GDOP optimised LEO constellation for Positioning Estimation

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***Abstract*—An emergence of new applications such as autonomous driving, UAVs, indoor positioning determination, low energy positioning, Robust Real Time Kinematics (RTK) estimation, infra referencing are demanding for robust, accurate, safe GNSS systems. On contrary conventional Medium Earth Orbit (MEO) or Geo Synchronised Orbit (MEO+GSO) based GNSS systems are facing limitation in terms of performance against growing threats such as spoofing, jamming and multipath.**

On the other hand, new space economy and race to launch Mega Constellations from private sectors such as Starlink, Oneweb, Kuiper, Iridium etc, for applications such as internet, IoT, telecommunication hint towards the Low Earth Orbit (LEO) satellites to be recognised as the promising positioning system in the near future where these piggybacked payloads can be used for Precise Positioning, Navigation and Timing (PNT) estimation. In fact, the versatility, small dimensions, short development period and high return-to-cost potential make LEO satellites attractive options not only for technology demonstration but also for navigation purpose. Moreover, considering relative proximity to the Earth compared to MEO satellites, high C/No ratio, high speed, the Doppler based positioning makes LEO systems good candidates for future positioning solutions to complement the existing GNSS and terrestrial navigation.

In this context, this paper presents the design of a dedicated LEO constellation optimized using a Genetic Algorithm (GA). The optimization aims to minimize the Geometric Dilution of Precision (GDOP). The paper demonstrates that the designed constellation provides good Geometrical Navigation Accuracy (GNAC) and global availability. The proposed constellation is a combination of sub-Walker constellations (hybrid) that can provide 100 % global coverage, with at least 4 visible satellites at given epoch. Using the proposed constellation, the position accuracy in a static and dynamic user scenario has been assessed.

***Index Terms*—LEO Positioning, GDOP, Global Availability, GNAC, Optimised LEO, GA.**

I. INTRODUCTION

The conventional Medium Earth Orbit (MEO) based Global Navigation Satellite System (GNSS) is prone to growing intentional and unintentional threats such as spoofing, jamming, multipath etc. On parallel, the emergence of new applications such as UAVs, Autonomous Driving etc, shows a clear tendency of even more demanding requirements for robust, accurate and safe GNSS system.

Current technological developments in small satellite industry and race to launch mega constellations from private sectors such as Starlink, Oneweb, Kuiper, Iridium for various applications such as internet, IoT, telephony, telecommunication hint towards the LEO satellites that can be recognised as the promising positioning system in the near future where these LEO constellations can be used for Precise PNT analysis and estimation.

There are several advantages where these LEO systems can be good candidates for future positioning solutions. The author [7] has shown that Doppler based positioning using LEO is complementary to the existing GNSS and terrestrial navigation. The relative proximity to the Earth compared to MEO satellites makes LEO signals 300 to 2400 times more powerful than GNSS signals. This strongly reduces static and persistent multipath in an urban canyon. The assessment study [4] of UHF/VHF has shown that lower altitude reduces space losses less than 10 dB w.r.t. MEO (includes antenna), lower frequency reduces free space losses with 24 dB (VHF) w.r.t. L Band. Similar studies show that the use of L/S/C bands can be used along with Ka/Ku to improve the performance. The comparison shown in [13] between MEO and LEO (Iridium) has proved that the spreading loss at zenith for LEO is -69 dB and for MEO is –97 dB while the multipath decorrelation time is 1 min and 10 min, respectively. Simulations with Signal of Opportunity (SoO) have shown that the convergence time for full operational capability multi GNSS PPP can be significantly shortened from 9.6 min to 7.0, 3.2, 2.1 or 1.3 min through augmentation with 2.4, 3.1, 6.3 or 9.5 visible LEO satellites respectively [10].

All the above mentioned studies are carried out using existing LEO mega constellations as SoO. Moreover considering the advantages of current LEO satellites for navigation purpose, constellation design is fundamental to improve the performance in terms of accuracy and availability. Several research have been done to design optimised constellations for different purposes. Author [2] shows the optimization of classical MEO constellation for global coverage and hybrid LEO+GSO constellation to provide a navigation service over Europe. In [5] author has designed hybrid constellations for broadband internet access that can be used for augmented navigation purpose with global coverage and at most three visible satellites. Research [12] has shown optimization with combined GA and semi-analytical approach for regional coverage to reduce computational load of GA. Similarly, the paper [8] describes the application of an evolutionary optimization method to design a regional communication satellite constellation.

However, none of the above works proposes an optimal design for a LEO constellation dedicated to navigational services. The paper presents the design of a dedicated LEO constellation using GA which aims to minimize the GDOP and maximize the global coverage with at least 4 visible satellites at given epoch.

