COMPLEX DYNAMICS OF FIRST SUPERIOR BARNSLEY FRACTAL

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Abstract.

Superior fractals have expanded the range of fractal applications, such as chaotic cryptography, in the creation of spectacular and lifelike computer visuals, fractal antenna, particle dynamics, social research, in the design of file compression systems, the construction of the internet's networks, and even in the diagnosis of some diseases. Indeed, this modest start has revealed a whole new dimension of fractal theory and its applications. This paper introduces new fractals using Barnsley functions and superior iterates. Barnsley has introduced three fractal functions to generate new escape time fractals which gives a new approach to the study of IFS (Iterated function systems). In addition to, this study also represents the complex dynamics of First Barnsley Fractals using superior iterates. In the existing literature, Julia sets and their generalizations have been developed using one-step feedback process (Picard iteration). In this paper two step feedback process (Mann iterates and superior iterates) has been introduced to study of Julia sets and obtained superior Julia sets.

Key words: Complex dynamics, Barnsley Fractals, Picard iteration, Mann iterates and superior iterates

1. INTRODUCTION

Michal F. Barnsley has given a new approach to fractal by using iterative function system [1-2] and [4]. In his book "Fractals Everywhere" [2] author has presented the concept of Barnsley fractal of type First, Second and Third. The images of Barnsley fractals are different from the classical fractals. This paper has presented a new approach to the Barnsley Fractals using superior iterates [5], [6], [7] and [8], named as Superior Barnsley Fractals. Further this paper has discussed the first Barnsley fractal and their behavior using superior iterates.

The two-step feedback method (Mann iterates and superior iterates) was used to examine Julia sets in this paper, and superior Julia sets were obtained. The IFS patterns, curves, spokes, and spirals symmetric along the y axis can be seen by zooming in on the superior Barnsley fracrtal.

2. SUPERIOR ORBIT

Let A be a subset of real or complex numbers and $f : A \to A$. For $x_0 \in A$, construct a sequence $\{x_n\}$ in A in the following manner

$$\begin{aligned} x_1 &= s_1 f(x_0) + (1-s_1)x_0 \\ x_2 &= s_2 f(x_1) + (1-s_2)x_1 \\ &\vdots \\ x_n &= s_n f(x_{n-1}) + (1-s_n)x_{n-1} \end{aligned}$$

where $0 < s_n \le 1$ and $\{s_n\}$ is convergent to a non-zero number.

The above mention sequence $\{x_n\}$ is known as the Mann sequence of iterates or superior sequence of iterates. Let z_0 be an arbitrarily element of C that can be chosen at random. Construct a $\{z_n\}$ sequence of points C in the following manner:

$$z_n = sf(z_{n-1}) + (1-s)z_{n-1,} n = 1, 2, 3....,$$

where f is a function over a subset of C and s is closed interval parameter lies in $[0,1]$. The above mention sequence $\{z_n\}$, represented by $SO(f, z_0, s)$, is the superior orbit for the complex-valued function f with an initial choice z_0 and parameter s. This may be symbolized by $SO(f, x_0, s_n)$. Notice that $SO(f, x_0, s_n)$ with $s_n = 1$ is $O(f, x_0)$. When $s_n = 1$, the superior orbit decreases to the normal Picard orbit, according to this observation.

The Barnsley Fractal functions are defined as:

(A) First Barnsley Fractal:

$z_{n+1} = c\, z_n - c$ ***where real (z) ≥ 0***

$= c\, z_n + c$ otherwise

(B) Second Barnsley Fractal:

$z_{n+1} = c\, z_n - c$ where imag(z) ≥ 0

$= c\, z_n + c$ otherwise

(C) Third Barnsley Fractal:

Here the value of c is (1.0, 0)

$z_{n+1} = z_n^2 - c$ if real(z) ≥ 0

$= z_n^2 - c + px$ otherwise

here px = real(c) * real (z_n)

3. ESCAPE CRITERION FOR BARNSLEY FRACTALS

After looking to the fractal formulas it has been observed that all the three Barnsley fractal formulas are of the form $\boldsymbol{z}^2+\boldsymbol{c}$, hence the escape criterion is defined as follows:

Let $B_c(\boldsymbol{z})$ be the Fractal Formula, then

$$
\begin{aligned}
&| B_c(\boldsymbol{z})| = |(\boldsymbol{c}\, \boldsymbol{z}_n + \boldsymbol{c}) | \\
&\geq |\boldsymbol{c}|\, |\boldsymbol{z}| - |\boldsymbol{c}| && \text{since}|\boldsymbol{z}| \geq |\boldsymbol{c}| \\
&\geq |\boldsymbol{c}|\, \{|\boldsymbol{z}| - 1\} \\
&> |\boldsymbol{z}|\, (|\boldsymbol{z}| - 1) \\
&\text{since } |\boldsymbol{z}| > 2, \text{ there is} \\
&> (1 +\lambda^n)\, |\boldsymbol{z}|
\end{aligned}
$$

It has been found that the escape criterion is the same as of quadratic polynomial.Raniand Kumar [6] has developed the phenomenon of Superior orbits for the Madelbrot and Julia Sets and named as Superior Mandelbrot and superior Julia Set respectively using Mann Iterates. After applying the same phenomenon it can be redefine three Barnsley Fractals named as Superior Barnsley Fractal. Fractal can be considered with the fusion technique [9-14] to improve the visualization in such manner.

