Application of Fast Fourier Transform to the Synthesis of Track Irregularities

Panya Kansuwan[1], Sedthawat Sucharitpwatskul[2] and Anchalee Manonukul[2]

[1]Mechanical Engineering Department, School of Engineering, King Mongkut's Institute of Technology Ladkrabang, Bangkok, Thailand

[2]National Metal and Materials Technology Center, National Science and Technology Development Agency, Pathumthani, Thailand

E-mail address: panya.ka@kmitl.ac.th, sedthaws@mtec.or.th, anchalm@mtec.or.th

Abstract

The theoretical analysis of railway vehicles' dynamic behavior requires track models that include track irregularities. The power spectrum density (PSD) function is a well-accepted mathematical expression suitable for the irregular random nature. Since there are various PSD parameters according to the impact on the vehicles, track engineering quantifies track quality into six classes in the USA but two categories in Germany. EN13848 considers track irregularity at different wavelengths, while UIC518 classified the track quality as QN1, QN2, and QN3 by a standard deviation depending on the speeds of the rail vehicles. Minimum separated lengths of errors are also recommended. This paper presents a unified algorithm to synthesize the track irregularity conforming to each standard when PSD is provided with a standard deviation of the data, sample length, and track length. The algorithm implements the parameters in Discrete Fourier Transformation (DFT) representation, giving the availability to use Fast Fourier Transformation (FFT) that can speed up the computational time.

Keywords. track irregularity, power spectral density, rail vehicle dynamics, Discrete Fourier Transform, Fast Fourier Transform

1. INTRODUCTION

One system input that influences the rail vehicle dynamics is the track on which the vehicle runs. The track geometry is not plenary but contains intentionally and unintentionally variations described in four irregularities, i.e., cross-level, alignment, vertical, and gage track profile [1, 2]. Different classes of track quality exist depending on the influence of the parameters on track/vehicle interaction aspects such as vehicle performance, passenger comfort, and safety issue analysis compared to a safety threshold. The Federal Railroad Administration (FRA) divides the track into six classes [3], whereas the German high-speed train engineers classify two classes of the track [2]. Depending on the speed of the vehicles, track quality in the European network is represented under three quality levels defined as QN1, QN2, and QN3 [4]. Due to the importance of the geometry variation, researchers and designers have studied two interrelated aspects of the subject, i.e., the measurement of track irregularities and the dynamic response behaviors of rail vehicles to the irregularity.

The first aspect states the design of the device structure to measure, record, and interpret the irregularity information. The track measurement devices range from human pushed trolleys to fully automated measuring Track Recording Vehicles (TRV). TRV equipped with sensors records data that requires a fast algorithm to transform data into helpful information. In all cases, PSD is the appropriate mathematical expression that concludes the track irregularity [5]. The latter aspect of the subject involves some methods to generate the track model at a specific level of required irregularity understudying. The main approaches to developing random rail irregularities are either time integration [6] or spatial integration methods [7]. A transfer function that relates the frequency domain of an irregularity to a white noise signal with constant PSD can be derived directly from a determined PSD function. After applying the appropriate integral transform, they obtained differential equations to be integrated numerically. The other approach employs a discrete-time method using a state-space transition matrix. The derived transfer function is the shape filter whose frequency response magnitude squared matches the PSD [8].

According to the Weiner-Khinchin theorem, two-sided PSD is the Fourier transform of autocorrelation of a time- domain function [9]. Given the PSD, one can determine the amplitude of the real signals using the application of the Inverse Fourier transform. In the discrete sense, the computation of the Fourier transform can be accomplished by the FFT algorithm, which speeds up the calculation of the integrals greatly. The Fourier transform methods or spectral methods provide an efficient computational tool in an extensive field of science and engineering. Indeed, the vast application is possible due to the FFT algorithm in approximating the integral numerically. This paper applies the FFT algorithm to create irregularity with the provided PSD data format. The method is suitable for transforming measuring data to classify track quality onboard in various classes of tracks in the USA and Europe.

