# On Ꝁ − Ǫ −Bipolar Fuzzy BCI-Ideals

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## Abstract.

This paper introduced the Ꝁ − Ǫ-BFBCI-Ids and Ꝁ − Ǫ-BFBCI-Imp-Ids with examples and properties are studied. In furthermore, discussed about Ꝁ − Ǫ −Bipolar Fuzzy Union and Intersection set as its various algebraic aspects.

**Keywords**. Fuzzy Set (FS), Fuzzy BCI-ideal(FBCI-Id), Ꝁ − Ǫ-Fuzzy Subset (Ꝁ − Ǫ − $FSb$), Ꝁ − Ǫ-Bipolar fuzzy set (Ꝁ − Ǫ-BFS), $\kappa - Q$-bipolar fuzzy Ideal (Ꝁ − Ǫ-BFI), Ꝁ − Ǫ-bipolar fuzzy BCI-Ideal (Ꝁ − Ǫ-BFBCI-Id) and Ꝁ − Ǫ-bipolar fuzzy BCI-Implicative Ideal (Ꝁ − Ǫ-BFBCI-Imp-Id).

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## 1. INTRODUCTION

[17]Zadeh L A described the notation of fuzzy sets in 1965. In 2004, Bipolar logic and bipolar fuzzy logic developed by [18]Yang Y. [19]Zimmermann H J initiated by the concept of Fuzzy set theory and its applications in 1985. In 1986, described the concept of Intuitionistic fuzzy sets by [1]Atanassov K T. [2]Hu Q P developed the concept of On BCI-algebras satisfying$(x * y) * z = x * (y * z)$ in 1980. [16]Nagarajan R, initiated by notation of a new structure and construction of $Q$- fuzzy groups in 2009. In 2019, Cubic intuitionistic structures applied to ideals of BCI-algebras developed by [15]Shum K P. [9]Aldhafeeri S depicted the concept of N-soft p-ideals of BCI-algebras in 2019. In 2009 introduced by the notation of BCI-Implicative ideals of BCI-algebras in [8]Meng J. [10]Jun Y B developed the concept of Hesitant fuzzy translations and extensions of subalgberas and ideals in BCK/BCI-algebras in 2017. Bipolar valued fuzzy sub algebras and bipolar

fuzzy ideals of BCK/BCI-algebras in 2009 developed by [5]Lee K J. [7]Liu Y L described the notation of fuzzy ideals in BCI-algebras in 2001. Bipolar valued fuzzy sets and their operations developed by [6]Lee K M in 2000. [14]Premkumar M develop the concept of On Fundamental Algebraic Attributes of $\omega - Q -$Fuzzy Subring, Normal Subring and Ideal in 2021. On $\kappa - Q$-Anti Fuzzy Normed Rings in 2021 described by [12]Prasanna A. [3]Iseki K initiated by the notation of BCI-algebras in 1980. $\kappa - Q -$Fuzzy Orders Relative to $\kappa - Q -$Fuzzy Subgroups and Cyclic group on various fundamental aspects depicted by [11]Premkumar M in 2020. [13]Premkumar M developed the concept of Fundamental Algebraic Properties on $\kappa - Q -$ Anti Fuzzy Normed Prime Ideal and $\kappa - Q -$ Anti Fuzzy Normed Maximal Ideal in 2021. In 1993, Closed fuzzy ideals in BCI-algebras depicted by [4]Jun Y B.

In this paper introduced by the new contribution of Algebraic Properties on $Ꝁ - Q̧$-BFBCI-Ids. And also described the new notation of $Ꝁ - Q̧$-BFBCI-Imp-Ids in BCI-Algebra and their results.

## 2. PRELIMINARIES

**Definition: 2.1**

An algebra $(G;*,0)$ of kind (2,0) is a BCI-algebra if it satisfies for all $x, y, z \in G$

