Dynamic Analysis and Shape Control of Membrane Structures

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Abstract

This research examines the dynamic analysis and shape control of membrane structures designed to function as space structures under a variety of loading conditions. The finite element method was used in commercially available software to perform the analysis. Square membrane structures have been analyzed under on-orbit loading conditions. The present analysis gives the results of various dynamic properties of membranes such as natural frequencies, mode shapes, etc. PVDF and PZT smart materials were used at optimized locations of membranes to minimize shape error. It has been observed that the shape error of the membrane is minimized significantly after using smart materials.

Keywords: Membrane Structures, Pre-Stressed, FEM, Natural Frequency, Mode Shape, Shape Error, Finite Element Method, Smart Materials.

1. INTRODUCTION

Membrane structures play a vital role in the fields of architecture, civil engineering, and space technology. In space, these structures are widely used as communication satellite antennas, space telescopes, solar arrays, etc. [1]. Membrane structures have the capability to play an advanced role in space-borne structures due to their inherent characteristics like ultra light weight, ability to stow in small volume, demanding small space requirements, high flexibility to change their shape etc. These structures show very similar properties to conventional materials like Aluminium, Carbon Fiber, etc. These membrane structures are very thin and possess negligible bending and compressive stiffness. So these thin structures need to be pre-tensioned to function as an engineering structural element [2, 3]. For the current study, an ultra-lightweight inflatable space-based membrane structure made of Kapton, Mylar, and Kevlar was used. Inflatable structures, often known as gossamer structures and are inflated with gas, usually air [4]. In space, ambient conditions are perilous and unpredictable. As a result, the challenges for the long-term survival of space-borne structures are complex. Space-borne structures like inflatable antennas and solar arrays are subjected to thermal distortion, bombarding particles (meteoroids) and huge vibrations while being launched. These unwanted elements reduce the performance of these structures. That’s why dynamic properties calculation is paramount for safety and accuracy of the system. Huge vibrations create large amount of out-of-plane displacement which affects the shape accuracy and the main concern with these structures is how they keep their shape intact. Shape accuracy creates the new opportunities in the field of space exploration and structures with high performance. One such technique is using smart materials, for instance, Lead Zirconate Titanate (PZTs), Polyvinylidene Fluoride (PVDFs), etc [5]. These materials will behave as sensors and actuators. These smart materials will help to reduce large deformations by controlling them and maintaining the shape as smoothly as possible. So PVDF and PZT are going to be used at optimal locations. This paper deals with FEM based numerical analysis of square membrane structures at various on-orbit loading conditions. Natural frequencies, mode shapes, and out of plane displacement have been calculated using the finite element method in ABAQUS. Out-of-plane displacement with and without using smart materials (PVDF and PZT) was also compared.

2. Membrane Materials Properties

For exploring space at a level of admiration with minimal cost and the utmost possible performance, researchers must use materials that possess high mechanical and thermal stability over a long period of time in their proposed orbital environment. All space-borne structures are comprised of a light and very thin film-based material that would be amenable to compact stowage volume and deployment in space. The achievement of light-weight, large-deployable, and compact launch volume structures poses a great challenge to the scientific community. Numerous thin polymeric films have been used in space applications. Table 1 lists the properties of the membrane.

Table 1. Membrane material properties [6]

Materials	Kapton	Mylar	Kevlar
Density[ρ](Kg/m3)	1430	1390	1400
Young’s Modulus[E](MPa)	2000	5000	100000
Poisson’s ratio[μ]	0.34	0.35	0.30
Thermal conductivity(w/mk)	0.12	0.14	0.04

3. Mathematical Modelling of Membrane Structure

A membrane assumed as a thin plate loaded with tension. It cannot sustain bending resistance and restoring forces arises only when subjected to tensile loading [7]. Membrane behaves similarly to a plate, while string behaves similarly to a beam. The perfect example of membrane is a drumhead.

