14. A Comparative Analysis for Realization of Limit-Cycle Free 2D Digital Filters with External Disturbance

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ABSTRACT

In this paper a comparative analysis for realization of limit-cycle free 2D digital filters with external disturbance has been done. We have done a comparative analysis of earlier reported criterion with an improved criterion which ensures asymptotic stability with guaranteed H_∞ performance with low computational complexity. In particular, improved criterion for elimination of overflow oscillations in two-dimensional (2-D) digital filters is described by Roesser model with saturation arithmetic and external interference. The criterion is represented in terms of linear matrix inequality (LMI). Specific examples are given to demonstrate the effectiveness of proposed criterion.

Keywords - *Two dimension digital filters, limit-cycle, Rosser model, linear matrix inequality*

INTRODUCTION

We analyze signals because it contains information and to obtain that information in more desirable form we perform signal processing. Thus signal processing is analysis, interpretation and manipulation of signals like sound, images, time varying measurement values and sensor data etc. [1]. When signals are transmitted, there is a possibility of signals being contaminated by some external noise. In order to retrieve the original signal at the receiver, suitable filters are to be used i.e. the signal is processed to obtain pure signal[1,2]**.** Digital signals are often desired to be processed to modify the characteristic carried out by such signals.

In idealized form, digital signal processing is the processing of discrete-time signals. Digital filter uses digital processer that performs for numeric computation on sampled values of signal to obtain the associated information with the signal [2,3]. The processer may be general purpose computer.

Digital filters are vital dynamical systems in signal processing that useful for handling out the discrete signals [2]. Due to enormous range of applications of digital filters, which include communications, seismographic data processing, speech processing, image processing, control systems etc. have more importance regarding its design and analysis [2-4]. It can be implemented in the field of geophysics specially in signal processing to acquire smooth seismic data for various types of complex geological structures beneath the earth surface [31]. In case of seismic survey, seismic noise (4D signal) is usually detected during acquisition of 2D signal [33]. To reduce that effect the 2D digital filters can be applied to obtain external noise free data.

A. TWO DIMENSIONAL DIGITAL SIGNAL PROCESSING

The field of two-dimensional (2-D) digital signal processing has been emerging rapidly in recent years. Images such as satellite photographs, radar and sonar maps, medical X-ray pictures and magnetic records are typical examples of 2-D signals that might need to be processed [2,3]. The types of processing that can be applied may range from enhancing the quality of signals to extracting certain useful features from them. A continuous 2-D signal is a physical quantity that is a continuous function of two real independent variables [3]. A discrete 2-D signal is a sampled version of continuous 2-D signal. 2-D discrete signal can be modified, reconstructed, reshaped or manipulated through filtering. This type of processing can be carried out by using 2-D digital filters [3,4].

2D digital filter can be characterized in terms of difference equations or state-space equations in two independent variables and in terms of matrices of transfer functions. When a 2-D digital filter is implemented in terms of either software on general purpose computer or dedicated hardware, signals must be stored and manipulated in registers of finite length [5]. When approximation step is carried out, transfer function coefficients are calculated, and they must be quantized before implementation of digital filter. The net effect of coefficient quantization is to introduce inaccuracies in amplitude response of the filter. Signal quantization can lead to other problems as well, such as generation of spurious parasitic oscillations, known as limit cycles [3, 5, 6, 28, 29].

Like 1-D digital filters and other types of systems, 2-D digital filters can be represented by state –space models [7-9]. In this approach a set of internal signals referred as state variables, is used to describe completely operation of filter. The technique is very useful in analysis, design and implementation of digital filters [10-13,26-30]. In this approach, the digital filters are characterized in terms of matrices, which are easy to manipulate. State space models for 2-D digital filters have been proposed by Attasi, Givone and Roesser, and Fornasini and Marchesini [3,5,8,9]. In this paper, Roesser model is used for the realization of the 2-D digital filter.

B. H_∞ FILTERING APPROACH

High-order digital filters are usually breakdown into various biquad filters at the time of hardware implementation. Among these biquad filters there exists some external interference which is unavoidable, and such interferences lead to disturbance [14-17]. Also in practical physical systems, statistical information on the signals is insufficient. When the statistical information about the external interference is insufficient, we can employ H_∞ filtering, energy-to-peak *($l_2 - l_\infty$)* filtering and peak-to-peak *(l_1)* filtering [18-19,29,30]. Such filtering techniques have been studied in filtering and control problems [20,26,27]. In H_∞ filtering approach, the external interference is assumed to be energy bounded and the energy-to-energy gain is minimized or below a prescribed level. In $l_2 - l_\infty$ filtering the external input interference signal assumed to be energy bounded and the output signal to be peak bounded [21-24].