(i) $\big((x * y) * (x * z)\big) * (z * y) = 0$

(ii) $\big(x * (x * y)\big) * y = 0$

(iii) $x * x = 0$

(iv) $x * y = 0 \; and \; y * x = 0 \Rightarrow x = y.$

**Definition: 2.2**

A $FS$ $\mu$ in $G$ is a FBCI-Id of $G$ if it satisfies for all $x, y, z \in G$

(i) $\mu(0) \geq \mu(x)$

(ii) $\mu(x) \geq min\{\mu(x * y), \mu(y)\}.$

**Definition: 2.3**

A $FS$ $\mu$ in $G$ is a FBCI-Imp-Id of $G$ if it satisfies for all $x, y, z \in G$

$$\mu\Big\{\Big(x * \big(y * (y * x)\big) * \Big(0 * \big(0 * (x * y)\big)\Big)\Big)\Big\} \geq min\Big\{\mu\Big(\Big(\big(((x * y, q) * y, q\big) * (0 * y, q), q\Big) * (z, q)\Big), \mu(z)\Big\}.$$

**Definition: 2.4**

Let $G$ and $Q$ be any two nonempty sets and $\kappa \in [0,1]$ and $\mu$ be a $\bar{Q} - FSb$ of a set $G$. The $FS$ $\mu^{\kappa}$ of $G$ is called the $\kappa - Q - FSb$ of $G$ is defined by

$\mu^{\kappa}(x, q) = (\mu(x, q), \kappa), \forall \; x \in G \; and \; q \in Q.$

## 3. ON Ꝁ − Q̧-BFBCI-IDS AND Ꝁ − Q̧-BFBCI-IMP-IDS IN BCI-ALGEBRA

**Definition: 3.1**

A $Ꝁ - Q̧$-BFS, $\tilde{Ą}$ in $G$ is called a $Ꝁ - Q̧$-BFBCI-Id of $G$. If its following conditions

(a) (i) $\mu_{\tilde{Ą}^{Ꝁ}}^{-}(0, q̧) \geq \{(\mu_{\tilde{Ą}}^{-}(\tilde{u}, q̧), Ꝁ)\}$

(ii) $\mu_{\tilde{A}^{Ꝁ+}}(0, q) \leq \{(\mu_{\tilde{A}^{+}}(\tilde{u}, q), Ꝁ)\}$

(b) (i) $\mu_{\tilde{A}^{Ꝁ-}}(\tilde{u}, q) \geq min\{(\mu_{\tilde{A}^{-}}(\tilde{u} * \ddot{v}, q), Ꝁ), (\mu_{\tilde{A}^{-}}(\ddot{v}, q), Ꝁ)\}$

(ii) $\mu_{\tilde{A}^{Ꝁ+}}(\tilde{u}, q) \leq max\{(\mu_{\tilde{A}^{+}}(\tilde{u} * \ddot{v}, q), Ꝁ), (\mu_{\tilde{A}^{+}}(\ddot{v}, q), Ꝁ)\}, \forall \tilde{u}, \ddot{v} \in G.$

**Definition: 3.2**

A Ꝁ − Ọ-BFS, $\tilde{A}$ in $G$ is called a Ꝁ − Ọ-BFBCI-Imp-Id of $G$ if it satisfies in above definition condition (a) and the following conditions

(i) $\mu_{\tilde{A}^{Ꝁ-}}\{(\tilde{u} * (\ddot{v} * (\ddot{v} * \tilde{u}, q), q) * (0 * (0 * (\tilde{u} * \ddot{v}, q), q), q), q)\} \geq min\left\{\left(\mu_{\tilde{A}^{-}}\left(\left(\left((\tilde{u} * \ddot{v}, q) * \ddot{v}, q\right) * (0 * \ddot{v}, q), q\right) * (z, q)\right), Ꝁ\right), (\mu_{\tilde{A}^{-}}(z, q), Ꝁ)\right\}$ and

(ii) $\mu_{\tilde{A}^{Ꝁ+}}\{(\tilde{u} * (\ddot{v} * (\ddot{v} * \tilde{u}, q), q) * (0 * (0 * (\tilde{u} * \ddot{v}, q), q), q), q)\} \leq max\left\{\left(\mu_{\tilde{A}^{+}}\left(\left(\left((\tilde{u} * \ddot{v}, q) * \ddot{v}, q\right) * (0 * \ddot{v}, q), q\right) * (z, q)\right), Ꝁ\right), (\mu_{\tilde{A}^{+}}(z, q), Ꝁ)\right\}, \forall \tilde{u}, \ddot{v}, z \in G.$