2. DESCRIPTION OF TRACK IRREGULARITIES

Many aspects of rail vehicle study require an analytical representation of the track geometry. The quality of the geometry could be described in vertical (v), alignment (a), cross-level (c), and gage variation (g) [1, 2]. These parameters are expressed in Fig. 1 and mathematically in (1) – (4). Among the irregularity, cross-level and alignment variation strongly influences lateral vibration, whereas the gage controls lateral stability. The other vertical irregularity has little impact on lateral vibration.

$$v = (z_L + z_R)/2 \tag{1}$$

$$a = (y_L + y_R)/2 \quad (2)$$

$$c = (z_L - z_R)/2b \quad (3)$$

$$g = (y_L - y_R)/2 \quad (4)$$

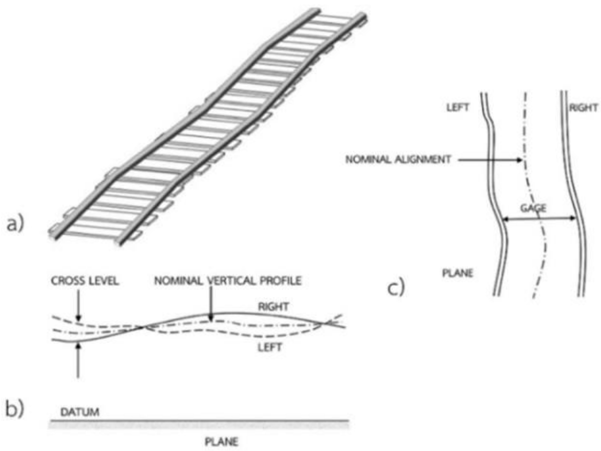

Figure 1 Track irregularity schematic diagram [1]

Due to the random nature of the quantities, PSD in the form of (5) is an appropriate statistical representation of the irregularity [5].

$$S(\Omega) = \frac{A}{\Omega^2} \quad (5)$$

The equations of the PSD described by German high- speed train applications [2] and ORE B176 [10] are expressed for cross-level (S_C), alignment (S_A), and vertical (S_V) irregularity in (6)-(8).

$$S_C(\Omega) = \frac{A_V \Omega_C^2 \Omega^2}{b^2(\Omega^2+\Omega_r^2)(\Omega^2+\Omega_C^2)(\Omega^2+\Omega_S^2)} \quad (6)$$

$$S_A(\Omega) = \frac{A_A \Omega_A^2}{(\Omega^2+\Omega_r^2)(\Omega^2+\Omega_C^2)} \quad (7)$$

$$S_V(\Omega) = \frac{A_V \Omega_C^2}{(\Omega^2+\Omega_r^2)(\Omega^2+\Omega_C^2)} \quad (8)$$

in which Ω_C = 0.8246 rad/m, Ω_r = 0.0206 rad/m, Ω_S = 0.4380 rad/m, and Ω_A = 0.8246 rad/m. The values of A_i are given in TABLE I for the lower and higher interference track classes [2, 11].

Table 1 Track Parameters [2, 11]

A_i	Lower	Higher
A_V $(m\,rad)$	4.032 × 10^{-7}	1.080 × 10^{-6}
A_A $(m\,rad)$	2.119 × 10^{-7}	6.124 × 10^{-7}

Fig. 2 shows the PSD of the alignment irregularity analytically obtained from (7) with the provided constants. The recommended wavelength considered in EN13848 to describe the geometric irregularity acceptable levels is in three different ranges of [3, 25], [25, 70], and [70, 200] m. The wavelengths are between 2 to 100 meters for a demonstration to cover all shorter and middle track lengths. This range covers the lowest sampling rate of 0.01 to 0.5 sample/m or the wavenumber (Ω) from 0.0628 to 3.1416 rad/m. However, for the other end of the sampling rate, if any supposed two separated irregular indicators are statistically 0.2 m, the corresponded Nyquist frequency is five samples per meter or 15.57 rad/m. In conclusion, the lowest sampling rate is randomly selected in 0.01 to 0.5 sample/m for each section, whereas the highest sampling rate is more than ten sample/m.