The membrane structures are analyzed by considering the following assumptions;

1. The effect of gravity is insignificant.
2. Displacement is small and takes place only in z-direction.
3. Membrane is very thin.
4. Pre-stress and mass density is considered to be uniform throughout the membrane structure.

5. The transverse shear stresses are insignificant.

A flat curve S bounds a region in *xy* plane to formulate the equation of motion for membrane structures, as shown in Fig. 1. Pressure loading acts normal to the plane surface of membrane (z-direction) denoted by $f(x, y, t)$ and p is the tension at a point.

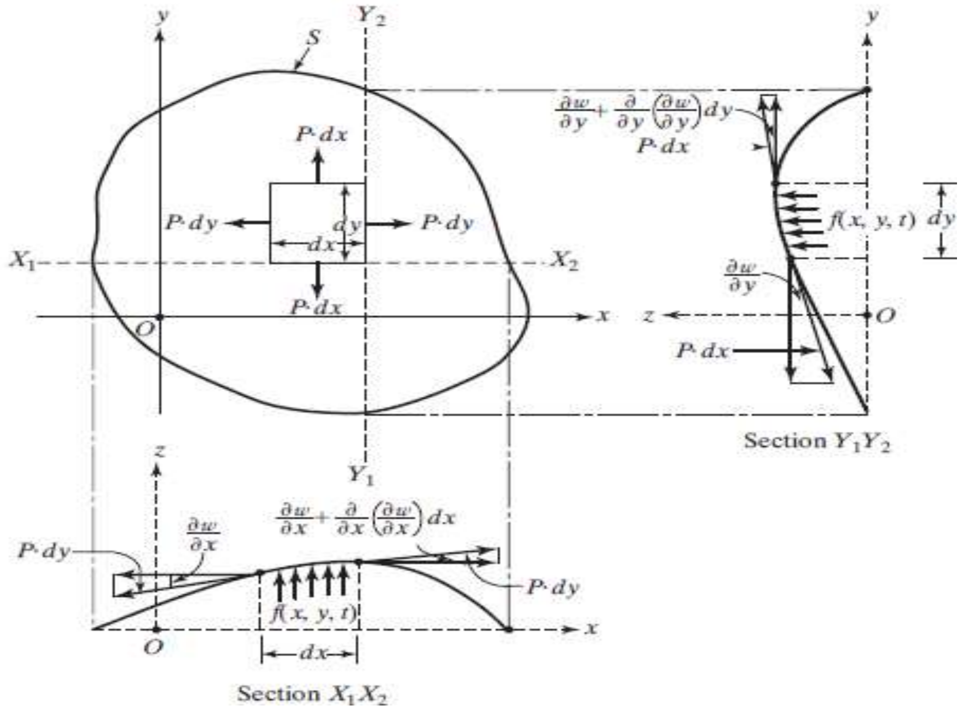


Figure 1. A membrane under uniform tension [7]

The magnitude of tension p typically remains invariable at every point of the membrane, making it completely visible in the drumhead. A very small elemental area *dxdy is considered in the x-y* plane, forces of magnitude *pdx* and *pdy* will act on the sides that are parallel to the *x* and *y*-axes, respectively, as shown in Fig. 1. A resultant force in the z direction emerges as a result of these two forces.

$$\left(p\frac{\partial^2 w}{\partial y^2}dxdy\right) \quad \text{and} \quad \left(p\frac{\partial^2 w}{\partial x^2}dxdy\right)$$

The pressure force acting normal to the plane surface of membrane is f (x, y, t) dx dy, and the inertia force is

$$\rho\ (x, y)\ p\frac{\partial^2 w}{\partial t^2}dxdy$$

where ρ (x, y) is the mass of membrane per unit area. The following is the equation of motion for the membrane's forced vibration

$$p\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)+f=\rho\frac{\partial^2 w}{\partial t^2} \qquad (1)$$