**Example: 3.2.1.**

Consider a BCI-Algebra$(G, *, 0)$, where $G = \{0, a, b, c\}$ and $*$ is given by the table

| $*$ | 0 | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| 0 | 0 | $a$ | $b$ | $c$ |
| $a$ | $a$ | 0 | $c$ | $b$ |
| $b$ | $b$ | $c$ | 0 | $a$ |
| $c$ | $c$ | $b$ | $a$ | 0 |

Let Ꝁ − Ọ-BFS in $G$ represented by

| $G$ | 0 | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $\mu_{\tilde{A}^{Ꝁ-}}$ | -0.8 | -0.8 | -0.5 | -0.5 |
| $\mu_{\tilde{A}^{Ꝁ+}}$ | 0.9 | 0.9 | 0.4 | 0.4 |

Then by routine calculations Ꝁ − Ọ-BFBCI-Imp-Id of $G$. ■

**Theorem: 3.3**

Any Ꝁ − Ọ-BFBCI-Imp-Id of $G$ is a Ꝁ − Ọ − BFI of $G$.

Proof:

Let, Ꝁ − Ọ-BFBCI-Imp-Id of $G$

Then,

(i) $\mu_{\tilde{A}}^{Ꝁ-}\{(\tilde{u} * (v̏ * (v̏ * \tilde{u}, ꝗ), ꝗ) * (0 * (0 * (\tilde{u} * v̏, ꝗ), ꝗ), ꝗ), ꝗ)\} \geq min\left\{\left(\mu_{\tilde{A}}^{-}\left(\left(\left(((\tilde{u} * v̏, ꝗ) * v̏, ꝗ\right) * (0 * v̏, ꝗ), ꝗ\right) * (z, ꝗ)\right), Ꝁ\right), \left(\mu_{\tilde{A}}^{-}(z, ꝗ), Ꝁ\right)\right\}$ and

(ii) $\mu_{\tilde{A}}^{Ꝁ+}\{(\tilde{u} * (v̏ * (v̏ * \tilde{u}, ꝗ), ꝗ) * (0 * (0 * (\tilde{u} * v̏, ꝗ), ꝗ), ꝗ), ꝗ)\} \leq max\left\{\left(\mu_{\tilde{A}}^{+}\left(\left(\left(((\tilde{u} * v̏, ꝗ) * v̏, ꝗ\right) * (0 * v̏, ꝗ), ꝗ\right) * (z, ꝗ)\right), Ꝁ\right), \left(\mu_{\tilde{A}}^{+}(z, ꝗ), Ꝁ\right)\right\}, \forall\, \tilde{u}, v̏, z \in G.$

Substitute $z$ by $v̏$ and $v̏$ by 0 to get

(i) $\mu_{\tilde{A}}^{Ꝁ-}\{(\tilde{u} * (0 * (0 * \tilde{u}, ꝗ), ꝗ) * (0 * (0 * (\tilde{u} * 0, ꝗ), ꝗ), ꝗ), ꝗ)\} \geq min\left\{\left(\mu_{\tilde{A}}^{-}\left(\left(\left(((\tilde{u} * 0, ꝗ) * 0, ꝗ\right) * (0 * 0, ꝗ), ꝗ\right) * (v̏, ꝗ)\right), Ꝁ\right), \left(\mu_{\tilde{A}}^{-}(v̏, ꝗ), Ꝁ\right)\right\}$ and

(ii) $\mu_{\tilde{A}}^{Ꝁ+}\{(\tilde{u} * (0 * (0 * \tilde{u}, ꝗ), ꝗ) * (0 * (0 * (\tilde{u} * 0, ꝗ), ꝗ), ꝗ), ꝗ)\} \leq max\left\{\left(\mu_{\tilde{A}}^{+}\left(\left(\left(((\tilde{u} * 0, ꝗ) * 0, ꝗ\right) * (0 * 0, ꝗ), ꝗ\right) * (v̏, ꝗ)\right), Ꝁ\right), \left(\mu_{\tilde{A}}^{+}(v̏, ꝗ), Ꝁ\right)\right\}, \forall\, \tilde{u}, v̏, z \in G.$

