An analysis of Julia sets and Noor iterations using a complex Mandelbrot iteration scheme

Deepak Negi
Amrapali Institutes of Technology and sciences, Haldwani, UK (India)
dsingh.negi23@gmail.com

Ashwini Kumar Saini
Govind Ballabh Pant Institute of Engineering & Technology, Pauri Garhwal ,UK (India)
ashu82004@gmail.com

Nitin Pandey
Amrapali Institutes of Technology and sciences, Haldwani, UK (India)
pnnitin@gmail.com

Suresh Chandra Wariyal
Amrapali Institutes of Technology and sciences, Haldwani, UK (India)
swariyal@gmail.com

Ravindra Sharma
Swami Rama Himalayan University, Jolly Grant, Dehradun (India)
Ravindrasharma97@gmail.com

ABSTRACT

In this study, we apply Noor iteration process with complex Mandelbrot set Called composite function, Julia set patterns that are fused to the composite function $f(\square) = \sqrt{\square^{3} + \square\square^{2} + 1}$ and discuss their dynamical behavior. The classy orbit structure of this function, whose Julia set encloses the entire complex plane, is purported using figurative dynamics. We also present the fixed point scrutiny using proposed iteration function, which using three parameters, and discussed their sequel graphical analysis of complications taking place in the function.

Keywords: Julia Set, Noor iteration, Mandelbrot sets and Feedback process.

1. INTRODUCTION

The fractals are most beautiful and full of real life applications because of their convoluted geometrical structure. Fractals calculation is affectionate of non-Euclidean having algorithms, we sculpt beautiful Julia's pattern which are not generated by any other generation model [6, 8].

Julia set is introduced by grate mathematician Goston Julia in 1918. He studies the iterated polynomials and defines the various set of examples of fractals. And this study is extended by the popular mathematician and researcher Benoit B Mandelbrot in 1975 called Mandelbrot set are most beautiful and classy fractural structure and relate to the different pattern which is not definable at that time such as heartbeat and irregular shape (coastal area).

There are many other procedure to examine fractals, one of them is most popular is the iterated function system which used to find the approximate fixed points of functions under appropriate conditions, the application of fixed point is applied to nonlinear phenomena in different area of science and arts such as computer graphic, biotechnology, physics and engineering etc.

2. PRELIMINARIES

2.1. Mandelbrot set

We select the initial point 0 since it is the single critical point of the quadratic equation $\square_c(z) = z^k + c$, and the Mandelbrot set B is defined as the set of all $c \in \square$ for which the orbit of point 0 is bounded, that is, $\mathrm{B} = \left\{ c \in \square : \left\{ \square_c^{\,k}(0) \right\}; k = 0,1,2,3 \ldots \textit{bounded} \right\}$ an analogous formulation is $\mathrm{B} = \left\{ c \in \square \left\{ \square_c^{\,k}(0)\, \textit{does not tends to} \, \infty \, \textit{as} \, n \to \infty \right\} \right\}$ [13,1].

2.2. Julia set

If $f : \square \to \square$ is a polynomial function with complex values, then the filled Julia set Q is

$$K(Q) = \left\{ z \in \square : \left| Q^k(z) \right|_{k=1}^{\infty} \textit{does not tend to} \, \infty, \textit{as} \, k \to \infty \right\},$$

where complicated space is$\square$ and $Q^k(z)$ is k^{th} iterate of Q the filled Julia set's boundary, ∂KQ ,is referred to as the "Julia set" [13, 1].