Eq. (1) gives the equation of free vibration if force f (x, y, t) = 0

$$c^2\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)=\frac{\partial^2 w}{\partial t^2} \qquad (2)$$

Where, $c=\left(\frac{p}{\rho}\right)^{1/2}$ and $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$

(3)

Equations (1) and (2) can be written in the following way

$$p\nabla^2 w+f=\frac{\partial^2 w}{\partial t^2} \qquad (4)$$

And

$$c^2\nabla^2 w=\frac{\partial^2 w}{\partial t^2} \qquad (5)$$

Equation (5), with c representing wave velocity, is sometimes known as the 2-D wave equation. Variable separable method is going to be used for free vibration condition. A square membrane with side a along x and y direction is considered. *w*(x, y, t) is assumed to be

$$w(x, y, t) = W(x, y)\, T(t) = X(x)\, Y(y)\, T(t) \qquad (E.1)$$

By using Eq. (E.1), we obtain

$$\frac{\partial^2 w}{\partial t^2}=w(x,y)T''(t)$$

$$\frac{\partial^2 w}{\partial x^2}=\frac{\partial^2 w(x,y)}{\partial x^2}T(t)$$

$$\frac{\partial^2 w}{\partial x^2}=\frac{\partial^2 w(x,y)}{\partial y^2}T(t)$$

Using variable separable method, we obtain

$$\frac{T''(t)}{T(t)} = c^2k^2 \quad \text{(i)}$$

$$\frac{X''(x)}{X(x)} = j^2 \quad \text{(ii)}$$

$$\frac{Y''(y)}{Y(y)} = k^2 - j^2 \quad \text{(iii)}$$

We get the solutions of Equations by solving Equations (i), (ii) and (iii)

$$T(t) = c_1 \cos ckt + c_2 \sin ckt \quad \text{(A)}$$

$$X(x) = c_3 \cos jx + c_4 \sin jx \quad \text{(B)}$$

$$Y(y) = c_5 \cos \sqrt{k^2 - j^2}\, y + c_6 \sin \sqrt{k^2 - j^2}\, y \quad \text{(C)}$$

The boundary and initial conditions can be used to determine the constants $c_1 to c_6$.

3.1 Initial and Boundary Conditions-

Initial and boundary conditions are required to determine a unique solution. Typically, the displacement and velocity of the membrane at time t = 0 are $w_0(x, y)$, $w_0'(x, y)$. Hence the initial conditions are specified by

$$w(x, y, 0) = w_0(x, y)$$

$$\frac{\partial w}{\partial t}(x, y, 0) = w_0'(x, y)$$

The boundary conditions are as follows:

1. If the square membrane is hinged at the all four corners, the boundary conditions will be-

$$w(0,0,t) = 0; w(a,0,t) = 0; w(0,a,t) = 0 \text{ and } w(a,a,t) = 0 \qquad t \geq 0$$

4. Results and Discussions

The finite element method (FEM) in ABAQUS is used to analyze free and forced vibrations of single-layer flat square-shaped membranes. The square membrane has a thickness (t) of 0.05 mm and side length of 200 mm. It is supported on knife edges, which essentially implies that all four corners are hinge supported. As shown in Fig. 2, all membrane models are pre-stressed by 10 N/m in the *x-y* plane. For meshing of the configuration, Quadrilateral membrane elements have been used (the M3D4 membrane element, which is a 4 node quadrilateral element). A membrane element is a surface or 2D element that transmits only in-plane forces. So it is necessary to pre-stress these elements to carry out the vibration analysis. According to the assumption, the modulus of elasticity of the membrane will be equal to E when the membrane is in tension and will be very small when subjected to compression. This explains why the membrane elements cannot endure any in-plane compressive loads. Static and linear perturbation analyses with pre-stress effects have been performed in ABAQUS. The effects of various parameters, such as pre-stress, boundary conditions, thickness, and materials behavior, are analyzed using FEM. The analysis has been performed on several models whose results are exposed below;