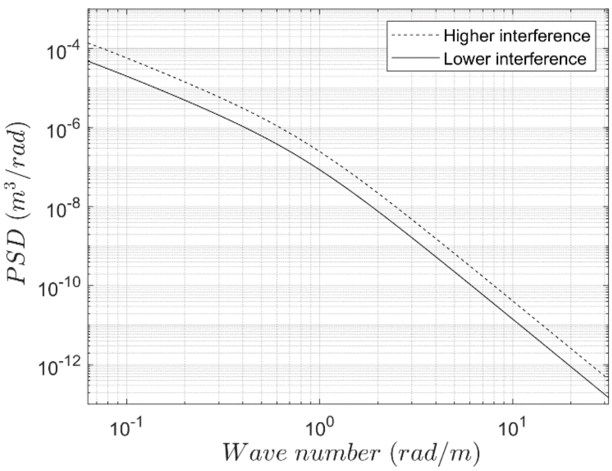

Figure 2 PSD of the two quality levels of alignment irregularity

3. THE CONTINUOUS REPRESENTATION OF THE PSD

Any track irregularity parameters can be described in a continuous function in the spatial domain as $h(x)$ or in the wavenumber domain as $H(s)$. The two quantities are by mean of the following Fourier transform pair as represented by (9) and (10).

$$H(s) = \int_{-\infty}^{\infty} h(x) e^{2\pi i s x} dx \tag{9}$$

$$h(x) = \int_{-\infty}^{\infty} H(s) e^{-2\pi i s x} ds \tag{10}$$

In order to get an expression of PSD, we rely on the correlation theorem of $g(x)$ and $h(x)$, which states that Fourier transformation of $corr(h,g)$ is $G(s)H^*(s)$. According to the Wiener-Khinchin theorem, Fourier transformation of the autocorrelation of $h(x)$ is $|H(s)|^2$ or two-sided PSD, $\Phi(s)$.

$$\Phi(s) = \int_{-\infty}^{\infty} corr(h,h)e^{-2\pi isx}dx = |H(s)|^2 \tag{11}$$

A one-sided PSD or $S(s)$ is defined in (12).

$$S(s) = \begin{cases} \Phi(0), & if\ s = 0 \\ 2\Phi(s), & if\ s \neq 0 \end{cases} \tag{12}$$

4. DISCRETE REPRESENTATION OF PSD IN A FINITE LENGTH

In a discrete form of $h(x)$, the signal values are sampled evenly every interval Δ in (13). For a finite length, L is equal to $N\Delta$ in which N is an integer and the power of 2. For a N consecutive samples, a discrete approximation of (9) is (14).

$$h_n = h(x_n) = h(n\Delta); n = 1,2,3, \ldots, N-1 \tag{13}$$

$$H(s_k) = \Delta \sum_{n=0}^{N-1} h_n e^{2\pi ikn/N} \tag{14}$$

in which $s_n = \frac{n}{N\Delta}, n = -\frac{N}{2}, \ldots, \frac{N}{2} - 1$. We can ignore the scale factor Δ. It is only considered when we need to interpret the result in a physical interval in space. As a result, the formula for the discrete Fourier transform for a set of pair of N samples becomes

$$H_k = \sum_{n=0}^{N-1} h_n e^{2\pi ikn/N} \tag{15}$$

$$h_n = \frac{1}{N}\sum_{k=0}^{N-1} H_k e^{-2\pi ikn/N} \tag{16}$$

A relevant total power to our application is the mean squared amplitude (σ^2), i.e.

$$\sigma^2 = \frac{1}{L}\int_0^L |h(x)|^2 dx \approx \frac{1}{N}\sum_{n=0}^{N-1}|h_n|^2$$

$$= \frac{1}{N^2}\sum_{k=0}^{N-1}|H_k|^2 = \frac{1}{N^2}\sum_{k=0}^{N/2} S_{\boldsymbol{k}} \tag{17}$$

As a result,

$$S_k = \frac{1}{N^2}\begin{cases} |H_0|^2 \\ |H_k|^2 + |H_{N-k}|^2, \ k = 1,2, \ldots, \left(\frac{N}{2} - 1\right) \\ 2\left|H_{N/2}\right|^2 \end{cases} \tag{18}$$