In this paper, we have done a comparative analysis of established criterion proposed in [25] with an improved criterion which ensures asymptotic stability with guaranteed H_∞ performance with low computational complexity. The paper is organized as follows. Section 2 introduces the system under consideration. A comparatively improved LMI criterion for H_∞ stability of two dimensional digital filters with saturation overflow nonlinearities and external interference is proposed in Section 3. In Section 4, a numerical example is given to illustrate the effectiveness of the proposed results. Finally, the paper is concluded in Section 5.

SYSTEM DESCRIPTION

Consider the following two dimensional digital filters:

$$\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = f\left\{\begin{bmatrix} y^h(i,j) \\ y^v(i,j) \end{bmatrix}\right\} + \begin{bmatrix} w^h(i,j) \\ w^v(i,j) \end{bmatrix} = \begin{bmatrix} f^h(y^h(i,j)) \\ f^v(y^v(i,j)) \end{bmatrix} + \begin{bmatrix} w^h(i,j) \\ w^v(i,j) \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} y^h(i,j) \\ y^v(i,j) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \mathrm{x} \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} \quad (2)$$

$$f^h(y^h(i,j)) = \left[f_1^h\left(y_1^h(i,j)\right), \dots, f_m^h\left(y_m^h(i,j)\right)\right]^T$$

$$f^v(y^v(i,j) = \left[f_1^v\left(y_1^v(i,j)\right), \dots, f_n^v\left(y_n^v(i,j)\right)\right]^T$$

$$y^h(i,j) = \left[y_1^h(i,j), \dots, y_m^h(i,j)\right]^T$$

$$y^v(i,j) = \left[y_1^v(i,j), \dots, y_n^v(i,j)\right]^T$$

Where

$x^h(i,j) \in R^m$ is horizontal state vector,
$x^v(i,j) \in R^n$ is vertical state vector,
$w^h(i,j) \in R^m$ is horizontal external interference,
$w^v(i,j) \in R^n$ is vertical external interference,
$y^h(i,j) \in R^m$ is horizontal output vector,
$y^v(i,j) \in R^n$ is vertical output vector.
$f^h(.)$ is horizontal overflow nonlinearity ,
$f^v(.)$ is vertical overflow nonlinearity .
$A_{11} \in R^{m\times m}$, $A_{12} \in R^{m\times n}$, $A_{21} \in R^{n\times m}$ and $A_{22} \in R^{n\times n}$ are state matrices.

The overflow arithmetic to be considered presently is the saturation arithmetic given by;

$$f_1^h\left(y_1^h(i,j)\right) = \begin{cases} 1 & if\ y_k^h(i,j) > 1 \\ y_k^h(i,j) & if -1 \le y_k^h(i,j) \le 1 \\ -1 & if\ y_k^h(i,j) < -1 \end{cases} \tag{3}$$

k=1,2,…,m,

$$f_1^v\left(y_1^v(i,j)\right) = \begin{cases} 1 & if\ y_k^v(i,j) > 1 \\ y_k^v(i,j) & if -1 \le y_k^v(i,j) \le 1 \\ -1 & if\ y_k^v(i,j) < -1 \end{cases} \tag{4}$$

$$k = 1,2,\dots,n$$

Given a level $\gamma > 0$, the purpose of presented work is to establish a new LMI criterion such that the 2-D digital filter (1)-(2) with $w^h(i,j) = 0, w^v(i,j) = 0$ is asymptotically stable and

$$\frac{\sum_{i=0}^{\infty}\ \sum_{j=0}^{\infty}\ \left[\ x^{h^T}(i,j)K_1x^h(i,j)+x^{v^T}(i,j)K_2x^v(i,j)\right]}{\sum_{i=0}^{\infty}\ \sum_{j=0}^{\infty}\ \left[w^{h^T}(i,j)K_1w^h(i,j)+w^{v^T}(i,j)K_2w^v(i,j)\right]} < \gamma^2 \tag{5}$$

Under zero boundary conditions for all nonzero $w^h(i,j)$ and $w^v(i,j)$ where K_1 and K_2 are positive symmetric matrices. Parameter γ is called the H_∞ norm bound or the interference attenuation level. In this case, the two dimensional digital filter (1)-(2) is said to be asymptotically stable with a guaranteed H_∞ performance γ.

MAIN RESULT

A H_∞ stability criterion for the 2-D digital filter (1)-(2) is given in the following theorem.