The paper is organized as follows: Section I reviews current LEO constellations used for PNT in recent. In section II, an analysis of some selected constellations is carried out which provides useful hints to define important properties of the constellation to be designed; the mathematical modelling of hybrid constellations is presented in section III. Section IV explains GA implementation with definition of fitness function, constraint function, selection of optimization variables and parameter selection for GA which includes population selection, generation size, probability of crossover, probability of mutation and search ranges. Section V and VI shows simulation and discussion of the achieved results respectively; conclusions are drawn in section VII.

II. PRELIMINARY CHOICE OF SOME ORBITAL PARAMETERS

Table I shows an overview of orbital parameters of mainstream GNSS [9] and some selected LEO mega constellations [9]. This is useful to get some initial hints on orbital parameters choices such as eccentricity, argument of perigee, altitude, type of configuration etc. In particular, a combination of circular sub-Walker constellation has been chosen. As a matter of fact, circular constellations are better for navigation and communication purpose due to the constant velocity in the given orbits. Therefore, eccentricity (e) and argument of perigee (w) are set to 0. The choice of hybrid configuration with combination of sub-Walker constellations is motivated by the fact that higher inclination orbits offer better coverage at poles and lower inclination offers greater coverage in equatorial areas [14]. For an instance, to get global coverage Starlink uses a hybrid constellation (different altitude, inclinations) as shown in I.

About the altitude LEO constellations are classified as follows:

1) **Low LEO orbit** (<**1000 Km):** Currently, there are more than 300 satellites at this altitude. Most of them with small and low cost platforms (Nanosat/Cubesat). The New Space Economy is focused here.
2) **High LEO orbit** (>**1000 Km):** At this altitude, there are less than 150 satellites, with moderately bigger platforms, higher reliability and larger lifetime compared to low LEO orbits.
3) **Higher orbit** (>**1200 Km):** There are very few satellites with medium size platforms, higher reliability and longer lifetime with respect to other two orbits.

As shown in the Table I, most of the selected LEO mega constellations are in orbit lower than 1200 Km. Therefore, Low LEO and High LEO orbits are crowded with large number of satellites. Moreover considering other effects such as distribution of space debris, the radiation environment, Total Ionizing Dose (TID) and other orbital perturbation, the higher orbit LEO are considered a better choice [5]. Therefore, we have set the altitude to 1250 Km.

III. MATHEMATICAL MODEL FOR LEO CONSTELLATION DESIGN

We aim to design a hybrid constellation. A Walker constellation is represented by the notation $a : i : T/P/F$ where: a is semi-major; i is the inclination; T is the total number of satellites per plane; P is the total number per orbital planes; F is the phasing between satellites in adjacent planes; $S = T * P$ is total number of satellites. A hybrid constellation is represented as follows based upon different inclinations:

- **Sub-Walker – I (Polar Orbits,** $i = 90°$ **)**
- **Equatorial Orbit** ($i = 0°$)
- **Sub-Walker – II (Optimize,** $0° < i < 90°$)

Mathematical modelling of polar, equatorial orbits and sub-Walker – II are presented in the next sub-sections.

A. Designing of Polar and Equatorial Orbit

Fig. 1 shows the single satellite coverage on the Earth's surface.

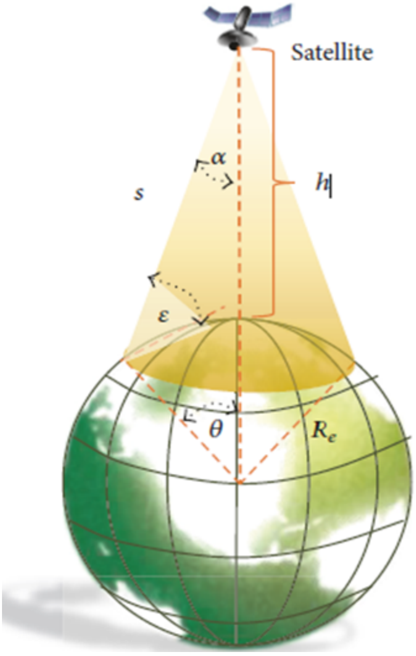

Fig. 1. Single Satellite Coverage with Earth's central angle of visibility [12]