4. SUPERIOR BARNSLEY FRACTALS

After applying the superior iterates the three Barnsley Fractal Functions (A), (B) and (C) can be rewritten as:

(A1)	$\boldsymbol{z}_{n+1}$	$= \boldsymbol{s}(\boldsymbol{c}\boldsymbol{z}_n - \boldsymbol{c}) + (1 - \boldsymbol{s})\, \boldsymbol{z}_n$	where real $(\boldsymbol{z}) \geq 0$
		$= \boldsymbol{s}(\boldsymbol{c}\boldsymbol{z}_n + \boldsymbol{c}) + (1 - \boldsymbol{s})\, \boldsymbol{z}_n$	otherwise
(B1)	z_{n+1}	$= \boldsymbol{s}(\boldsymbol{c}\boldsymbol{z}_n - \boldsymbol{c}) + (1 - \boldsymbol{s})\, \boldsymbol{z}_n$	where imag $(\boldsymbol{z}) \geq 0$
		$= \boldsymbol{s}(\boldsymbol{c}\boldsymbol{z}_n+ \boldsymbol{c}) + (1 - \boldsymbol{s})\, \boldsymbol{z}_n$	otherwise
(C1)	z_{n+1}	$= \boldsymbol{s}(\boldsymbol{z}_n^2 - \boldsymbol{c}) + (1 - s)\, \boldsymbol{z}$	where real $(\boldsymbol{z}) \geq 0$
		$= \boldsymbol{s}(\boldsymbol{z}_n^2 - \boldsymbol{c} + \boldsymbol{px}) + (1 - \boldsymbol{s})\, \boldsymbol{z}_n$	otherwise
		where px = real ($\boldsymbol{c}$) * imag ($\boldsymbol{z}_n$)	

This paper analysis the First Barnsley Fractal.

5. ANALYSIS OF FIRST SUPERIOR BARNSLEY FRACTAL

It has been observed that the first superior Barnsley fractal as a sequence of beautiful spiral for $\boldsymbol{f}\ (\boldsymbol{a}\ ,\boldsymbol{b},\ \boldsymbol{s})$ = (1.025, -0.093, 1.0) see Figure. 1. For $\boldsymbol{f}\ (\boldsymbol{a},\ \boldsymbol{b},\ \boldsymbol{s})$= (1.263, -0.989, 0.1) and $\boldsymbol{f}\ (\boldsymbol{a},\ \boldsymbol{b},\ \boldsymbol{s})$= (1.025, 1.229, 0.5) the symmetry has been observed around $\boldsymbol{y}$ axis see Figure. 1, 3 and 5. It has been observed the chaotic nature of points for first superior Barnsley fractal see Figure. 2, 4 and 6 where the orbit value is changes periodically rather than conversing to any fixed point.

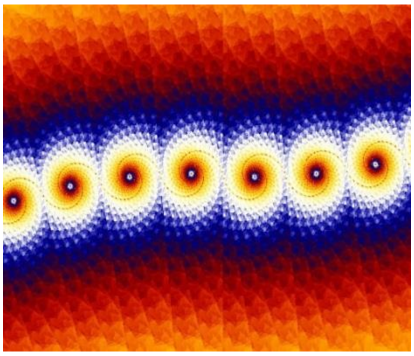


Figure. 1. Superior Barnsley Fractal for *f (a, b, s)*= (1.025, -0.093, 1.0)

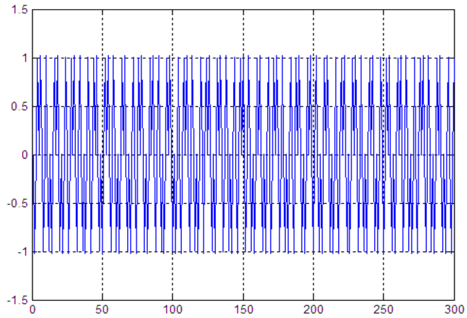
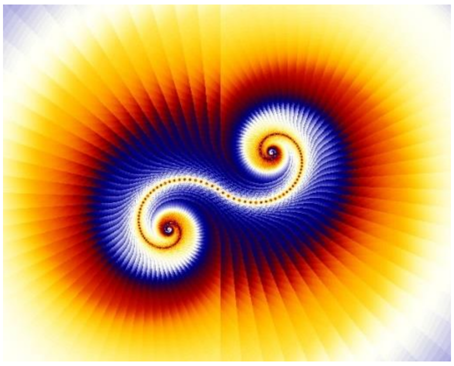