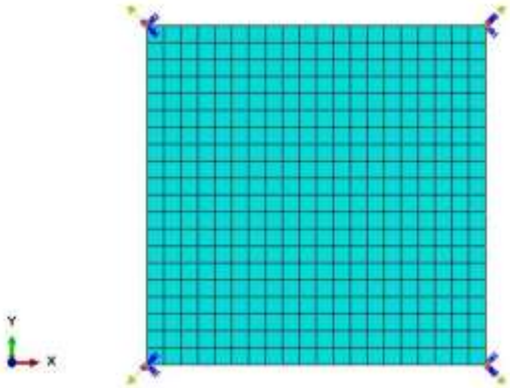


Figure 2. Meshed square membrane showing BCs

4.1 Square membrane with free vibration

Table 2 presented the computational results of natural frequencies obtained by the finite element analysis. The analysis is performed for various space materials. The graph between mode number and natural frequency is plotted for all three materials and shown in Fig. 3. Fig. 4 shows the various mode shapes of kapton membrane.

Table 2. Comparison of Natural frequencies of various square membranes

Mode No.	**Kapton (Hz)**	**Myla r(Hz)**	**Kevlar (Hz)**
1	1.573	2.523	11.246
2	1.573	2.523	11.246
3	2.477	3.973	17.704z
4	2.477	3.973	17.704
5	2.477	3.973	17.704

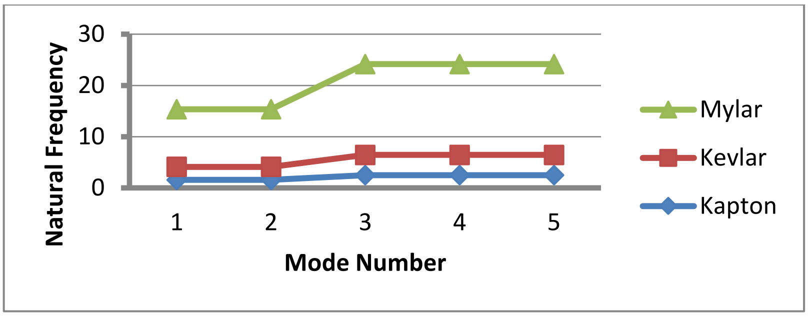
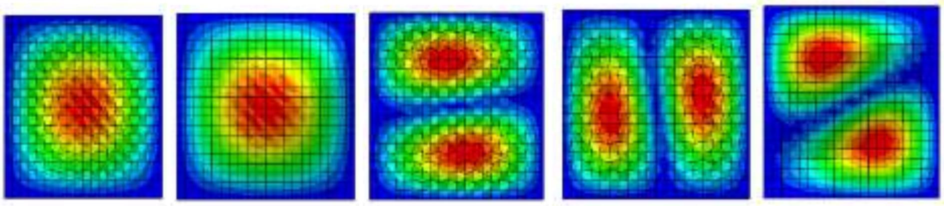


Figure 3. Graphical representation of natural frequencies for different membrane materials



i) Mode 1 ii) Mode 2 iii) Mode 3 iv) Mode 4 v) Mode 5

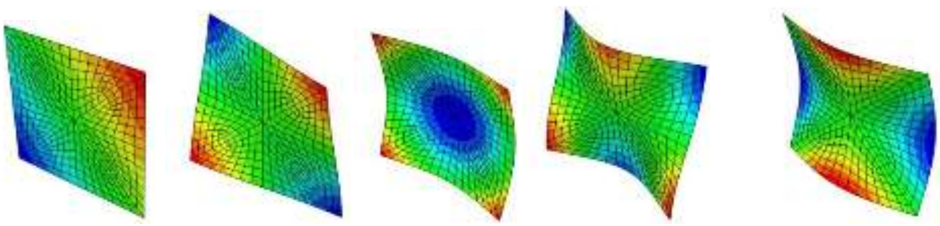
Figure 4. Different mode shapes of the Kapton square membrane

