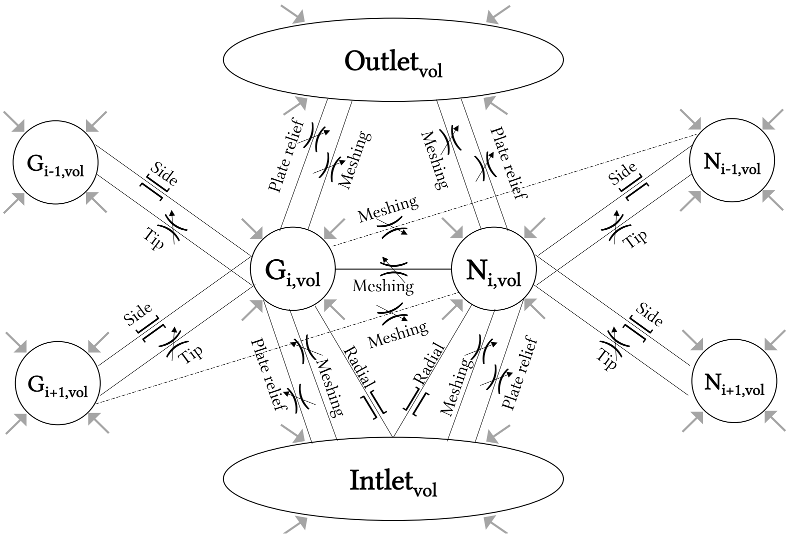

Figure 5.1. Schematized internal connection modeled in the *Surface Tool*

As it shown, the connections between the displacement chamber volumes ($G_{i,vol}$ - $N_{i,vol}$) and the plate relief grooves are modeled as variable orifice. The opening and closing timings paired with the cross-sectional area values of these connections are the characteristics that mainly affect the EGP performance and behavior.

The previously described *Surface Tool*, as said, estimates the volume of a displacement chamber as function of the rotation angle, $\varphi$. The new procedure implements a subroutine to find the rotation angle value that correspond the minimum chamber volume. This identified angle, $\varphi_{DC\,MIN}$, is of utmost importance since represents the angle when the instantaneous commutation between the delivery and suction pressure takes place. The optimization of the wear plate design in this area is fundamental for preventing unwanted pressure spikes.

After that, an iterative procedure can start. Since the procedure is under-development, at this stage only a simple rectangular shaped groove designs could be tested for both the delivery and inlet groove. These shapes are characterized by a series of parameters, as shown in Figure 5.2. The parameters are normalized respect to the gear module *m* and vary between an interval of values, which can be seen in the Table 2.