5. DFT SYNTHESIS METHOD FOR ALIGNMENT IRREGULARITY

Given the power spectral density either from track measurement system or track classification spectrum, a selection of S_k in the discrete sense should represent $S(\Omega_k)$ for the equivalent power of the bin covering the wavenumber range of $\Delta\Omega = 2\pi/L$. Consequently, S_k is calculated in the integral form over the window function centered of its $S(s_k)$.

$$S(\Omega_k) = S_k = 2\Phi(\Omega_k) \approx \frac{1}{\Delta\Omega}\int_{\Omega_k-\frac{\Delta\Omega}{2}}^{\Omega_k+\frac{\Delta\Omega}{2}} S(\Omega)d\Omega \tag{19}$$

in which $k = 1, \dots, N/2$. We assume that the total power of the signal is zero; thus, $\int_{-\infty}^{\infty} a(x)dx = 0$ or $S(s_0) = S_0 = 0$. The magnitude of a discrete quantity in two-sided PSD can be derived directly from S_k as

$$|H_k| = \sqrt{\frac{1}{2}S_k}\ , k = 0, \dots, N/2 \tag{20}$$

Since h_n is a real signal, this leads to $H_{-k} = H_k^*$. To create a set of stochastic data from the one-sided PSD, a random phase angle set, φ_k, of $N/2$ data is generated and substituted into (21), by which we can generate the signal for the whole wavelength range $k = 0,1,2, \dots, N/2$.

$$H_k = |H_k|(\cos(\varphi_k) + i\sin(\varphi_k)) \tag{21}$$

A spatial signal, h_n is generated after we apply the Inverse FFT algorithm to (21).

6. DESCRIPTION OF THE PROCEDURE FOR ALIGNMENT IRREGULARITY

The section presents the approach to synthesize the track alignment PSD given by the German high-speed railway description. The total energy can be determined from (17), of which the square root is the standard deviation corresponding to the track quality level stated in Table II. The interchange of the irregularity expression in Germany and the USA to be qualified as UIC518 become possible by finding a scale factor A_{QN} from (22) given $Sqn_{\boldsymbol{k}} = A_{QN}S_{\boldsymbol{k}}$ by substitute (20) in (17)

$$A_{QN} = (\sigma_{QN}/\sigma_{ref})^2 \tag{22}$$

in which $\sigma_{ref} = \frac{1}{N}\sqrt{\sum_{k=0}^{N/2} S_{\boldsymbol{k}}}$ and σ_{QN} is given in [4]

The development of the process is described by stating the total track length L_{total} having p section, each of which has a constant length L_i and $L_{total} = \sum_{i=1}^{p} L_i$

$$L_{total} = \{L_1\ L_2\ L_3 \dots L_p\} \tag{23}$$

L_i is also divided into k sections; thus, $L_i = \sum_{j=1}^{k} lm_j$

$$L_i = \{lm_1\ lm_2\ lm_3 \dots lm_k\},\ lm_j \in [lm_{min}, lm_{max}] \tag{24}$$

in which lm_1 to lm_{k-1} have a random length except lm_k is determined by the remaining length. Each section has its own sample length (sa_i) and quality level (qn_i)

$$sa_i = \{sl_1\ sl_2\ sl_3 \dots sl_k\}_i, sl_j \in [sl_{min}, sl_{max}] \tag{25}$$

$$qn_i = \{\sigma_1\ \sigma_2\ \sigma_3 \dots \sigma_k\}_i, \sigma_j \in [\sigma_{min}, \sigma_{max}] \tag{26}$$

After setting up the requirement, we have a restriction for each lm_k to be sampled by N data to be the power of 2. This can be implemented by $m_j = ceil(log_2(lm_j/sl_j))$ and $N_j = 2^{m_j}$. Equation (25) is revised by the expression, $sl_j = lm_j/sl_j$. The algorithm for each section lm_j for alignment is as follows