4.2 Square membrane with forced vibration

The forced vibration analysis will focus on the steady-state behavior of the kapton membrane due to transversely applied periodic loading at the centre of the surface. The applied load is in the form of;

$$F = Fo*\sin(\omega t)$$

The magnitude of loading is 10 N and 500 Hz is the frequency applied. Fig. 5 shows the various mode shapes of kapton membrane. Fig. 6 shows the plot between transverse displacement (units in cm) and mode number.



i) Mode 1 ii) Mode 2 iii) Mode 3 iv) Mode 4 v) Mode 5

Figure 5. Different mode shapes of the Kapton square membrane during periodic loading

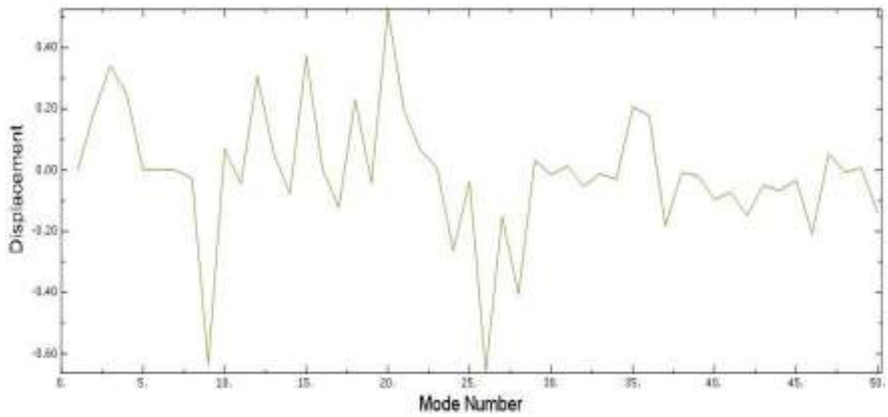


Figure 6. Displacement response with mode number

4.3. Square membrane with forced vibration attached with one smart material

PVDFs and PZTs are smart materials that are linked to the membrane near the centre (Node No.85), as shown in Fig. 7. Loading conditions will be similar to those in the prior scenario. The smart material's shape deforms as a result of the voltage applied to it. PVDF and PZT undergo induced deformation, which aids membrane recovery from a damaged state. Fig. 8 shows a plot of transverse displacement (units in mm) vs. mode number for kapton coupled to PVDF and PZT. As seen in Fig. 8, PVDFs are substantially more efficient than PZTs since PVDFs have less transverse displacement.