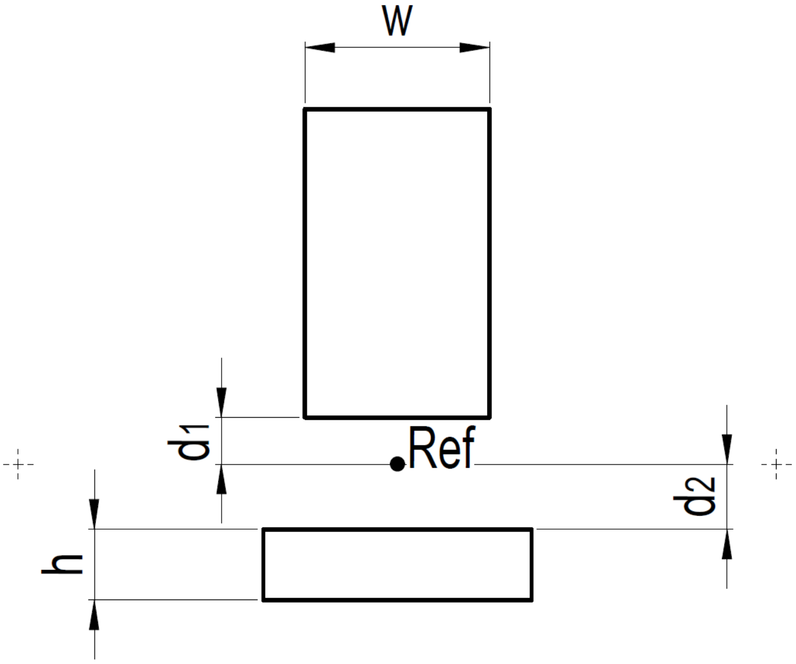

Figure 5.2. Reference parameters for the groove shape design search

**Table 2. GROOVES PARAMETRIZATION**

| *Description* | *Value* | *Min* | *Max* |
|---|---|---|---|
| Delivery Groove width | W | 1 | 3 |
| Suction Groove height | h | 0.5 | 2 |
| Delivery Groove reference distance | $d_1$ | 0 | 3 |
| Suction Groove reference distance | $d_2$ | 0 | 3 |

The designs obtained are tested against a series of target functions:

$$TF_1 = \left|\varphi_{DC\_MIN} - \varphi_{GRV}\right| \tag{5.1}$$

$$TF_2 = \frac{1}{n}\sum_{i=1}^{n} A\left(\varphi_{DC\_MIN} \mp n\Delta\varphi\right) - A_g\left(\varphi_{DC\_MIN} \mp n\Delta\varphi\right) \tag{5.2}$$

The target function $TF_1$ in (5.1) has the objective to obtain higher volumetric efficiency. To achieve this objective, the $TF_1$ has to be minimized, that means minimize the angular difference between the rotation angle value of minimum chamber volume $\varphi_{DC\ MIN}$ and the characteristics rotation angle value for the grooves $\varphi_{GRV}$. In particular, for the delivery groove, $\varphi_{GRV}$ represents the rotation angle value of connection closure between the groove and the delivery chamber, while $\varphi_{GRV}$ represents the rotation angle value of connection opening between the groove and the delivery chamber for the suction groove.

The target function $TF_2$ in (5.2) has the objective of minimize the internal pressure peaks occurring during the meshing in proximity of $\varphi_{DC\ MIN}$. To accomplish this target, $TF_2$ has to be minimized, that is the difference between the cross-sectional area of the groove $A_g$ (in dark red in Figure 9) and the projection area of the displacement chamber volumes on the lateral side $A$ (in green in Figure 9) for a number n of points with an angular pitch distance from $\varphi_{DC\ MIN}$ equal to Δφ. This ensures a more gradual passage between the high and low pressure zones around the $\varphi_{DC\ MIN}$, reducing the pressure peaks. The minus or plus sign in (5.2) are respectively for the delivery or suction groove.

In this way, the design that minimize both target functions represent the best choice for the simplified geometry utilized. In the next steps, the procedure will be implemented to be executed for typical groove geometries. With this implementation, EgeMATor MP+ will be not only able to fully characterize EGPs with helical gears but also to optimize them by acting on the wear plates geometries. It can be appreciated from the table that the new design has a small advantage regarding the mean delivery flow, exhibiting a value increase of about 0.5%, pointing the way to a small improvement of the volumetric efficiency.