Constellation	Application	Altitude Per Orbit (Km)	Mean Velocity (Km/s)	Period (min)	Planes per Orbit	Satellites per Plane	Total Number of Sat	Inclination (deg)	e	Frequency Bands Downlink
GPS	Navigation	20200	3.88	720	6	4	24	55	0	L1 - 1575.42 Mhz L2 -1227.60 MHz
GALILEO	Navigation	23222	3.66	845	3	6	24+6	56	0	E -1176-1207 MHz
Starlink	Global Internet Broadband	335.9	7.5576	91-112	9	77	11943	42	0	K-band: 17.8-18-5 GHz 18.8-19.3 GHz 19.7-20.2 GHz V-band: 37.5-42.0 GHz
		340.8	7.5839		7	354		48		
		345.6	7.6098		9	283		53		
		550	7.4984		24	66		53		
		1110	7.2139		32	50		53.8		
		1130	7.2696		8	50		74		
		1275	7.2128		5	75		81		
		1325	7.1679		6	75		70		
Oneweb	Global Internet Broadband	1200	7.24	110	18	36	358	87.9	0	Ku-Ka Bands
Iridium Next	Narrowband Communication	780	7.4628	97	6	11	66	86	0	L-band: 1.616-1.63 GHz K-band: 19.3-19.7 GHz
LEO 288*		1000	7,37	105	12		192	90	0	Not defined
LEO 192*		600	9.97	96	12			90		Not defined
LEO 192*		1000	7,37	105	12		288	90		Not defined

TABLE I

OVERVIEW OF MAINSTREAM APPLICATIONS, ORBITAL PARAMETERS, TYPE OF FREQUENCY BANDS OF SELECTED CONSTELLATIONS, * SOME SELECTED LITERATURE CONSTELLATIONS [10]

Based upon the sensor half angle and maximum swath width of the satellite, a coverage circle with a radius can be calculated by the following Eq.1, [12]:

$$\theta = \arccos\left(\frac{R_e + h}{R_e}\cos\epsilon\right) - \epsilon \tag{1}$$

where θ is the Earth's central angle of visibility viewed from its center, R_e is the radius of the Earth, h is the altitude of satellite, ϵ is the elevation mask angle.

For a given constellation, to get a continuous and uniform coverage over desire point, the minimum number of P and S can be deduced by the Eq.2 & Eq.3, [8]:

$$P_{min} = \left\lceil \frac{2\pi}{\theta} \right\rceil \tag{2}$$

$$S_{min} = \left\lceil \frac{\pi}{\theta} \right\rceil \tag{3}$$

In order to define the complete configuration for both polar and equatorial orbits, S and P in both orbits at given angle ϵ are given by Eq.4, [5]:

$$\Big\{(P_p - 1).\arcsin[\tan(\alpha)\cos(\pi/S_e)] + (P_p + 1)\arcsin\left[\frac{\sin(c)\cos(\pi/s_e)}{\cos(\alpha)}\right]\Big\}\eta = \pi \tag{4}$$

where;

$$c = \arccos\left[\frac{\cos(\alpha)}{\cos(pi/S_p)}\right] \tag{5}$$

P_p are the number of polar planes, S_p are the number of polar satellites in P_p, S_e are satellites in equatorial planes, α is sensor half cone angle, η is the multiplication factor set to 1. Non-linear Eq.4 can be solved using conditions from Eq.2 & Eq.3. Using Eq.1 to Eq.5 the complete configuration of polar and equatorial orbit is designed.

B. Designing of Sub-Walker - II

A Walker configuration can be sub divided into the following configuration based upon the satellite distribution of the Right Ascension of the Ascending Node (RAAN) between the planes of the constellation [1]:

- **Delta configurations:** This configuration have orbital planes distributed evenly over a span of 360° in RAAN.
- **Star configurations:** This configuration have orbital planes distributed over a span of 180° in RAAN.

In order to narrow the scope of optimization and increase efficiency of GA, the optimization is performed over a sub-Walker - II with delta configuration with inclined orbits. Eq.6 shows the modelling for sub-Walker II [5]:

$$\begin{cases} a_{k,j} = a_0 \\ e_{k,j} = e_0 \\ i_{k,j} = i_0 \\ \Omega_{k,j} = \Omega_0 + \frac{360^0}{P} * (k-1) \\ w_{k,j} = w_0 \\ M_{k,j} = M_0 + \frac{360^0}{P*S} * F * (k-1) + \frac{360^0}{S}(j-1) \end{cases} \tag{6}$$

where, k & j are the indices of k^{th} satellite in its j^{th} orbit plane, Ω is the $RAAN$, w is the Argument of Periapsis, M is the Mean Anomaly (MA). Suffix 0 represents nominal reference satellite. Initial Ω_0 and M_0 is set to 0 for the reference satellite of each sub constellation.

IV. OPTIMIZATION USING GENETIC ALGORITHM

GA is an optimization method inspired by the Principle of Natural Selection (Darwin's theory of Natural Evolution). It is population based technique. From population, a GA selects individuals with good chances of reproduction (best fitness function) and reproduces the new generation of individuals using operations such as crossover and mutation. In GA solutions are terms as chromosomes. There are two types of chromosomes, one is parent chromosome and other is offspring chromosome. From parents chromosomes new solutions are generated where as from offspring chromosomes contains newly generated solutions. Better candidate has more chance to survive in an environment of limited resources. Good solutions are retained where as bad solutions are eliminated. The process is repeated several times until it runs a certain number of generations or until a solution considered as optimum is reached [6].