For $j = 1\ to\ k$

$m = ceil(log_2(lm_j/sl_j))$

$N = 2^m$

$sl_j = \frac{Lm_j}{N}$

$xsub_j = \frac{Lm_j}{N}\{0\ 1\ 2 \dots\ N-1\}$

$\omega sub_j = \frac{2\pi}{Lm_j}\{0\ 1\ 2 \dots\ N/2\}$

$\Omega_l = \omega sub_{jl}$

$S = \left\{S_0\ S_1\ S_2 \dots S_{\frac{N}{2}}\right\}$

$$S_l = \begin{cases} 0; & l = 0 \\ S(\Omega_l) = \frac{A\Omega_2^2(\Omega_l^2+\Omega_1^2)}{\Omega_l^4(\Omega_l^2+\Omega_2^2)}; & l = 1,2,\dots,\frac{N}{2} \end{cases}$$

$\sigma_{ref} = \frac{1}{N}\sqrt{\sum_{k=0}^{\frac{N}{2}} S_{\boldsymbol{k}}}$

Generate a random phase angle (Uniform distribution)

$$\varphi = \left\{\varphi_1\ \varphi_2 \dots \varphi_{\frac{N}{2}}\right\},\quad \varphi_l \in [0,2\pi)$$

Generate stochastic wave length complex number

$$A = \{A_0\ A_1\ A_2 \dots A_{N-1}\},$$

$$A_l = \begin{cases} 0, \quad l = 0 \\ \sqrt{\frac{1}{2}S_l}e^{-i\varphi_l x}, l = 1,2,\dots,\frac{N}{2} \\ \sqrt{\frac{1}{2}S_{N-l}}e^{-i\varphi_{N-l}x}, l = \frac{N}{2}+1,\dots,N-1 \end{cases}$$

Generate a random number a_{QN} (Normal distribution)

$$asub_j = a_{QN}\ IFFT[A]$$

end

Our algorithm can specify the separation error length and section length. In this demonstration, we set the sample length to be less than 0.1 m to capture the minimum separated errors at 0.2 m and random section length between 2 to 100 m. The section length is uniformly selected except for the last one, which has a restriction to complete the total length and is greater than the minimum section range. Nevertheless, our Matlab scripts can change the section length range according to EN13848. Once the above algorithm is implemented for lower interference track quality, the algorithm has been verified by comparing the back-calculated one-sided PSD of each section to their original data in Fig. 2.

A pseudo-stochastic function is quickly introduced by randomly choosing a_{QN} with a provided normal distribution. We set the mean equal to σ_{QN}/σ_{ref} and the standard deviation is equal to 5% of the mean. For demonstration, five km-length tracks with different alignment qualities of QN1 and QN2 are generated according to the German PSD function. The standard deviation at $80 < V \leq 120$ km/h is 1.20 mm for QN1 and 1.50 mm for QN2 [4]. An additional parameter of the track is the range of the irregularity wavelength of [2, 100] m and sample length of 0.2 m. There are 359 track sections within the wavelength range. A cut-off 500 m-length of the generated track is shown in Fig. 3. From (2) and (4), we are then able to synthesize the lateral displacement of the right/left rail for further study in the field of railway vehicle dynamics.

To verify the algorithm, we use (17) to evaluate σ of each section. There are 359 sections for the generated 5-km test track. Fig. 4 shows that the distribution matches the Normal distribution in which the average value complies with the requirement of track quality QN1 at 1.2 mm and QN2 at 1.5 mm. This verifies our algorithm to generate track irregularity at a required track quality.

7. CONCLUSION

The Discrete Fourier Transformation representation of the random irregularity offers an attractive computational form to be implemented using the Fast Fourier Transform algorithm. In addition, the parameters involved can be interpreted in various track quality requirements that relate to the method used to measure the track digitally. The track wavelength stated in EN13848 is the lowest sampling frequency, while the standard deviation stated in UIC518 is interpreted as the root mean square of the amplitude of the irregularity. The irregularity synthesized by the algorithm has the flexibility to conform with both UIC518 and EN13848 upon the given PSDs. This work has to be implemented to further study the influence of the irregularity on railway vehicles dynamics and the track quality measuring system verification.

8. ACKNOWLEDGMENT

This work (P1950662) is supported by Research Development Innovation Management for National Strategic and Network Division, National Science and Technology Development Agency, Thailand.