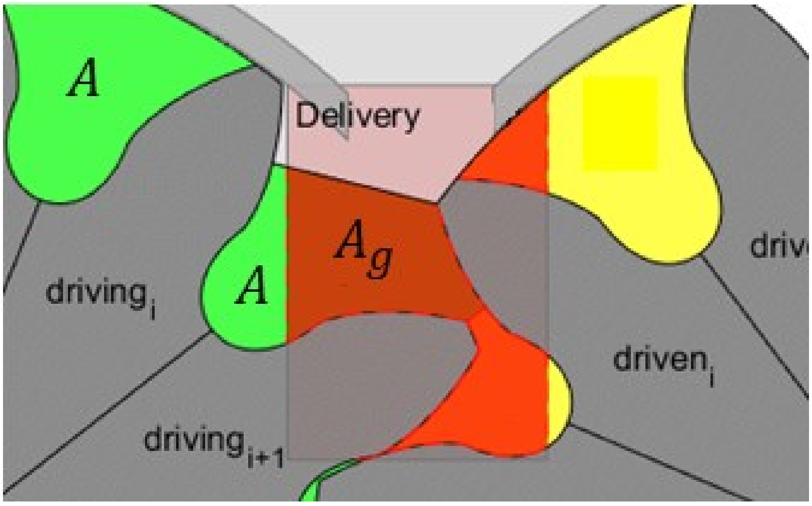

Figure 5.3. Areas evaluated in the optimal groove design methodology

## 6. CONCLUSIONS

A novel lumped parameter approach for studying external helical gear pumps has been described in this article. The tool, called *EgeMATor MP+*, has been developed to extend the functionalities of *EgeMATor*, a tool previously developed by the authors for the simulation of EGPs with spur gears.

Two important capabilities have been implemented. The first one was developed to include the bearing reactions and, consequently, to estimate the gears' eccentricity. This feature, developed in MATLAB®, substitutes a previous one executes by using a 3D-CFD approach with higher computational time. Secondly, the tool has been implemented to be applied to EGPs with helical gears.

The new tool has been run on a reference EGP geometry with helical gears. Model results have been compared with experimental data with errors always below 3%.

Finally, an under-development procedure has been firstly presented; it will allow the optimization of the wear plate designs avoiding pressure spikes in the transition area between suction and delivery.

## NOMENCLATURE

| *Acronym* | *Description* |
|---|---|
| EGP | External Gear Pump |
| CFD | Computational Fluid Dynamic |
| EGM | External Gear Machine |

| *Symbol* | *Description* | *Unit* |
|---|---|---|
| $A$ | Area of displacement chamber | [m$^2$] |
| $A_g$ | Cross-sectional area of groove | [m$^2$] |
| $b$ | Axial length of the gears | [m] |
| $F$ | First coefficient of the Vogelpohl equation | [-] |
| $G$ | Second coefficient of the Vogelpohl equation | [-] |
| $h$ | Hydrodynamic film thickness | [m] |
| $h^*$ | Non-dimensional hydrodynamic film thickness | [-] |
| $L$ | Bearings axial length | [m] |
| $M_v$ | Vogelpohl parameter | [-] |
| $p$ | Pressure | [Pa] |
| $p^*$ | Non-dimensional pressure | [-] |

| | | |
|---|---|---|
| $Q_{ref}$ | Reference flow rate | [L/min] |
| $R$ | Bearings radius | [m] |
| $U$ | Bearings entraining velocity | [m/s] |
| $x^*$ | Non-dimensional x-coordinate of the hydrodynamic film | [-] |
| $y^*$ | Non-dimensional y-coordinate of the hydrodynamic film | [-] |
| $z$ | Axial extension of the slices | [m] |
| $z_1$ | Axial start position for the reference profile | [m] |
| $z_2$ | Axial finish position for the reference profile | [m] |

| ***Greek letter*** | ***Description*** | ***Unit*** |
|---|---|---|
| $\beta$ | Total helix rotation angle | [rad] |
| $\varphi$ | Gear profile rotation angle | [rad] |
| $\varphi_1$ | Axial profile angle start position for the reference profile | [rad] |
| $\varphi_2$ | Axial profile angle finish position for the reference profile | [rad] |
| $\varphi_{DC\ MIN}$ | Gear profile rotation angle of instantaneous commutation (minimum chamber volume) | [rad] |
| $\varphi_{GRV}$ | Groove rotation angle characteristics | [rad] |
| $\Delta\varphi$ | Angular pitch value for optimization function | [Pa · s] |
| $\eta$ | Dynamic viscosity of the hydrodynamic film | [-] |

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## Biographies

**Pietro Mazzei** received his B.Sc and M.Sc of Mechanical Engineering degree at Federico II of Naples University, Italy in 2021. He received his Ph.D. degree on Industrial

Engineering in 2023, focusing on the development of numerical models to analyze and optimize the performances and the noise emissions of external gear machines. Currently he is employed at CIRA (Centro Italiano Ricerche Aerospaziali) on the development of hybrid LTA vehicle as a pressure and thermal modeling expert.