A. *Fitness Function and Constraints Functions*

The first step of the optimization is the definition of the fitness function and constraints functions. The fitness function is defined to take into account both the minimization of the GDOP and the maximization of global visibility.

1) GDOP

In navigation, precise positioning accuracy of user depends upon the geometrical configuration of the satellites i,e. Dilution of Precision (DOP). Several definitions of the DOP exist [3]:

  a) Geometric (GDOP): $\left(\sqrt{\sigma_x^2+\sigma_y^2+\sigma_z^2+\sigma_t^2}\right)/\sigma$
  b) Position (PDOP): $\left(\sqrt{\sigma_x^2+\sigma_y^2+\sigma_z^2}\right)/\sigma$
  c) Horizontal (HDOP): $\left(\sqrt{\sigma_x^2+\sigma_y^2}\right)/\sigma$
  d) Vertical (VDOP): $\left(\sqrt{\sigma_z^2}\right)/\sigma$
  e) Time (TDOP): $\left(\sqrt{\sigma_t^2}\right)/\sigma$

where, σ is root mean square (rms) pseudorange error of a satellite, $\sigma_x, \sigma_y, \sigma_z$, are rms errors of the user position, σ_t is user clock bias error, which is assumed to be known. An evenness in satellite geometry is important to improve accuracy. Table II shows that typically ideal GDOP is less than 1. Excellent values for GDOP are in the range 1-2 and good ones are in the range 2-5. Mainstream GNSS has on the average excellent GDOP [4]. The objective of the optimization is to achieve performance in terms of GDOP that range between ideal and excellent values.

GDOP	Rating
< 1	Ideal
$1 < 2$	Excellent
$2 < 5$	Good
$5 < 10$	Moderate

TABLE II
GDOP RATINGS CLASSIFICATION [9]

2) Visibility of at least 4 satellites at global level

To evaluate the user position, there is a need of at least four pseudoranges from visible satellites at given epoch. In case of more than 4 visible satellites accuracy in user location estimation can be further improved. Pseudorange errors are also function of elevation angle, free space losses and varies from satellite to satellite. In this paper, the LEO constellation is optimised for different elevation angles assuming no atmospheric losses. In order to calculate 100% visibility of satellites at global level as a function of design variables, some ground points of interest must be selected. Assuming that the Earth is perfectly symmetrical along both hemispheres, 18 Points are selected from $0°$ - $90°$ latitude and $0°$ longitude as shown in fig. 2. A 10 × 10 grid is generated along each point with further assumption of zero ellipsoidal height.

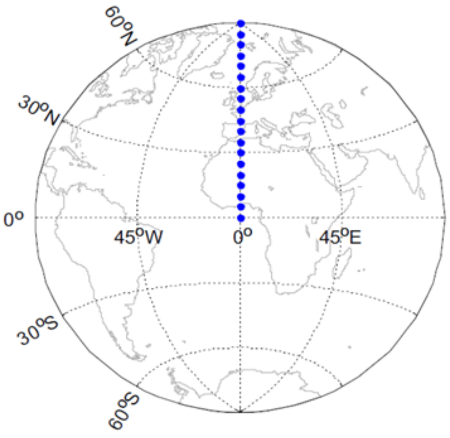

Fig. 2. Mapping of selected ground points for global coverage [5]

Therefore, we have defined the following fitness function:

$$J = min\big((1-\beta).f_1(x) + \beta.(1-f_2(x))\big), x \in C \tag{7}$$

where $f_1(x)$ is the GDOP, $f_2(x)$ is a function of the visibility and β is the parameter to define the weights for $f_1(x)$, $f_2(x)$. C represents the following constraints for optimization.

$$\begin{cases} \delta(x) = 100\% \\ F_o \leq P_o - 1 \\ P_o \in Z, \\ S_o \in Z, \\ F_o \in Z, \\ P_o.S_o = T_o \end{cases} \tag{8}$$

being $\delta(x)$, the global visibility of at least 4 satellites function of P_o, T_o, F_o, which are the important factors for evenness in satellite configuration. Z is an integer number. For a given set of satellites, different combinations of P_o, T_o, F_o and i_o are possible to design a constellation. Therefore, the optimization variables are:

$$x = [S_o, P_o, F_o, i_o] \tag{9}$$

Suffix o stands for optimization. Using the mathematical modelling from section III, the optimization process is finally performed using the GA toolbox of MATLAB.