$\Rightarrow \mu_{\tilde{A}}^{Ꝁ-}(\tilde{u}, ꝗ) \geq min\{(\mu_{\tilde{A}}^{-}(\tilde{u} * v̏, ꝗ), Ꝁ), (\mu_{\tilde{A}}^{-}(v̏, ꝗ), Ꝁ)\}$ and $\mu_{\tilde{A}}^{Ꝁ+}(\tilde{u}, ꝗ) \leq max\{(\mu_{\tilde{A}}^{+}(\tilde{u} * v̏, ꝗ), Ꝁ), (\mu_{\tilde{A}}^{+}(v̏, ꝗ), Ꝁ)\}, \ \forall\, \tilde{u}, v̏ \in G.$

Hence, $Ꝁ -$ Q̧-BFBCI-Id of $G$. The converse of theorem 3.3 is not true as proved by the following example. ■

**Example: 3.3.1**

Consider a BCI-Algebra$(G,*,0)$, where $G = \{0, a, b, c\}$ and $*$ is given by the table

| $*$ | **0** | ***d*** | ***e*** | ***f*** |
|---|---|---|---|---|
| **0** | 0 | 0 | 0 | $f$ |
| ***d*** | $d$ | 0 | 0 | $f$ |
| ***e*** | $e$ | $e$ | 0 | $f$ |
| ***f*** | $f$ | $f$ | $f$ | 0 |

Let $Ꝁ - Ꝋ$-BFS in $G$ represented by

| $G$ | **0** | $d$ | $e$ | $f$ |
|---|---|---|---|---|
| $\mu_{\tilde{A}}^{Ꝁ-}$ | -0.6 | -0.4 | -0.4 | -0.4 |
| $\mu_{\tilde{A}}^{Ꝁ+}$ | 0.8 | 0.7 | 0.7 | 0.7 |

Then not a $Ꝁ - Ꝋ$-BFBCI-Imp-Id of $G$, as defined by

$\mu_{\tilde{A}}^{Ꝁ+}\{(d * (e * (e * d, ꝗ), ꝗ) * (0 * (0 * (d * e, ꝗ), ꝗ), ꝗ), ꝗ)\} = \mu_{\tilde{A}}^{Ꝁ+}(d, ꝗ) = -0.4 \nleq -0.6 = max\left\{\left(\mu_{\tilde{A}}^{+}\left(\left(\left((d * e, ꝗ) * e, ꝗ\right) * (0 * e, ꝗ), ꝗ\right) * (0, ꝗ)\right), Ꝁ\right), \left(\mu_{\tilde{A}}^{+}(0, ꝗ), Ꝁ\right)\right\} = \mu_{\tilde{A}}^{Ꝁ+}(0, ꝗ)$. ■

**Proposition: 3.4**

Let, $Ꝁ - Ꝋ$-BFS in $G$ is a $Ꝁ - Ꝋ$-BFBCI-Id of $G$, if and only if for all $\tilde{u}, \ddot{v}, z \in G$, $(\tilde{u} * \ddot{v}, ꝗ) * (z, ꝗ) = (0, ꝗ) \Rightarrow$

$(i)$ $\mu_{\tilde{A}}^{Ꝁ-}(\tilde{u}, ꝗ) \geq min\{(\mu_{\tilde{A}}^{-}(\ddot{v}, ꝗ), Ꝁ), (\mu_{\tilde{A}}^{-}(z, ꝗ), Ꝁ)\}$ and

$(ii)$ $\mu_{\tilde{A}}^{Ꝁ+}(\tilde{u}, ꝗ) \leq max\{(\mu_{\tilde{A}}^{+}(\ddot{v}, ꝗ), Ꝁ), (\mu_{\tilde{A}}^{+}(z, ꝗ), Ꝁ)\}$.

**Proposition: 3.5**

Let, $Ꝁ - Ꝋ$-BFS in $G$ is a $Ꝁ - Ꝋ$-BFBCI-Id of $G$, if and only if for all $\tilde{u}, \ddot{v}, z \in G$, $(\tilde{u} * \ddot{v}, ꝗ) = 0 \Rightarrow$

(i) $\mu_{\tilde{A}}^{Ꝁ-}(\tilde{u}, ꝗ) \geq \mu_{\tilde{A}}^{Ꝁ-}(\ddot{v}, ꝗ)$ and

(ii) $\mu_{\tilde{A}}^{Ꝁ+}(\tilde{u}, ꝗ) \leq \mu_{\tilde{A}}^{Ꝁ+}(\ddot{v}, ꝗ)$.