A. THEOREM 1

For a given level γ>0, if there exist symmetric positive definite matrices $K_1\ \ K_2, L, and\ M,$ positive diagonal matrices $D_1$and D_2 such that

$$\begin{bmatrix} \phi_{1,1} & \phi_{1,2} & A_{11}^TD_1 & A_{21}^TD_2 & \mathbf{0} & \mathbf{0} \\ \phi_{2,1} & \phi_{2,2} & A_{12}^TD_1 & A_{22}^TD_2 & \mathbf{0} & \mathbf{0} \\ D_1A_{11} & D_1A_{12} & L-2D_1 & \mathbf{0} & L & \mathbf{0} \\ D_2A_{21} & D_2A_{22} & \mathbf{0} & M-2D_2 & \mathbf{0} & M \\ \mathbf{0} & \mathbf{0} & L & \mathbf{0} & L-\gamma^2I & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & M & \mathbf{0} & M-\gamma^2I \end{bmatrix} \prec 0 \tag{6}$$

where

$\varphi_{1,1} = K_1 - L,\ \varphi_{1,2} = \mathbf{0},\ \varphi_{2,1} = \mathbf{0},$
$\varphi_{2,2} = K_2 - M$

Then the 2-D digital filter (1)-(2) is asymptotically stable with guaranteed H_∞ performance γ.

B. COMPARISON WITH THE EXISTING CRITERION

In section 3, a criterion for H∞ stability of 2-D interfered digital filters is presented. The criterion ensures elimination of overflow oscillations in 2-D digital filters described by Roesser model. In [25] the criterion is reported with eight unknown variables however the proposed criterion ensures asymptotic stability with guaranteed H_∞ performance considering six unknown variables. Thus the proposed criterion is numerically simpler as compared to the criterion proposed in [25].

ILLUSTRATIVE EXAMPLE

To illustrate the applicability of proposed theorem given in section 3, we now consider the following example.

Example 1

Consider the 2D digital filter described by (1)-(2) with following parameters

$$A_{11} = \begin{bmatrix} -0.1 & 0.9 \\ 0.39 & 0.4 \end{bmatrix} \qquad A_{12} = \begin{bmatrix} 0.01 & -0.5 \\ 0 & 0.35 \end{bmatrix}$$

$$A_{21} = \begin{bmatrix} -0.15 & 0.1 \\ 0.3 & -0.1 \end{bmatrix} \qquad A_{22} = \begin{bmatrix} 0.1 & 0.23 \\ 0.8 & 0.03 \end{bmatrix}$$

$$\boldsymbol{w^h}(i,j) = \begin{bmatrix} n^1(i,j) \\ n^2(i,j) \end{bmatrix} \qquad \boldsymbol{w^v}(i,j) = \begin{bmatrix} n^3(i,j) \\ n^4(i,j) \end{bmatrix}$$

where $n^1(i,j), n^2(i,j), n^3(i,j)$ and $n^4(i,j)$ are mutually independent white noises with mean 0 and variance 0.1. Let the disturbance attenuation level $\gamma = 0.4$

By solving LMI (6) using Matlab LMI toolbox we obtain the following feasible solutions:

$$K_1 = \begin{bmatrix} 0.0013 & -0.0001 \\ -0.0001 & 0.0013 \end{bmatrix} \qquad K_2 = \begin{bmatrix} 0.0066 & -0.0027 \\ -0.0027 & 0.0038 \end{bmatrix}$$

$$\mathrm{L} = \begin{bmatrix} 0.0191 & -0.0013 \\ -0.0013 & 0.0294 \end{bmatrix} \qquad \mathrm{M} = \begin{bmatrix} 0.0596 & 0.0050 \\ 0.0050 & 0.0307 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} 0.0296 & 0 \\ 0 & 0.0296 \end{bmatrix} \qquad D_2 = \begin{bmatrix} 0.0618 & 0 \\ 0 & 0.0618 \end{bmatrix}$$

Thus according to theorem 1, the system under consideration is asymptotically stable.

Example is taken from [25] and it was shown that the given 2-D digital filter described by (1)-(2) is feasible for 6 unknown variables as compared to 8 unknown variables that are taken in [25].

CONCLUSIONS

A comparative evaluation of presented criterion with the one proposed in [25] is made. It is found that the criterion assures improvement over the criterion proposed in [25] as it shows reduction in numerical complexity. The usefulness of the presented criterion is demonstrated with the help of numerical example. The possible extension of presented approach to establish a criteria for H_∞ stability of m-D (m>2) interfered digital filters appears to be an interesting problem for future investigation. These digital filters can be implemented in the seismic survey to minimize the noise during acquiring of data.

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