B. Parameters selection for GA

GA tool box is setup as Table III. GA is computationally demanding hence, search is started with initial population 50 and gradually increased based upon Pareto graph. For extensive search, the population size is set to 300, 400 & 500. Similarly number of generations are chosen from 30, 40, 50 & 100. Optimization variables S_o, P_o, F_o must be integers, whereas i_o can be non-integer. To solve this mixed-integer constrained optimization problem, Mixed Integer Nonlinear Programming (MINLP) is used. The probabilities of crossover and mutation are 0.8 & 0.194 respectively [5]. To evaluate the score of each chromosome, a penalty function is used internally instead of the objective function due to the presence of the constraints. The penalty value is equal to the objective value for a feasible individual, while it is equal to the sum of the objective value of the worst feasible individual plus the violation score for an infeasible individual. Additionally, the optimization procedure is set to not terminate until the end of the generation.

Parameters	value
Population Size	300/400/500
Max generations	30/40/50/100
MINLP	S,P,F
Probability of crossover	0.8
Probability of mutation	0.194
Penalty	5

TABLE III
GA PARAMETER SETTINGS

The search ranges for optimization variables are shown in Table IV. Scenarios S1, S2, S3, S4 represent ϵ angles 7°, 10°, 15°, 20° respectively with search ranges are chosen as per the reference [5]. Scenarios S5 and S6 are for ϵ angle 20° with search range for P_o from 10 - 20, F_o from 1 - 15 and i_o from 70° - 90°. S5 explicitly represents only Walker constellation. Selection of search ranges follow the constraint defined in eq.8.

X/ϵ	S1(7°)	S2(10°)	S3(15°)	S4(20°)	S5(20°)	S6(20°)
S_o	4-12	4-12	4-15	4-12	4-15	4-15
P_o	1-10	1-10	1-10	1-10	10-20	10-20
F_o	1-9	1-9	1-9	1-9	1-15	1-15
i_o	45°-60°	45°-60°	45°-60°	45°-60°	70°-90°	70°-90°

TABLE IV
SEARCH RANGES FOR OPTIMIZATION VARIABLES FOR EACH CASE

V. SIMULATION

Initially, a two body propagator is simulated at the epoch 1 January, 2000 for 2 complete orbits, which defines the simulation period. Visibility is calculated only when the angle between the given ground point and satellite is less than the Earth's central angle. At given epoch, if the number of visible satellites at a given location is at least 4, the GDOP is estimated. If the mean of GDOP for all points is higher than 5 or the satellite does not cover any target latitude, the algorithm automatically assigns a high value to the fitness function.

The fitness function of each individual solution is evaluated for every generation, and while the evaluation does not hit the stopping criteria, the selected parents continue to meet and reproduce new generations via crossover and mutation. The stopping criteria is set as default in MATLAB (tolerance) and testing of all generations and population to minimize the objective function.

VI. RESULTS AND DISCUSSION

The GA algorithm has been applied using the ϵ angle 7°, 10°, 15°, 20° obtaining the satellites configurations S1, S2, S3, S4, S5, S6 described in Table V. Parameters P_p, S_p, S_e are obtained based upon the condition from Eq.4. While S_o, P_o, F_o, i_o are the achieved optimised parameters obtained for the sub-Walker - II constellation. Case S5 shows a complete Walker constellation without polar and equatorial orbits. The obtained results are sensitive to the different search ranges and GA parameter settings. Hence, cases S4, S5, S6 have varying configurations for the same elevation angle.

Param	S1	S2	S3	S4	S5	S6
S_p	6	6	7	7	0	7
P_p	7	8	9	9	0	9
S_e	7	8	9	10	0	10
S_o	5	7	13	10	16	16
P_o	9	10	8	13	10	10
F_o	1	3	5	5	8	8
i_o	50.37°	46.58°	46.52°	50.50°	70.23°	70.23°
T_{sat}	94	120	176	203	160	233

TABLE V
OBTAINED CONFIGURATION FOR HYBRID CONSTELLATION FOR DIFFERENT ELEVATION ANGLES

The obtained constellations from Table V are then simulated in STK. In Fig. 3 only cases S1, S2, S3, S4 are shown. S5 and S6 could be added but the results are very similar and the readability of the Fig.3 would decrease. Moreover, we have calculated the position accuracy on the following two scenarios:

1) Static user located in Rome
2) Dynamic user travelling from Rome to Milan by Uniform Rectilinear Motion (URM) with constant velocity of about 96.3 Km/h.

For both scenarios, geometric ranges are imported in MATLAB from the visible satellites at each epoch. A standard normal distribution noise of 1 m, 5 m, 10 m & 20 m is added to simulate code measurements for positioning estimation using Least Square (LS) solution. Velocity estimation is done using the rate of change of position estimation. In this way, the performance in terms of navigation accuracy for both cases is determined using the optimized LEO constellations. Different graphs are plotted for all cases.