**Definition: 3.6**

Let, two $Ꝁ - Ꝋ$-BFSs in $G$. Then the union denoted by $\mu_{\tilde{A}_1}^{Ꝁ-} \cup \mu_{\tilde{A}_2}^{Ꝁ-}$ and $\mu_{\tilde{A}_1}^{Ꝁ+} \cup \mu_{\tilde{A}_2}^{Ꝁ+}$ is $max\left\{\mu_{\tilde{A}_1}^{Ꝁ-}, \mu_{\tilde{A}_2}^{Ꝁ-}\right\}$ and $min\left\{\mu_{\tilde{A}_1}^{Ꝁ+}, \mu_{\tilde{A}_2}^{Ꝁ+}\right\}$.

**Definition: 3.7**

Let, two $Ꝁ - Ꝋ$-BFSs in $G$. Then the intersection denoted by $\mu_{\tilde{A}_1}^{Ꝁ-} \cap \mu_{\tilde{A}_2}^{Ꝁ-}$ and $\mu_{\tilde{A}_1}^{Ꝁ+} \cap \mu_{\tilde{A}_2}^{Ꝁ+}$ is $min\left\{\mu_{\tilde{A}_1}^{Ꝁ-}, \mu_{\tilde{A}_2}^{Ꝁ-}\right\}$ and $max\left\{\mu_{\tilde{A}_1}^{Ꝁ+}, \mu_{\tilde{A}_2}^{Ꝁ+}\right\}$.

**Theorem: 3.8**

Let, two $Ꝁ - Ꝋ$-BFSs in $G$ and two $Ꝁ - Ꝋ$-BFBCI-Imp-Id of $G$. Then $\mu_{\tilde{A}_1}^{Ꝁ-} \cup \mu_{\tilde{A}_2}^{Ꝁ-}$ and $\mu_{\tilde{A}_1}^{Ꝁ+} \cup \mu_{\tilde{A}_2}^{Ꝁ+}$ is a $Ꝁ - Ꝋ$-BFBCI-Imp-Ids of $G$.

Proof:

Let two $Ꝁ - Ꝋ$-BFBCI-Imp-Ids of $G$.

Then,

(i) $\mu_{\tilde{A}_1}^{Ꝁ-}(0, ꝗ) \geq \mu_{\tilde{A}_1}^{-}((\tilde{u}, ꝗ), Ꝁ)$ and $\mu_{\tilde{A}_2}^{Ꝁ-}(0, ꝗ) \geq \mu_{\tilde{A}_2}^{-}((\tilde{u}, ꝗ), Ꝁ)$

(ii) $\mu_{\tilde{A}_1}{}^{Ꝁ+}(0,q) \leq \mu_{\tilde{A}_1}{}^{+}\big((\tilde{u},q),Ꝁ\big)$ and $\mu_{\tilde{A}_2}{}^{Ꝁ+}(0,q) \leq \mu_{\tilde{A}_2}{}^{+}\big((\tilde{u},q),Ꝁ\big)$.

Therefore

$$max\left\{\mu_{\tilde{A}_1}{}^{Ꝁ-},\mu_{\tilde{A}_2}{}^{Ꝁ-}\right\}(0,q) \geq max\{\mu_{\tilde{A}_1}{}^{-}\big((\tilde{u},q),Ꝁ\big),\mu_{\tilde{A}_2}{}^{-}\big((\tilde{u},q),Ꝁ\big)\} \text{ and}$$

$$min\left\{\mu_{\tilde{A}_1}{}^{Ꝁ+},\mu_{\tilde{A}_2}{}^{Ꝁ+}\right\}(0,q) \leq min\left\{\mu_{\tilde{A}_1}{}^{+}\big((\tilde{u},q),Ꝁ\big),\mu_{\tilde{A}_2}{}^{+}\big((\tilde{u},q),Ꝁ\big)\right\}$$