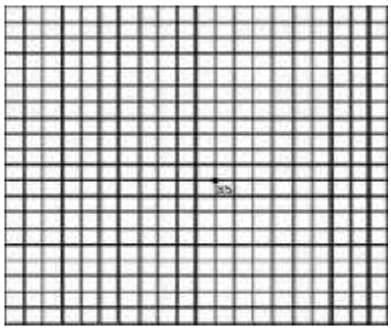


Figure 7**.** Meshed square membrane showing node 85

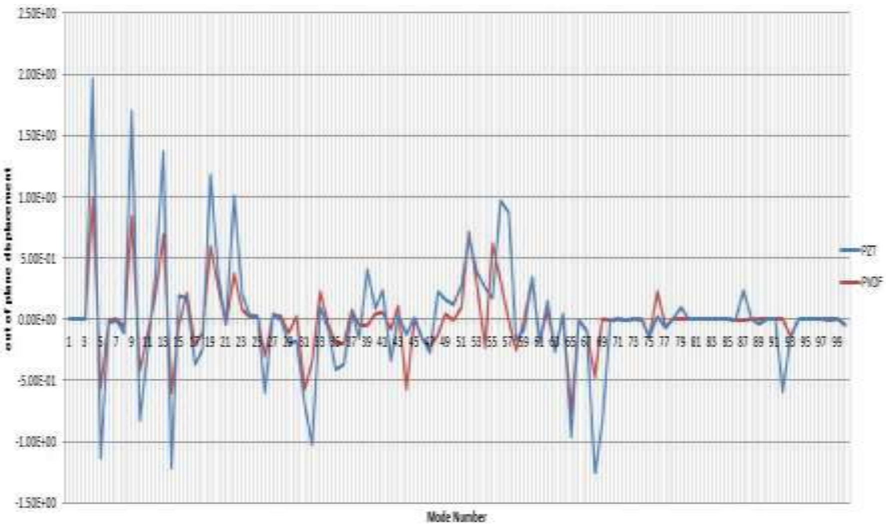


Figure 8. Out-of-plane displacement corresponding to mode numbers

4.4 Square membrane attached with eight smart materials

This model is the same as the previous one. The only change is the placement of PVDF materials into the membrane structure. PVDFs are tied at eight locations, as shown in Fig. 9.

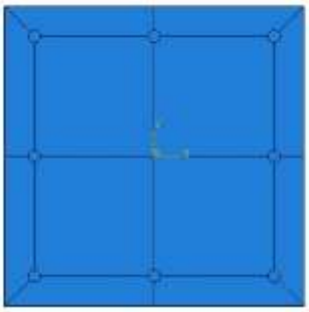


Figure 9. Square membrane coupled with eight PVDFs

Fig. 10 evidently shows that the use of smart materials at optimum locations minimizes the out-of-plane displacement substantially.

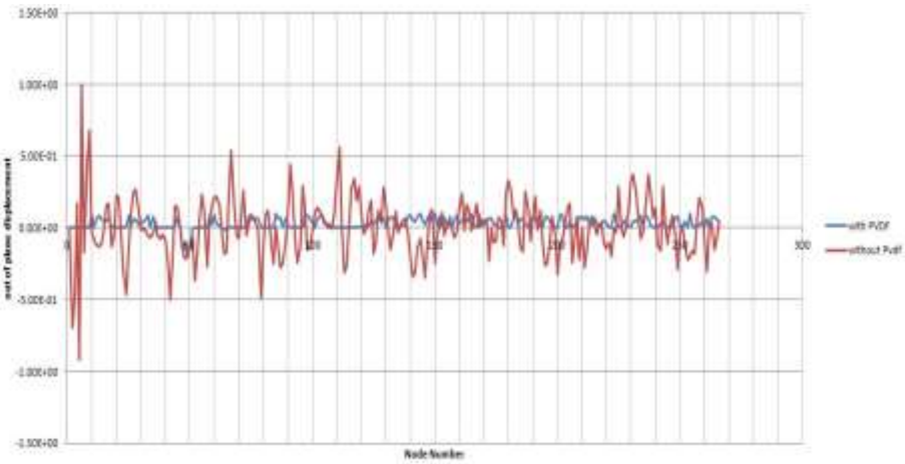


Figure 10. Out-of-plane displacement using PVDF and without PVDF

5. Conclusion

Kapton, Kevlar, and Mylar have emerged as the most viable prospects as space materials, replacing traditional materials such as aluminum, carbon fiber reinforced plastic (CFRP), and so on, due to their ultra-lightweight and flexibility. FEM analysis has been carried out on square membrane with the help of ABAQUS. Free and forced vibration condition applied on membrane. At these loading conditions, various natural frequencies, mode shapes and out-of-plane displacement was collected.Membrane structures are flexible and this inherent property of membrane structure makes it incompetent to abide bending or compressive stresses and this leads to shape deformation (wrinkle formation). At this stage, membrane structure needs to recover its original smooth shape for proper signal transmission. Thanks to PVDF and PZT for controlling the shape exceptionally well. Smart material PVDF and PZT have enough capability to improve the surface accuracy.

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Biographies

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