Fig. 4 shows the visibility of average number of satellites at an interval of 60 seconds over latitude from 0° to 90° North.

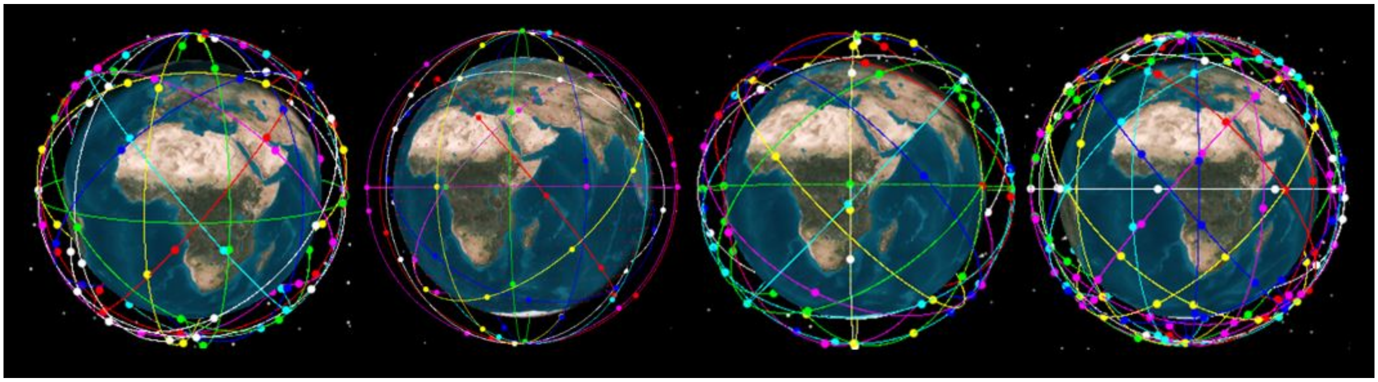

Fig. 3. Cases S1 (7°), S2 (10°), S3 (15°), S4 (20°) with respective elevation mask angles from left-right

Case S6 has maximum number of average visible satellites due to maximum number of satellites in the constellation. For S5 average number of visible satellites are lesser than S6 due to absence of polar and equatorial orbits. S1, S2, S3 shows constant results in terms of availability over entire region. On an average at least 5 satellites are visible at each latitude. The good visibility is achieved at lower latitude. Case S4 has the highest visibility among all solutions at lower latitude while at higher latitude, the visibility is reduced but it is still sufficient.

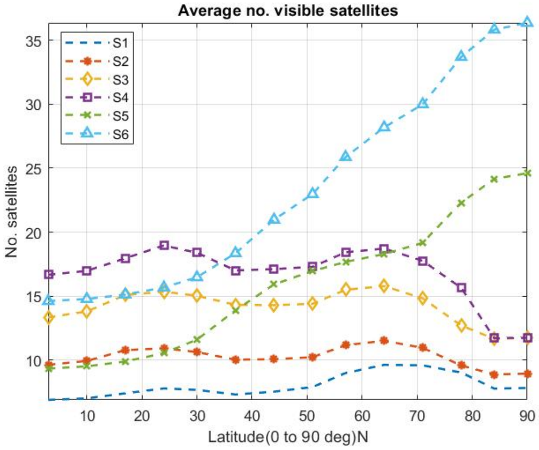

Fig. 4. Average number of visible satellites over latitude

Fig. 5 shows the GDOP over the latitude (0° - 90°) North. For all configurations GDOP is less than 2 except for S1. For Cases S3, S4, S6 average GDOP is about 1.5, which means that GDOP results are in the excellent range (Table III). For S6, GDOP further goes below 1 after 55° latitude, so ideal GDOP value can be achieved. S1 has worst GDOP among all configurations but it is still in the range of good GDOP (Table III). Similar results are achieved for HDOP, PDOP, VDOP, TDOP.

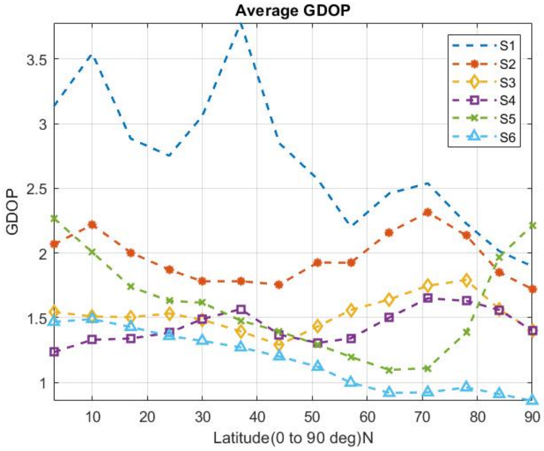

Fig. 5. GDOP over latitude

Similarly, GNAC is computed for the optimized configurations as shown in Fig. 6, which has a similar behaviour as the GDOP (Fig. 5). S6 GNAC is the lowest among all cases. It goes below 5m after 57° latitude. S3, S4, S6 performance are better i.e. less than 8m. Similarly position (PNAC), horizontal (HNAC), verticle (VNAC), time (TNAC) navigation accuracy show similar results. Based upon the assumption of symmetry of the Earth, Fig. 4, Fig. 5, Fig. 6 show symmetric behavior for latitude (0° - 90°) South.