For all $\tilde{u},\ddot{v} \in G$ *and* $q \in Q$,

$$\mu_{\tilde{A}_1}{}^{Ꝁ+}\{(\tilde{u} * (\ddot{v} * (\ddot{v} * \tilde{u},q),q) * (0 * (0 * (\tilde{u} * \ddot{v},q),q),q),q)\} = \mu_{\tilde{A}_1}{}^{+}\left\{\Big(\big(((\tilde{u} * \ddot{v},q) * \ddot{v},q\big) * (0 * \ddot{v},q),q\Big),Ꝁ\right\},$$

$$\mu_{\tilde{A}_1}{}^{Ꝁ-}\{(\tilde{u} * (\ddot{v} * (\ddot{v} * \tilde{u},q),q) * (0 * (0 * (\tilde{u} * \ddot{v},q),q),q),q)\} = \mu_{\tilde{A}_1}{}^{-}\left\{\Big(\big(((\tilde{u} * \ddot{v},q) * \ddot{v},q\big) * (0 * \ddot{v},q),q\Big),Ꝁ\right\} \text{ and}$$

$$\mu_{\tilde{A}_2}{}^{Ꝁ+}\{(\tilde{u} * (\ddot{v} * (\ddot{v} * \tilde{u},q),q) * (0 * (0 * (\tilde{u} * \ddot{v},q),q),q),q)\} = \mu_{\tilde{A}_2}{}^{+}\left\{\Big(\big(((\tilde{u} * \ddot{v},q) * \ddot{v},q\big) * (0 * \ddot{v},q),q\Big),Ꝁ\right\},$$

$$\mu_{\tilde{A}_2}{}^{Ꝁ-}\{(\tilde{u} * (\ddot{v} * (\ddot{v} * \tilde{u},q),q) * (0 * (0 * (\tilde{u} * \ddot{v},q),q),q),q)\} = \mu_{\tilde{A}_2}{}^{-}\left\{\Big(\big(((\tilde{u} * \ddot{v},q) * \ddot{v},q\big) * (0 * \ddot{v},q),q\Big),Ꝁ\right\}$$

Thus, $min\left\{\mu_{\tilde{A}_1}{}^{Ꝁ+},\mu_{\tilde{A}_2}{}^{Ꝁ+}\right\}\{(\tilde{u} * (\ddot{v} * (\ddot{v} * \tilde{u},q),q) * (0 * (0 * (\tilde{u} * \ddot{v},q),q),q),q)\} = min\left\{\mu_{\tilde{A}_1}{}^{+}\left(\Big(\big(((\tilde{u} * \ddot{v},q) * \ddot{v},q\big) * (0 * \ddot{v},q),q\Big),Ꝁ\right),\mu_{\tilde{A}_2}{}^{+}\left(\Big(\big(((\tilde{u} * \ddot{v},q) * \ddot{v},q\big) * (0 * \ddot{v},q),q\Big),Ꝁ\right)\right\} = min\left\{\mu_{\tilde{A}_1}{}^{+},\mu_{\tilde{A}_2}{}^{+}\right\}\left\{\Big(\big(((\tilde{u} * \ddot{v},q) * \ddot{v},q\big) * (0 * \ddot{v},q),q\Big),Ꝁ\right\}$,

and

$$\begin{aligned} max\left\{\mu_{\tilde{A}_1}{}^{Ꝁ-},\mu_{\tilde{A}_2}{}^{Ꝁ-}\right\}&\{(\tilde{u} * (\ddot{v} * (\ddot{v} * \tilde{u},q),q) \\ &* (0 * (0 * (\tilde{u} * \ddot{v},q),q),q),q)\} \\ &= max\left\{\mu_{\tilde{A}_1}{}^{-}\left(\Big(\big(((\tilde{u} * \ddot{v},q) * \ddot{v},q\big)\right.\right. \\ &* (0 * \ddot{v},q),q\Big),Ꝁ\Big),\mu_{\tilde{A}_2}{}^{-}\Big(\Big(\big(((\tilde{u} * \ddot{v},q) * \ddot{v},q\big) \\ &\left.\left.* (0 * \ddot{v},q),q\Big),Ꝁ\right)\right\} \\ &= max\{\mu_{\tilde{A}_1}{}^{-},\mu_{\tilde{A}_2}{}^{-}\}\left\{\Big(\big(((\tilde{u} * \ddot{v},q) * \ddot{v},q\big)\right. \\ &\left.* (0 * \ddot{v},q),q\Big),Ꝁ\right\}. \end{aligned}$$

That is $\mu_{\tilde{A}_1}{}^{Ꝁ-} \cup \mu_{\tilde{A}_2}{}^{Ꝁ-}$ and $\mu_{\tilde{A}_1}{}^{Ꝁ+} \cup \mu_{\tilde{A}_2}{}^{Ꝁ+}$ is $Ꝁ - Q$-BFBCI-Imp-Ids of $G$. ■