2.3. Noor orbit

Let us, take into consideration an iteration sequence $\{x_n\}$ for the starting point $x_0 \in X$, such that the question is.

$$\begin{Bmatrix} x_{k+1} : x_{k+1} = (1-\delta)\, x_k + \delta_k T y_k ; \\ y_k = (1-\phi_k)\, x_k + \phi_k T z_k ; \\ z_z = (1-\varphi)\, x_k + \varphi_k T x_k ; \quad k = 0,1 \ldots \end{Bmatrix}$$

The sequences away from 0 and converge $\delta_k, \phi_k, \varphi_k \in [0,1]$ and $\{\delta_k\}, \{\phi_k\}, \{\varphi_k\}$. The aforementioned repetitions are known as the Noor orbit, which is characterized by NO, a function of five tuple $(T, x_0, \delta_k, \phi_k, \varphi_k)$. [7]

3. PORPOSED ALGORITHM FOR COMPOSITE MANDELBROT SET

A Mandelbrot set is produced by using the function $f(z) = z^k + c$ where $k \geq 2$, however when creating a composite function, we use the method $f(z) = \sqrt{z^n + cz^2 + 1}$ where $n \geq 3$ see fig1. Here, we are creating a composite function using the iterative Noor technique and the function $f(z) = \sqrt{z^3 + cz^2 + 1}$. The equation displays a basic composite function for $n = 3$ and $\alpha, \beta, \gamma = 1$. Here, we see that when the values of α, β, γ are changed, the wings in the composite Mandelbrot set appear at the beginning, middle, and end points [10, 4, 12]. Likewise, when the values of α and γ are changed, the wings in the composite Mandelbrot set appear at the middle, while the wings in the composite set appear at the starting point when the values of β are changed [5]. And the wings are near the tip of the composite Mandelbrot set when we adjust the α, β, γ values see in fig 2,3,4,5.

Fig1: Composite Mandelbrot set for $\alpha, \beta, \gamma = 1$, $n = 3$

Fig2: Composite Mandelbrot set for $\alpha, \beta, \gamma = .5$, $n = 3$

Fig3: Composite Mandelbrot set for $\alpha = .5, \beta, \gamma = 1$, $n = 3$

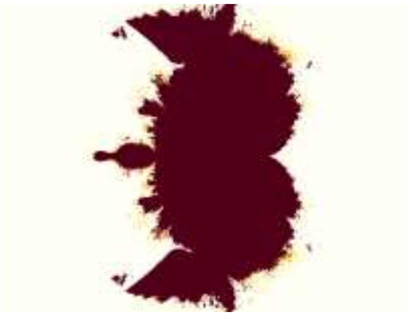

Fig4: Composite Mandelbrot set for $\beta = .5, \alpha, \gamma = 1,\ n = 3$

Fig5: Composite Mandelbrot set for $\alpha, \beta = 1, \gamma = .5,\ n = 3$

3.1 Corresponding Julia sets:

We now have some stunning composite Julia sets with nested and fold symmetry that typically resembles a bush and a dragon curve that alludes to this fractal's most well-known appearance, "The Harter-Heighway Dragon"[9]. These beautiful pictures and realistic-looking natural objects, such bird nests see fig.(8, 9), Peacock wing eye see fig.(6, 7), which we thoroughly examined and discovered to have different points of attachment[11]. When we further examined these, we discovered that they had various points and were giving fixed points, indicating that they were generated by the same formula but were distinct in nature because they followed a dynamical system. Each variable has a unique characteristic that sets it apart from the others, although their origins are the same.

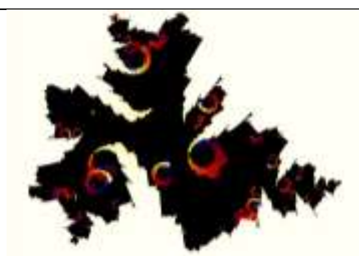	
Fig6: Composite Julia set for $\alpha, \beta = 1, \gamma = .5$, $n = 3\ c = -2.07, 0.62I$	Fig7: Composite Julia set for $\alpha, \beta = 1, \gamma = .5$, $n = 3\ c = -1.6989, -0.9972I$
	
Fig8: Composite Julia set for $\alpha, \beta = 1, \gamma = .5$, $n = 3\ c =$ -0.84,1.49I	Fig9: Composite Julia set for $\alpha, \beta = 1, \gamma = .5$, $n = 3\ c =$ -2.69,-2.59I

3.2 Fixed Points Analysis

A useful framework for examining a variety of nonlinear phenomena appearing in the practical sciences, such as complex graphics, geometry, biology, and physics, is provided by fixed point theory. Fractals and other intricate graphic forms were found to be fixed points in some set maps [2]. Fractals can be thought of as mathematical structures that are similar to themselves and have enough symmetry and resemblance that even very small portions of the overall structure are geometrically similar to it [3]. We selected α, β, γ between zero and one $\alpha = \beta = \gamma \in [0,1]$ for the computational investigation.