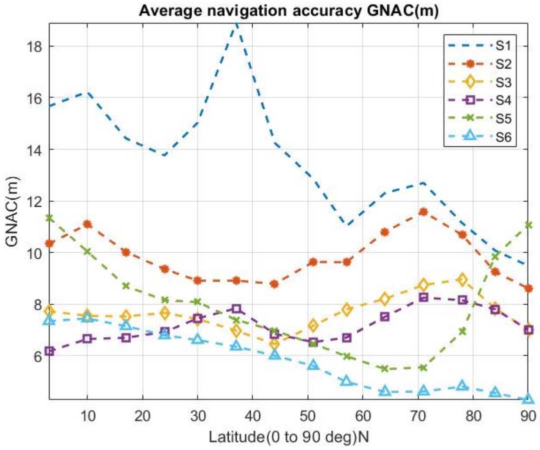

Fig. 6. GNAC over latitude

Stationary and dynamic scenarios are simulated in STK for

60 minutes. Fig. 7, Fig. 8 and Fig. 9 show results using only S5 case. Fig. 7, shows average GDOP for both scenarios. Dynamics user is moving with velocity 96.3 Km/h so, GDOP is shows more variation with respect to stationary one. Average GDOP for both scenarios is about 1.187.

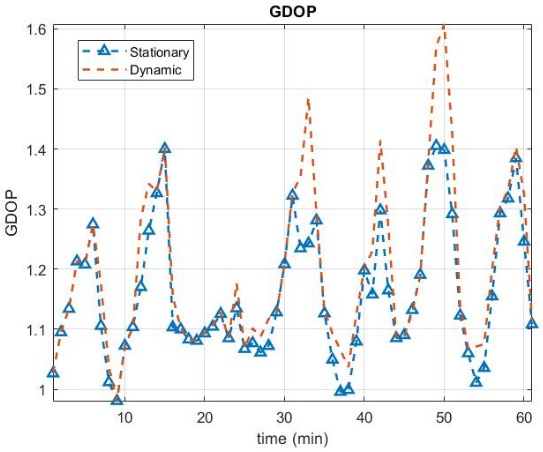

Fig. 7. GDOP for stationary and dynamics user for 60 min

Fig. 8 and Fig. 9 represents GNAC for both scenarios respectively. Each scenario shows GNAC values for uncertainties 1 m, 5 m, 10 m, 20 m in code measurements. For lower uncertainties, GNAC is on an average 5 m, and 8 m for static and dynamics user scenarios respectively.

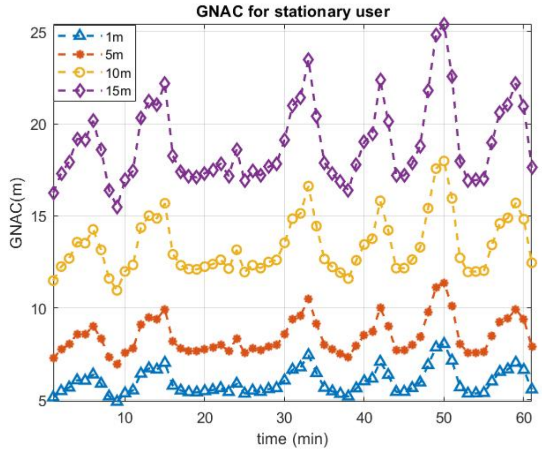

Fig. 8. GNAC for stationary user for 60 min

Further, Fig. 10 shows errors in LS positioning estimation (x,y,z) of stationary user using LEO (S5) and MEO (GPS) measurements. The graph is for worst case scenarios with the standard normal distribution noise of 10 m. The average absolute position error in x, y, z directions is 5.7857 m, 3.4933 m, 6.5981 m for S5 whereas that for GPS is 10.294 m, 3.7825 m, 4.7727 m respectively.