**Theorem: 3.9**

Let, two $Ꝁ - Q$-BFSs in $G$, and two $Ꝁ - Q$-BFBCI-Imp-Ids of $G$. Then $\mu_{\tilde{A}_1}{}^{Ꝁ-} \cap \mu_{\tilde{A}_2}{}^{Ꝁ-}$ and $\mu_{\tilde{A}_1}{}^{Ꝁ+} \cap \mu_{\tilde{A}_2}{}^{Ꝁ+}$ is a $Ꝁ - Q$-BFBCI-Imp-Ids of $G$.

Proof:

Let, two $Ꝁ - Q$-BFBCI-Imp-Ids of $G$

Then,

(i) $\mu_{\tilde{\mathbb{A}}_1}{}^{Ҝ-}(0,ꝗ) \geq \mu_{\tilde{\mathbb{A}}_1}{}^{-}\big((ũ,ꝗ),Ҟ\big)$ and $\mu_{\tilde{\mathbb{A}}_2}{}^{Ҝ-}(0,ꝗ) \geq \mu_{\tilde{\mathbb{A}}_2}{}^{-}\big((ũ,ꝗ),Ҟ\big)$

(ii) $\mu_{\tilde{\mathbb{A}}_1}{}^{Ҝ+}(0,ꝗ) \leq \mu_{\tilde{\mathbb{A}}_1}{}^{+}\big((ũ,ꝗ),Ҟ\big)$ and $\mu_{\tilde{\mathbb{A}}_2}{}^{Ҝ+}(0,ꝗ) \leq \mu_{\tilde{\mathbb{A}}_2}{}^{+}\big((ũ,ꝗ),Ҟ\big)$.

Therefore

$$min\left\{\mu_{\tilde{\mathbb{A}}_1}{}^{Ҝ-},\mu_{\tilde{\mathbb{A}}_2}{}^{Ҝ-}\right\}(0,ꝗ) \geq min\left\{\mu_{\tilde{\mathbb{A}}_1}{}^{-}\big((ũ,ꝗ),Ҟ\big),\mu_{\tilde{\mathbb{A}}_2}{}^{-}\big((ũ,ꝗ),Ҟ\big)\right\} \text{ and}$$
$$max\left\{\mu_{\tilde{\mathbb{A}}_1}{}^{Ҝ+},\mu_{\tilde{\mathbb{A}}_2}{}^{Ҝ+}\right\}(0,ꝗ) \leq max\left\{\mu_{\tilde{\mathbb{A}}_1}{}^{+}\big((ũ,ꝗ),Ҟ\big),\mu_{\tilde{\mathbb{A}}_2}{}^{+}\big((ũ,ꝗ),Ҟ\big)\right\}$$

For all $ũ, v̏ \in G$ $and$ $ꝗ \in Ǫ$,

$$\mu_{\tilde{\mathbb{A}}_1}{}^{Ҝ+}\{(ũ*(v̏*(v̏*ũ,ꝗ),ꝗ)*(0*(0*(ũ*v̏,ꝗ),ꝗ),ꝗ),ꝗ)\} =$$
$$\mu_{\tilde{\mathbb{A}}_1}{}^{+}\left\{\Big(\big((ũ*v̏,ꝗ)*v̏,ꝗ\big)*(0*v̏,ꝗ),ꝗ\Big),Ҟ\right\},$$
$$\mu_{\tilde{\mathbb{A}}_1}{}^{Ҝ-}\{(ũ*(v̏*(v̏*ũ,ꝗ),ꝗ)*(0*(0*(ũ*v̏,ꝗ),ꝗ),ꝗ),ꝗ)\} =$$
$$\mu_{\tilde{\mathbb{A}}_1}{}^{-}\left\{\Big(\big((ũ*v̏,ꝗ)*v̏,ꝗ\big)*(0*v̏,ꝗ),ꝗ\Big),Ҟ\right\} \text{ and}$$
$$\mu_{\tilde{\mathbb{A}}_2}{}^{Ҝ+}\{(ũ*(v̏*(v̏*ũ,ꝗ),ꝗ)*(0*(0*(ũ*v̏,ꝗ),ꝗ),ꝗ),ꝗ)\} =$$
$$\mu_{\tilde{\mathbb{A}}_2}{}^{+}\left\{\Big(\big((ũ*v̏,ꝗ)*v̏,ꝗ\big)*(0*v̏,ꝗ),ꝗ\Big),Ҟ\right\},$$
$$\mu_{\tilde{\mathbb{A}}_2}{}^{Ҝ-}\{(ũ*(v̏*(v̏*ũ,ꝗ),ꝗ)*(0*(0*(ũ*v̏,ꝗ),ꝗ),ꝗ),ꝗ)\} =$$
$$\mu_{\tilde{\mathbb{A}}_2}{}^{-}\left\{\Big(\big((ũ*v̏,ꝗ)*v̏,ꝗ\big)*(0*v̏,ꝗ),ꝗ\Big),Ҟ\right\}$$