Table 1: Orbit of $\lambda(z)$ for $c = -2, 0.4I$ Fixed point for $\alpha = .5, \beta, \gamma = 1$

Total Iteration: n	$\lvert\lambda(z)\rvert$
1	0.4756
2	0.7308
3	0.8177
4	0.7762
.	.
.	.
.	.
54	0.6445
55	0.6445
56	0.6445

Table 2: Orbit of $\lambda(z)$ for $c = -2.1488, 0.0008I$ Fixed point for $\beta = .5, \alpha, \gamma = 1$

Total Iteration: n	$\lvert\lambda(z)\rvert$
1	0.5109
2	0.5470
3	0.5877
4	0.6128
.	.
.	.
.	.
21	0.6262
22	0.6262
23	0.6262

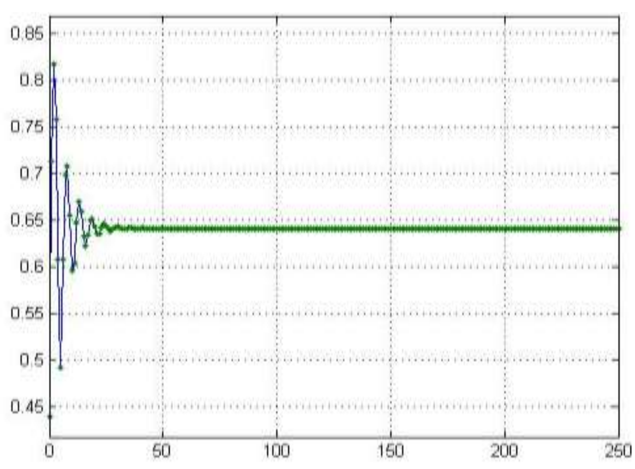

Fig. 10: Observation: In this case, we see that after 54 iterations, the value converges to a fixed point.

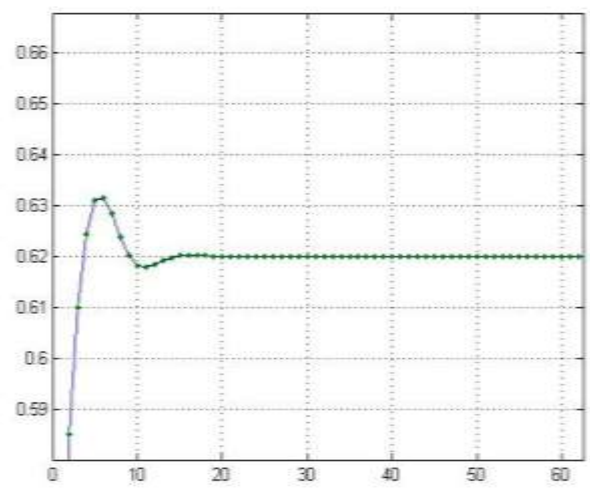

Fig. 11: Observation: Here, we note that after 21 iterations, the value converges to a fixed point.

4. CONCLUSION

The Noor orbit demonstrates that the boundary of the fixed point region is similar to natural features such as bird nests and certain types of peacock wing structures. This is demonstrated by geometrical and numerical analysis of composite Julia sets and composite Mandelbrot sets for the Noor iteration. The escape time computing method hides the intricate structure of these fractals. A variety of orbit traps are developed for the Noor iteration method. The study shows that these sets are completely original and fundamentally different from other known Mandelbrot sets.

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