Figure. 2. Orbit of Superior Barnsley Fractal for *f (a, b, s)*= (1.025, -0.093, 1.0)

Table 1. For *f (a, b, s)*= (1.025, -0.093, 1.0)

Number of iteration i	| F(z) |
288	0.024404
289	0.99999
290	1.48E-05
291	1.025
292	0.025609
293	0.99875
294	0.001281
295	1.0237
296	0.024279
297	1.0001
298	0.00011668
299	1.0249

(Intentionally, several intermediary iterations have been skipped)



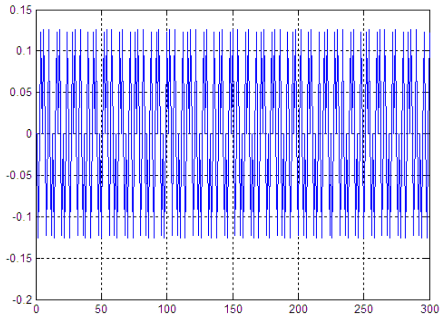


Figure. 3. Superior Barnsley Fractal for *f (a, b, s)*= (1.263, -0.989, 0.1)

Figure. 4. Orbit of Superior Barnsley Fractal for *f (a, b, s)*= (1.263, -0.989, 0.1)

Table 2. For *f (a, b, s)*= (1.263, -0.989, 0.1)

Number of iteration i	| F(z) |
288	0.003153
289	0.12306
290	6.78E-07
291	0.1263
292	0.003321
293	0.12289
294	0.00017627
295	0.12612
296	0.003136
297	0.12308
298	1.85E-05
299	0.12628

(Intentionally, several intermediary iterations have been skipped)

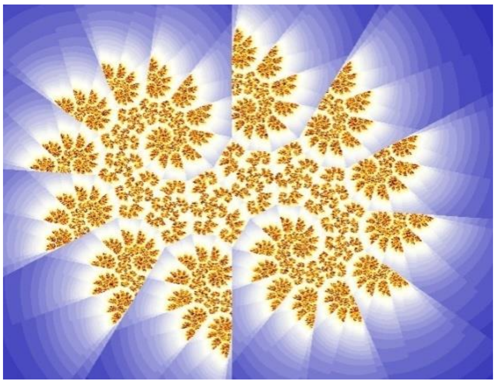


Figure. 5. Superior Barnsley Fractal for *f (a, b, s)*= (1.025, 1.229, 0.5)

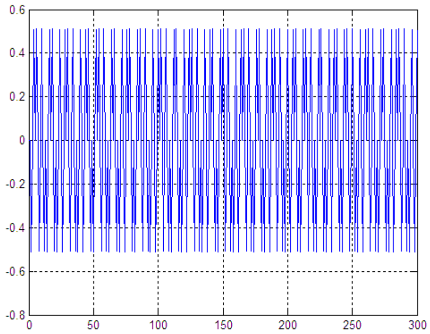


Figure. 6. Orbit of Superior Barnsley Fractal for *f (a, b, s)*= (1.025, 1.229, 0.5)

Table 3. For *f (a, b, s)*= (1.025, 1.229, 0.5)

Number of iteration i	| F(z) |
288	0.006249
289	0.50617
290	2.89E-09
291	0.5125
292	0.0064062
293	0.50601
294	0.00016115
295	0.51234
296	0.006241
297	0.50618
298	8.21E-06
299	0.51249

(Intentionally, several intermediary iterations have been skipped)

6. CONCLUSION

This paper has been presented the three Barsnley Fractals using superior iterates. The results of the Barnsley fractals and superior Barnsley fractals are illustrated in Figure. 1 to 6. On zooming the superior Barnsleyfracrtal we find the IFS patterns, ***curves***, ***spokes*** and ***spirals*** symmetric along ***y*** axis. Further by using the Mann iterates we are able to get the superior Barnsley fractals for the values of ***c*** more than –2 to +2. The work in this study can be expanded to find uses of Superior Orbits in optimising the noise and perturbations of the Mandelbrot set, as well as some novel applications and analysis using Fractal theory in fields such as medicine, engineering, and the arts.

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Biographies

Dr. Sunil Shukla is having a teaching and research experience of approximately 15 Years. He has earned his Ph.D in 2013 from Uttarakhand Technical University, Dehradun. He has supervised M. Phil scholars at "DeshBhagat University", MandiGobindgarh (Punjab). Many of his papers have been published in national and international journals. He has also attended a number of conferences, workshops and Faculty Development Programs.