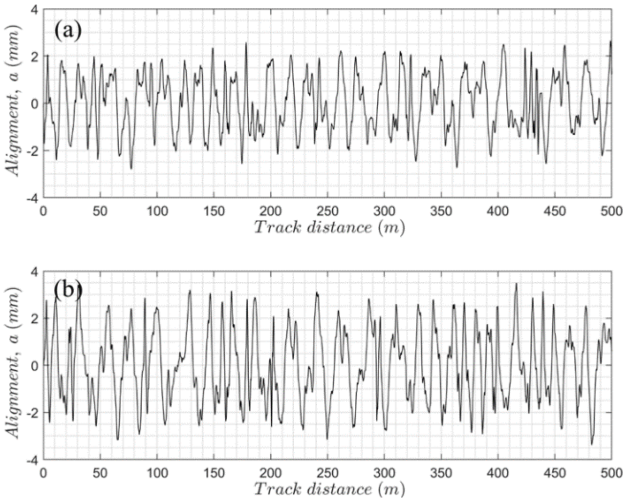

Figure 3 The track alignment synthesis: (a) track quality level QN1 (b) track quality level QN2

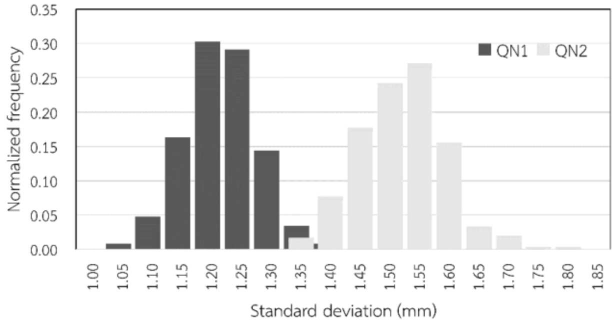

Figure 4 The standard deviation distribution of the generated track for classes QN1 and QN2

9. REFERENCES

[1] V.K. Garg and R.V. Dukkipat, “Dynamics of Railway Vehicle Systems.” 1984, Ontario: Academic Press Canada.

[2] P. Meinke and A. Mielcarek, “Design and evaluation of trucks for High-Speed wheel/rail application,” in Dynamics of High-Speed Vehicles, W.O. Schiehlen, Editor. 1982, SPRINGER-VERLAG.

[3] A. Hamid and T.L. Yang, “Analytical descriptions of Track Geometry Variations.” 1983, FRA-ORD-83-03.

[4] UIC 518, “Testing and approval of railway vehicles from the point of view of their dynamic behavior-Safety-Track Fatigue-Ride Quality.” 2009.

[5] J.C. Corbin and W.M. Kaufman, “Classifying Track by Power Spectral Density.” Mechanics of Transportation Suspension Systems, ASME AMD, 1975. 15: p. 1-20.

[6] R.C. White, D.A. Limbert, J.K. Hedrick and N.K. Cooperrider, “Guideway-Suspension Tradeoffs in Rail Vehicle Systems,” in Arizona State University Engineering Research Center Report. 1978.

[7] J.E. Dzielski and J.K. Hedrick, “Energy Dissipation Due to Vehicle/Track Interaction.” Vehicle System Dynamics, 1984. 13: p. 315-337.

[8] R.H. Fries and B.M. Coffey, “A state-space approach to synthesis of random vertical and cross-level rail irregularity.” Journal of Dynamic System, Measurement, and Control, 1990. 112: p. 83-87.

[9] W.H. Press, S.A. Teukolsky, W.T. Vetterling and B.P. Flannery, “Numerical Recipes in Fortran 77: The Art of Sceintific Computing.” 2nd ed. 2001.

[10] ORE B 176, “Bogies with steered or steering wheelsets, Report No. 1: Specifications and preliminary studies. Vol. 2. Specification fr a bogie with improved curving characteristics.” 1989.

[11] M. Dumitriu, “Numerical synthesis of the track alignment and application. Part I: The synthesis method.” Transport problems, 2016. 11(1): p. 19-28.