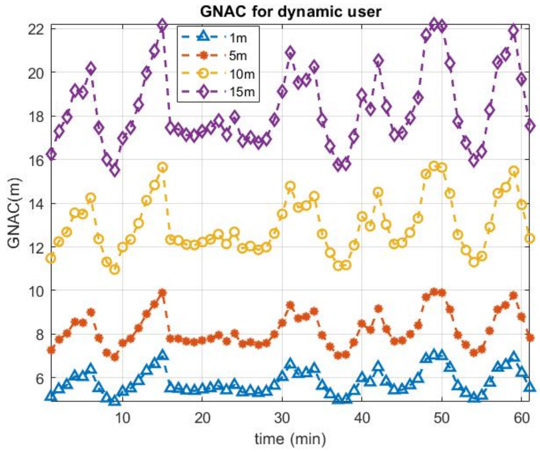

Fig. 9. GNAC for dynamics user for 60 min

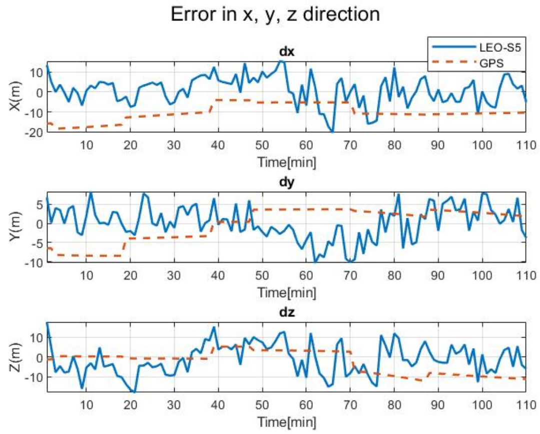

Fig. 10. LS estimation error in x,y,z directions

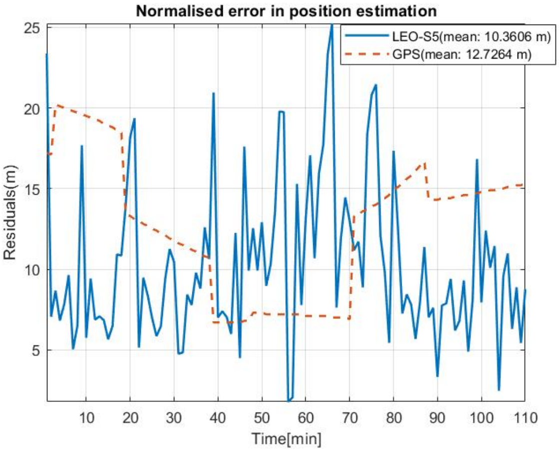

Fig. 11. Normalised error in positioning estimation

Fig. 11 shows the normalized positioning estimation error. The average error in square root of all 4 components i.e. x, y, z, t is 10.3606 m and 12.7264 m for S5 and GPS respectively. It is assumed that in the both cases ephimeris of satellites are already known. Errors in the case of S5 seem to be more fluctuating than GPS due to the rapid change in geometry of

LEO (faster velocity) and more visible satellites at given epoch than GPS. Moreover these errors in positioning estimation also depends upon the level of standard normal distribution measurement noise. Similarly, positioning estimation using LS for other cases (S1, S2, S3, S4, S6) is found to be satisfactory for different level of standard normal distribution noise.

VII. CONCLUSION AND FUTURE WORK

The paper has presented the design of a dedicated PNT LEO constellation optimized to minimize the GDOP and maximize the satellite visibility at global level. The achieved optimised constellation provides good navigation accuracy (GNAC) and global availability. The proposed solution is a combination of sub-Walker constellations which can provide 100% global coverage with at least 5 visible satellites at given epoch. Higher orbit zone (>1200 Km) is a good choice for these constellations. The GDOP values of all obtained solutions are in the excellent range ($1 < 2$). Cases S3, S4 and S6 can be good choices for improving the performance as GDOP is lower than 1.5. S6 is best among all with GDOP in ideal range but number of satellites in constellation are maximum. Positioning estimation for static and dynamic scenarios is effective using proposed constellations. For higher latitude GDOP is further improved to 1.187 for S5. Even for worst case (std noise of 10 m), final average error in positioning estimation is 10.3606 m and 12.7264 m for LEO and MEO respectively.

GA is stochastic approach but computationally demanding. Final results depend upon various factors such as population size, generation size, step size, orbital propagator, mutation and crossover probabilities, search ranges etc. In future, analysing the LEO signals considering factors such as type of signal, clock synchronization biased, ephemeris etc, which is considered to be known already may yield more meaningful results and make this problem closer to a real case. Other considerations about orbital parameters such as altitude, RAAN, mean anomaly are considered same for complete hybrid constellation which should be changed slightly to avoid collisions between satellites. Moreover, subsequent increase in mainstream GNSS satellites, optimised LEO based navigation can be complimentary solution/standalone solution in presence of threats. It would be also interesting to investigate the integration of LEO+MEO, LEO+INS signals or LEO code+Doppler measurements to improve the accuracy in Urban Canyon environment.

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