Thus, $max\left\{\mu_{\tilde{\mathbb{A}}_1}{}^{Ҝ+},\mu_{\tilde{\mathbb{A}}_2}{}^{Ҝ+}\right\}\{(ũ*(v̏*(v̏*ũ,ꝗ),ꝗ)*(0*(0*(ũ*v̏,ꝗ),ꝗ),ꝗ),ꝗ)\} = max\left\{\mu_{\tilde{\mathbb{A}}_1}{}^{+}\left(\Big(\big((ũ*v̏,ꝗ)*v̏,ꝗ\big)*(0*v̏,ꝗ),ꝗ\Big),Ҟ\right),\mu_{\tilde{\mathbb{A}}_2}{}^{+}\left(\Big(\big((ũ*v̏,ꝗ)*v̏,ꝗ\big)*(0*v̏,ꝗ),ꝗ\Big),Ҟ\right)\right\} = max\left\{\mu_{\tilde{\mathbb{A}}_1}{}^{+},\mu_{\tilde{\mathbb{A}}_2}{}^{+}\right\}\left\{\Big(\big((ũ*v̏,ꝗ)*v̏,ꝗ\big)*(0*v̏,ꝗ),ꝗ\Big),Ҟ\right\}$,

and

$$\begin{aligned} min\left\{\mu_{\tilde{\mathbb{A}}_1}{}^{Ҝ-},\mu_{\tilde{\mathbb{A}}_2}{}^{Ҝ-}\right\}&\{(ũ*(v̏*(v̏*ũ,ꝗ),ꝗ) \\ &*(0*(0*(ũ*v̏,ꝗ),ꝗ),ꝗ),ꝗ)\} \\ &= min\left\{\mu_{\tilde{\mathbb{A}}_1}{}^{-}\left(\Big(\big((ũ*v̏,ꝗ)*v̏,ꝗ\big)\right.\right. \\ &\left.*(0*v̏,ꝗ),ꝗ\Big),Ҟ\right),\mu_{\tilde{\mathbb{A}}_2}{}^{-}\left(\Big(\big((ũ*v̏,ꝗ)*v̏,ꝗ\big)\right. \\ &\left.\left.*(0*v̏,ꝗ),ꝗ\Big),Ҟ\right)\right\} \\ &= min\left\{\mu_{\tilde{\mathbb{A}}_1}{}^{-},\mu_{\tilde{\mathbb{A}}_2}{}^{-}\right\}\left\{\Big(\big((ũ*v̏,ꝗ)*v̏,ꝗ\big)\right. \\ &\left.*(0*v̏,ꝗ),ꝗ\Big),Ҟ\right\}. \end{aligned}$$

That is $\mu_{\tilde{\mathbb{A}}_1}{}^{Ҝ-} \cap \mu_{\tilde{\mathbb{A}}_2}{}^{Ҝ-}$ and $\mu_{\tilde{\mathbb{A}}_1}{}^{Ҝ+} \cap \mu_{\tilde{\mathbb{A}}_2}{}^{Ҝ+}$ is $Ҟ - Ǫ$-BFBCI-Imp-Ids of $G$. ■

## 4. CONCLUSIONS

During in this paper, we acquainted a $\kappa - Q$-BFBCI-Id of Fuzzy BCI -algebra which is discussed with illustrative examples and proposition of Algebras and also investigated $\kappa - Q$-BFBCI-Imp-Ids . In further future work define as Doubt $\kappa - Q$-BFBCI-Id and $\kappa - Q$-BFBCI-Imp-Ids.

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